Palacký University
Olomouc

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External Referee's Report on the Ph.D. Thesis

Nonclassicality of Measurement Statistics of Quantum Electromagnetic Radiation

by **Vadym Kovtoniuk**

The thesis *Nonclassicality of Measurement Statistics of Quantum Electromagnetic Radiation*, by Vadym Kovtoniuk, is devoted to the theoretical investigation of nonclassicality criteria that allow one to certify the nonclassicality of a quantum state from tomographically incomplete measurement statistics. This is a very important and timely problem in quantum optics, with potential applications in optical quantum technologies. Despite decades of research on this subject, important new results can still be obtained, as this thesis clearly demonstrates.

The thesis consists of an Abstract, an Introduction, five main chapters, Conclusions, a Bibliography, and a list of the author's publications and conference presentations. The thesis is based on four publications in *Physical Review A* and one arXiv preprint (accepted for publication in Phys. Rev. A). Vadym Kovtoniuk is the first author of four of these five papers. The thesis is well structured and nicely integrates the results previously reported in the author's publications. This is a theoretical work firmly grounded in rigorous mathematical analysis. Concrete examples illustrating the application of the general results are provided, which I greatly appreciate. The thesis is highly relevant not only to theoreticians but also to experimentalists, who can apply the derived nonclassicality criteria to experimental data.

The first chapter of the thesis introduces the necessary background on optical nonclassicality and quantum measurements. It also provides an overview of important results from convex geometry that are utilized in the subsequent chapters. In the second chapter, the author introduces tight inequalities that reveal nonclassicality. Particular attention is paid to polynomial coherent-state response functions. In this case, general nonclassicality criteria can be formulated using known results on the finite Hausdorff moment problem. These criteria represent complete and tight nonclassicality criteria for click statistics recorded with a balanced array detector.

In Chapter 3, the author applies the general results and techniques to specific types of detectors. The author considers photon-number-resolving detectors, arrays of binary detectors where approximate photon counting is accomplished by multiplexing, and photocounting with Josephson photomultipliers. This analysis is further extended in Chapter 4 to detectors with several settings. Specifically, unbalanced homodyne detection with variable displacement is considered, as well as photocounting with an on-off detector with tunable detection efficiency.

In Chapter 5, the concept of latent optical nonclassicality is introduced and analyzed. Finally, the Conclusions chapter contains a brief summary of the main results and an outlook on possible directions for future research.

Vadym Kovtoniuk clearly demonstrates the ability to carry out cutting-edge theoretical research in quantum optics, and the obtained results may have a significant impact on the field by enabling experimentalists to perform new tests of nonclassicality. The thesis contains original scientific results, meets all the requirements for a doctoral dissertation, and I recommend awarding Vadym Kovtoniuk the degree of Doctor of Philosophy in the specialty 104 Physics and Astronomy.

Below I provide a list of questions and comments that could be addressed during the defense:

1) On page 13 it is claimed that the *Q* function cannot reveal nonclassicality on its own. However, the *Q* function encodes full information on the quantum state so nonclassicality can be analyzed based on the knowledge of *Q*. For instance, if $Q(\alpha)=0$ for some finite α, then it implies negativity of the Wigner function, since the *Q* function is a convolution of the Wigner function with a Gaussian.

2) Can you provide more details on the model of the Josephson photomultiplier that yields the quantity $W(m_j, n_j|m_{j-1})$ utilized in Eq. (3.52)?

3) You introduce the noise-penalized violation in Eq. (3.66). Would not it be better to consider there a ratio of the witness and the standard error instead of their difference?

4) The caption of Figure 3.5 refers to inequalities (3.17) and (3.19) that were derived for the case $N=3$. However, also results for $N=5$ are plotted in Figure 3.5. Please explain.

5) In the chapter 4.2.2 the author restricts the attention to the regime in which the boundary curve ∂C is convex. How would the tight inequalities (tight nonclassicality criteria) look like for non-convex curve ∂C? What would change?

6) Most of the derived nonclassicality criteria should be applicable also to multimode states provided that detectors sensitive to total photon number of the state are employed, because multimode coherent states exhibit Poisson statistics of the total photon number. What about the unbalanced homodyne detection? Does it require a strictly single-mode nature of the probed state? How would a limited interference visibility at the beam splitter BS influence the unbalanced homodyne detection scheme?

I have also several additional minor remarks:

1) Ref. [48] is cited as a method for testing nonclassicality. In this classic paper, A. Lvovsky showed how to perform Maximum Likelihood estimation of a quantum state of light from measured homodyne data, which is not a specific method for testing nonclassicality. It is true, though, that once the density matrix is reconstructed in Fock basis, its nonclassicality can be directly investigated.

2) Measurements with a single on/off detector with tunable detection efficiency were considered and investigated in the following two papers, which could have been cited:
D. Mogilevtsev, *Diagonal element inference by direct detection*, Opt. Commun. **156**, 307 (1998).
A. R. Rossi, S. Olivares, and M. G. A. Paris, *Photon statistics without counting photons*, Phys. Rev. A **70**, 055801 (2004).

3) Times t, t_1, ..., t_N appear on left-hand side of Eq. (2.27) However, only t and t_1 are present on the right-hand side of this formula.

4) The expression on the right-hand side of Eq. (4.19) should be checked. Is there really a minus sign in front of d^2 in the exponent?

Prof. Mgr. Jaromír Fiurášek, Ph.D.
Department of Optics
Faculty of Science
Palacký University Olomouc

UNIVERZITA PALACKÉHO V OLOMOUCI
PŘÍRODOVĚDECKÁ FAKULTA
KATEDRA OPTIKY
17. listopadu 1192/12, 779 00 Olomouc
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