REPORT
on the PhD thesis of Mykyta Dmytrovych Klen

“Statistical models and temporal coherence of quantum light
in the turbulent atmosphere”

Submitted for defense for the degree of Doctor of Philosophy
in the field of study 10 — Natural Sciences (104 – Physics and Astronomy)

Quantum optics is crucial for the advancement of modern science because it explains the fundamental interaction between light and matter at the level of individual photons. It serves as a foundation of many modern technologies. Quantum light enables completely secure communication through Quantum Key Distribution, which instantly detects any attempts at eavesdropping. In quantum computing, photons serve as qubits that transmit and process information at unprecedented speeds without overheating. Quantum-enhanced sensors use entangled light to enable ultra-precise biological imaging and to detect gravitational waves with microscopic accuracy. Photonic quantum networks serve as the backbone of a future quantum internet, linking distant quantum processors. Light propagation through the atmosphere normally experiences atmospheric turbulence, which is one of the main obstacles to the functioning of quantum communication channels. Research in this thesis was motivated mainly by the necessity of the correct statistical description of the quantum light transmission through turbulent media.

The first Section of the thesis is devoted to a detailed literature review, main definitions, and description of the numerical method, which is the *phase screen method*. Original results are presented in Sections 2-6. The thesis ends with Conclusions. Below, I will review in more detail the original results.

Sections 2 and 3 are strongly connected and are based on the same paper. Section 2 is devoted to the validation of different models of the probability density of transmittance (PDT). Transmittance is the ratio of the received and radiated power, and, obviously, it is not a constant in the turbulent atmosphere. The refractive index is random, and, consequently, the transmittance is a random quantity as well. It is important to understand how accurately different physically motivated models describe the quantum light propagation. The Author performed a very thorough analysis that included the direct numerical simulations as well as validation of the following models: beam-wandering, elliptical-beam, truncated lognormal, beta-distribution, and total probability. The main outcome is that the beta-distribution model appears to be the best approximation. In addition, the Author demonstrated the importance of the aperture size within the choice of the model. Section 3 discusses statistical properties of the beam parameters, such as the beam center coordinate, the beam-spot shape, and the semi-axes of the elliptical beam. It was shown that the beam center and its width appear to be almost uncorrelated in the regimes of weak and intermediate turbulence. If the turbulence is strong, they are not independent. The analysis of the distribution of the semi-axes of the elliptic beam shows deviation from the bivariate Gaussian dependence, which is especially visible in the strong turbulence regime. Hence, previous assumptions on the statistical independence of the different parameters should be revised.

Section 4 is related to the previous sections and is, in some sense, a consequence of them. Since certain physically motivated models of the beam statistics show misspecification bias, it is possible to complement these models with the matching of transmittance moments. The developed procedure was applied to the circular beam model. The obtained PDT and Kolmogorov statistics vs aperture radius for the different models, as well as direct numerical simulation, clearly show significant improvement produced by the momentum matching. However, the moment matching does not appear to be the best model in the regime of strong turbulence, as it loses to the beta distribution model.

In Section 5, the time correlations between the transmitted quantum states are studied. The Author examines the previous assumption of the almost time-independent pulses. The point is that on the *ms* time scale, the separate pulses would be correlated due to the turbulence. By computing the two-time PDT that depends on the first and second transmittance values, as well as the Pearson correlation coefficient vs wind-driven shift and the conditional PDT, the Author demostrated that optical pulses cannot be considered as independent, especially if the wind-driven shift between the pulses is small.

Practical applications are discussed in Section 6, which emanates naturally from the previous section. The main focus here is on how the various channel properties affect the degree of quantumness of the light transmission, and, consequently, the functioning of the communication protocol. Quantum entanglement was examined in the continuous variable setup (so-called two-state squeezed vacuum model) and the discrete variable setup (two-qubit system). In the continuous variable case, the Simon criterion was used, and it was shown that entanglement survives for the wind-driven shifts up to 10 m/s even in the strong turbulence regime. In the discrete variable case, the entanglement survives for the time separations between the pulses of the order of 10 *ms*, rather strong turbulence with Rytov parameter exceeding 10. However, it degrades significantly if the deterministic loss is taken into account.

The thesis is not free from drawbacks, and I list them below. However, these points of critique are not raised to serve as a basis for a rejection of this thesis, but rather as suggestions that can be used to refine or nuance certain points in it.

(i) The Author uses the Rytov parameter to characterise the strength of turbulence of the atmospheric channels. However, the thesis would benefit from an introductory discussion about the real meteorological quantities, such as temperature or pressure, that correspond to the values of the Rytov parameter being discussed. Especially because the Authors devoted a lot of effort towards discussing the physically realistic parameters.

(ii) This work relies extensively on the Taylor frozen hypothesis. While it is widely accepted for the weak turbulence, recent research on the strong turbulence in the atmospheric surface layer (which is up to 100-150 meters above the ground), this hypothesis does not always work [see Physics of Fluids **34**, 076602 (2022), DOI: https://doi.org/10.1063/5.0097729]. Therefore, the limitations of the Taylor frozen hypothesis for quantum communications within the atmospheric surface layer should be addressed more carefully.

The next comments are more about how certain results are presented.

(iii) The purpose of Figure 1.1 is not clear. Is it an illustrative example of a random function? Even if it is so, it was not explained how it was calculated.

(iv) This comment concerns Figures 3.1-3.3.

(a) Figures 3.1 and 3.2 need labels like (a) and (b) or legends so that the reader can easily distinguish between the collimated and focused beam cases.

(b) In Figures 3.1-3.2, the dependencies for the collimated and focused beams are almost identical, and the deviations from the Gaussian law are not clearly seen. It would be more beneficial to superimpose two dependencies and to provide a zoomed area to highlight the most interesting areas. The latter is also true for Figure 3.3.

(c) The quality of the data presentation is low. For example, Figure 3.3 seems to correspond to Figure 9 of the paper [I]. While in the paper the figure looks clearly as a histogram, ine the one in the thesis, the separate columns cannot be distinguished, and the histogram-like nature of the figure is lost.

The last two comments are rather technical.

(v) On page 53, line 9 from the bottom, there is a typo in the reference. I do not understand what "[^@bachmann2025]" stands for.

(vi) The term "*spatial frequency*" is used several times throughout the thesis (pages 38,51) instead of the more conventional term "*wave number*".

As I have pointed out above, these points of criticism should not prevent the doctoral degree from being awarded. In summary, it was an exciting experience to read this thesis and the results expressed in it. They are new, original, are based on 3 publications in Physical Review A and one proceeding paper of the prestigious conference, and, most importantly, they significantly move forward the field of quantum optical signal transmission in disordered media.

No violations of academic integrity were found in the thesis and scientific publications of the Author. In terms of relevance, practical significance, scope of research, and novelty of results, the thesis "Statistical models and temporal coherence of quantum light in the turbulent atmosphere" complies with all requirements of the Order of the Ministry of Education and Science of Ukraine No. 40 (12.01.2017), "On approval of requirements for the formatting of dissertations", and the "Procedure for awarding the degree of Doctor of Philosophy and revoking a decision of a one-time specialized scientific council of a higher education institution or a scientific institution on awarding the degree of Doctor of Philosophy", approved by the Resolution of the Cabinet of Ministers of Ukraine No. 44 (12.01.2022). The Author, Mykyta Dmytrovych Klen, deserves to be awarded the degree of Doctor of Philosophy in the field of study 10 "Natural Sciences" (104 - "Physics and Astronomy").

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