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ДИСЕРТАЦІЯ

**Моделювання еволюції сильновзаємодійної матерії у
високоенергетичних зіткненнях важких ядер**

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_____ В. М. Шаповал

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АНОТАЦІЯ

Шаповал В. М. Моделювання еволюції сильновзаємодійної матерії у високоенергетичних зіткненнях важких ядер.

Дисертація на здобуття наукового ступеня доктора фізико-математичних наук за спеціальністю 01.04.02 – теоретична фізика (104 – фізика і астрономія, галузь знань 10 – природничі науки). — Інститут теоретичної фізики ім. М. М. Боголюбова НАН України, Київ, 2025.

Дисертацію присвячено теоретичному дослідженню просторово-часової структури та характеру еволюції систем, що народжуються в релятивістських зіткненнях важких іонів (ядер), з'ясуванню властивостей нових видів сильновзаємодійної матерії (таких, як кварк-глюонна плазма), що утворюються на проміжних стадіях зіткнення, та аналізу механізмів формування кінцевих адронних спостережуваних в області м'якої фізики. Дослідження в основному проводилися в інтегрованій гідрокінетичній моделі ядро-ядерних зіткнень (іНКМ), що дозволяє реалістично моделювати процес еволюції утвореної системи в рамках п'яти послідовних стадій — формування початкового стану, передрівноважна динаміка, гідродинамічне розширення, розпад згустка неперервної матерії на систему частинок (партиклізація) та адронний каскад. Увага в роботі приділяється таким актуальним проблемам, як початкові умови еволюції системи, рівняння стану кварк-глюонної матерії на гідродинамічному етапі, аналіз просторово-часової структури утворюваних систем, опис даних експериментів на сучасних прискорювачах частинок щодо м'яких адронних спостережуваних, роль адронної стадії зіткнення у формуванні цих спостережуваних, проблема миттєвого або поступового замороження імпульсних спектрів і виходів частинок, оцінка тривалості різних стадій зіткнення, аналіз взаємодій між баріонами у кінцевому стані тощо.

Перший розділ присвячено систематичному опису в інтегрованій гідрокінетичній моделі експериментальних даних з області м'якої фізики для зіткнень ядер свинцю на Великому адронному колайдері (ЛНС) при енергії 5.02 TeV на нуклонну пару і порівнянню результатів та параметрів моделювання із тими, що мають

місце для зіткнень при енергії 2.76 TeV, а також для зіткнень ядер ксенону при енергії 5.44 TeV та ядер золота на колайдері RHIC при енергії 200 GeV на нуклонну пару. Розрахунки проводилися при двох варіантах рівняння стану кварк-глюонної матерії з відповідними температурами партиклізації. Аналізувався вплив адронної стадії еволюції системи на кінцеві спостережувані, а також залежність результатів обчислень від окремих параметрів моделі.

У підрозділі 1.1 аналізується залежність відношень виходів частинок різних сортів від рівняння стану кварк-глюонної матерії у моделі та від наявності непружних адрон-адронних взаємодій на фінальній стадії зіткнення при енергії LHC 2.76 TeV на нуклонну пару. У підрозділі 1.2 розглянуто калібрування моделі для опису зіткнень при енергії LHC 5.02 TeV на нуклонну пару з використанням двох різних рівнянь стану для кварк-глюонної фази, а також порівняння розрахованих у моделі чисел частинок, їх відношень, спектрів поперечного імпульсу, v_n -коефіцієнтів тощо з експериментальними даними. Зроблено передбачення для фемтоскопічних радіусів піонів і каонів. Підтверджено відсутність m_T -скейлінгу між піонними і каонними радіусами, при наявності k_T -скейлінгу за не надто малих поперечних імпульсів пари, що узгоджується з попередніми обчисленнями у гідрокінетичній моделі. Результати моделювання при двох розглянутих рівняннях стану виявляються близькими за умови відповідного перенормування параметру ϵ_0 , що визначає початкову густину енергії в центрі системи. Також виявлено, що адронна стадія зіткнення у моделі компенсує можливу різницю між результуючими спостережуваними (зокрема, відношеннями чисел частинок), яка може мати місце в кінці гідродинамічної стадії при використанні різних рівнянь стану кварк-глюонної фази. Підрозділ 1.3 присвячено аналізу залежності фемтоскопічних масштабів від поперечної маси в області найбільш низьких її значень ($m_T < 200$ MeV/c). Виявлено відхилення поведінки радіусів від степеневого закону в розглянутому інтервалі. Натомість, вдається єдиною формулою описати залежність радіусів від поперечної швидкості пари, що дає можливість екстраполяції відомої поведінки радіусів при більш високих імпульсах пари на область низьких його значень. У підрозділах 1.4–1.6 аналізується залежність результатів

обчислень у інтегрованій гідрокінетичній моделі від початкового поперечного профілю густини енергії та від часових параметрів моделі. Результати моделювання слабо залежать від часів термалізації та релаксації, які описують темпи переходу від початкового нерівноважного стану системи до стану, сумісного з гідродинамічним описом. Натомість, на результуючі імпульсні спектри та інші спостережувані впливає значення початкового часу τ_0 , який відтак слід вважати головним часовим параметром моделі. Підтверджено, що величина результуючих фемтоскопічних радіусів залежить не лише від множинності утворених частинок, а й від початкової геометрії ядер, що зіштовхуються. Значення параметра α , що регулює співвідношення між внесками моделей бінарних зіткнень та поранених нуклонів до початкового профілю густини енергії, зростає при значному збільшенні енергії зіткнення (завдяки зростанню перерізу непружного нуклон-нуклонного розсіяння), а також при зменшенні розмірів ядер, які беруть участь у зіткненні.

У *другому розділі* детально аналізується проблема вибору між двома картинами еволюції системи в ході зіткнення та відповідними сценаріями формування адронних спостережуваних — “миттєвий фріз-аут”, тобто замороження хімічного складу та імпульсних спектрів майже одразу після партиклізації, або “поступовий фріз-аут”, що передбачає довгу постгідродинамічну стадію, впродовж якої утворена система адронів еволюціонує як єдине ціле, і між частинками відбуваються інтенсивні пружні і непружні взаємодії. Робляться оцінки ефективних часів емісії піонів і каонів, а також оцінка тривалості гідродинамічної стадії і загального часу життя системи для зіткнень при різних релятивістських енергіях.

Підрозділ 2.1 присвячено дослідженню впливу взаємодії резонансів $K^*(892)$ і $\phi(1020)$ та продуктів їх розпаду з адронним середовищем на постгідродинамічній стадії зіткнення на кінцеві спостережувані. Було виявлено сильний вплив такої взаємодії на виходи короткоживучих K^* у центральних зіткненнях і відсутність аналогічного впливу на виходи довгоживучих ϕ . У підрозділах 2.2–2.4 в рамках моделі іНКМ знайдено оцінки на ефективні часи емісії π - та K -мезонів у зіткненнях ядер свинцю на LHC при енергіях 2.76 TeV і 5.02 TeV та у зіткненнях ядер золота на RHIC при енергії 200 GeV на нуклонну пару. Виявлено, що при всіх

енергіях каони випромінюються пізніше, ніж піони. Це можна пояснити сильним впливом взаємодій між адронами на фінальній стадії зіткнення, зокрема перерозсіань і рекомбінації продуктів розпаду резонансів K^* , на результуючі каонні спостережувані. В підрозділі 2.5 запропоновано метод оцінки загального часу життя системи, сформованої у релятивістському ядро-ядерному зіткненні, на основі фітування спектрів і фемтоскопічних масштабів в області малих поперечних імпульсів. Отримані значення часу життя для зіткнень на LHC свідчать про значний вплив довгої адронної стадії на результуючі спостережувані. Також в рамках моделі iHKM було оцінено тривалість гідродинамічної стадії для зіткнень за різних енергій LHC та RHIC. Отримані оцінки для різних енергій зіткнення є близькими завдяки інтенсивному, суттєво тривимірному гідродинамічному розширенню системи та схожим співвідношенням між розмірами системи та величиною швидкості звуку в усіх розглянутих випадках.

Третій розділ присвячено кореляційному аналізу сильної взаємодії між баріонами у кінцевому стані.

У підрозділі 3.1 на основі обчислень у гідрокінетичній моделі та підходів Куніна-Пратта і Ледніцкі-Любошица запропоновано просту аналітичну модель для опису кореляційних функцій $\bar{p}\Lambda$ та $p\bar{\Lambda}$ з урахуванням залишкових кореляцій. В рамках даної моделі вдалося вирішити проблему видимої двократної відмінності між ефективними радіусами джерела емісії пар $p\Lambda$ і $p\bar{\Lambda}$ у експериментальному аналізі колаборації STAR для зіткнень за найвищої енергії колайдера RHIC. Також було визначено довжину розсіяння сильної взаємодії в парах $\bar{p}\Lambda$ та $p\bar{\Lambda}$. У підрозділі 3.2 на основі результатів кореляційного аналізу для RHIC в рамках розвинутого підходу було зроблено передбачення кореляційних функцій для випадку енергій LHC. У підрозділі 3.3 аналогічні передбачення зроблено для пар $p\Xi$, в яких, окрім сильної, наявна також електромагнітна взаємодія між баріонами.

Ключові слова: релятивістські зіткнення важких іонів, кварк-глюонна плазма, сильновзаємодійна матерія, м'яка фізика, адронні спостережувані, кореляційна фемтоскопія, взаємодії у кінцевому стані, гібридні моделі ядро-ядерних зіткнень, релятивістська гідродинаміка, адронний каскад.

ABSTRACT

Shapoval V. M. Simulation of strongly interacting matter evolution in high-energy collisions of heavy nuclei.

Thesis for the Doctor of Sciences degree in Physics and Mathematics in specialty 01.04.02 – theoretical physics (104 – physics and astronomy, academic branch 10 – natural sciences). — Bogolyubov Institute for Theoretical Physics of the National Academy of Sciences of Ukraine, Kyiv, 2025.

The thesis is devoted to the theoretical study of the space-time structure and the character of the evolution of systems formed in relativistic heavy-ion (nuclear) collisions, to the investigation of the properties of new strongly interacting matter forms (such as quark-gluon plasma) created during the intermediate stages of the collision, and to the analysis of the mechanisms of final hadronic observables formation in the soft physics domain. The research was primarily conducted within the integrated HydroKinetic Model (iHKM) for nucleus-nucleus collisions, which allows for realistic modeling of the evolution of the created system through five successive stages: initial state formation, pre-equilibrium dynamics, hydrodynamic expansion, particlization (conversion of the continuous matter into a system of particles), and the hadronic cascade. The study addresses several topical problems, including the initial conditions for the system's evolution, the equation of state of quark-gluon matter at the hydrodynamic stage, the analysis of the space-time structure of the created systems, the description of experimental data from modern particle accelerators concerning soft hadronic observables, the role of the collision hadronic stage in the formation of these observables, the problem of sudden versus continuous freeze-out of particle yields and momentum spectra, the estimation of different collision stages' duration, the analysis of baryon-baryon final-state interactions, and more.

Chapter 1 is dedicated to a systematic description within the integrated hydrokinetic model of soft physics experimental data from lead-lead collisions at the Large Hadron Collider (LHC) at a nucleon pair energy of 5.02 TeV, and to a comparison of simulation results and respective model parameters with those for lead-lead colli-

sions at 2.76 TeV, xenon-xenon collisions at 5.44 TeV, and gold-gold collisions at the RHIC collider nucleon pair energy of 200 GeV. Calculations were carried out using two variants of the quark-gluon matter equation of state with corresponding thermalization temperatures. The influence of the hadronic phase of system's evolution on final observables was studied, as well as the dependence of simulation results on specific model parameters.

Section 1.1 analyzes how the ratios of yields of various particle species depend on the quark-gluon matter equation of state in the model and on the presence of inelastic hadron-hadron interactions during the final stage of the collision at the LHC energy of 2.76 TeV per nucleon pair. Section 1.2 discusses the calibration of the model for describing collisions at the LHC energy of 5.02 TeV per nucleon pair using two different equations of state of the quark-gluon phase, as well as comparison of calculated particle yields, their ratios, transverse momentum spectra, and v_n coefficients with experimental data. Predictions for femtoscopic radii of pions and kaons are presented. The violation of m_T -scaling between pion and kaon radii and the presence of k_T -scaling for not too small pair transverse momenta is observed, in accordance with the previous calculations in the hydrokinetic model. Modeling results for the two considered equations of state turn out to be close if the parameter ϵ_0 , determining the initial energy density at the center of the system, is appropriately retuned. It is also shown that the hadronic stage in the model compensates for possible differences in the final observables (in particular, particle number ratios) that may arise at the end of the hydrodynamic stage due to utilization of different quark-gluon matter equations of state. Section 1.3 is devoted to analyzing the dependence of femtoscopic scales on the transverse mass in the low- m_T region ($m_T < 200$ MeV/ c). Deviations from a power-law behavior of radii in this region are observed. Instead, a unified formula describing the dependence of the radii on the pair transverse velocity is found, enabling extrapolation of known radius behavior at higher pair momenta to the lowest-momentum region. Sections 1.4–1.6 analyze the dependence of simulation results in the integrated hydrokinetic model on the initial transverse energy density profile and on the time-related parameters of the model. Model results show weak dependence on the thermalization and the relaxation

times, which characterize the rate of transition from the initial non-equilibrium state to one compatible with hydrodynamic evolution. In contrast, the resulting momentum spectra and other observables are noticeably influenced by the initial time τ_0 value, which should thus be considered the primary time parameter of the model. It is confirmed that the magnitude of resulting femtoscopic radii depends not only on the multiplicity of produced particles but also on the initial geometry of the colliding nuclei. The parameter α , which governs the relative contribution of binary collision and wounded nucleon models to the initial energy density profile, increases at significant collision energy increase (due to the related increase in the nucleon-nucleon inelastic cross-section), and also with decreasing size of the colliding nuclei.

Chapter 2 presents a detailed analysis of the problem of choosing between two scenarios of system evolution in course of a collision and the corresponding models for hadronic observables formation — “sudden freeze-out”, meaning the chemical composition and momentum spectra freeze almost immediately after particlization, and “continuous freeze-out”, which assumes a long post-hydrodynamic stage during which the created hadronic system still evolves collectively and intense elastic and inelastic hadron-hadron interactions take place. Estimates are made for effective emission times of pions and kaons, as well as for the duration of the hydrodynamic phase and total system lifetime at various relativistic collision energies.

Section 2.1 focuses on the influence of interactions of $K^*(892)$ and $\phi(1020)$ resonances and their decay products with the hadronic medium at the post-hydrodynamic stage of the collision on final observables. It was found that such interactions strongly affect the yields of short-lived K^* resonances in central collisions, while the yields of long-lived ϕ mesons are not similarly affected. Sections 2.2–2.4 provide estimates, within the iHKM model, for the effective emission times of π - and K -mesons in Pb+Pb collisions at the LHC energies 2.76 and 5.02 TeV and in Au+Au collisions at the RHIC energy 200 GeV per nucleon pair. It was found that at all considered energies, kaons are emitted later than pions, which is unexpected from a hydrodynamic point of view. This result can be explained by the strong influence of interactions between hadrons at the final collision stage (in particular, rescattering and recombination of

K^* decay products) on resulting kaon observables. Section 2.5 proposes a method for estimating the total lifetime of the system formed in a relativistic nucleus-nucleus collision based on fits to spectra and femtoscopic radii at low transverse momenta. The obtained lifetime values for LHC collisions demonstrate a significant influence of the long hadronic phase on the final observables. The duration of the hydrodynamic stage for different LHC and RHIC collision energies is also estimated within the iHKM framework. These estimates turn out to be close for different energies due to the intense, essentially three-dimensional hydrodynamic expansion and similar ratios of system size to speed of sound in all considered cases.

Chapter 3 is devoted to the correlation analysis of strong final-state interactions between baryons.

In Section 3.1, based on calculations in the hydrokinetic model and the Koonin-Pratt and Lednický-Lyuboshitz approaches, one proposes a simple analytical model for describing the correlation functions of $\bar{p}\Lambda$ and $p\bar{\Lambda}$ pairs with residual correlations taken into account. This model resolves the problem of the apparent twofold difference in the effective source radii of $p\Lambda$ and $p\bar{\Lambda}$ pairs observed in the STAR Collaboration's experimental analysis of the top RHIC energy collisions. The strong interaction scattering length for $\bar{p}\Lambda$ and $p\bar{\Lambda}$ pairs was also determined. Section 3.2 uses correlation analysis results for RHIC to predict the corresponding correlation functions for LHC energies. Section 3.3 provides analogous predictions for $p\Xi$ pairs, where, in addition to strong interaction, electromagnetic interaction between baryons is present.

Key words: relativistic heavy ion collisions, quark-gluon plasma, strongly interacting matter, soft physics, hadronic observables, correlation femtoscopy, final-state interactions, hybrid models of nucleus-nucleus collisions, relativistic hydrodynamics, hadronic cascade.

СПИСКИ НАУКОВИХ ПРАЦЬ ЗДОБУВАЧА ЗА ТЕМОЮ ДИСЕРТАЦІЇ

СПИСОК НАУКОВИХ ПРАЦЬ, В ЯКИХ ОПУБЛІКОВАНО ОСНОВНІ НАУКОВІ РЕЗУЛЬТАТИ ДИСЕРТАЦІЇ

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ПЕРЕЛІК УМОВНИХ ПОЗНАЧЕНЬ

- AGS — англ. *Alternating Gradient Synchrotron* — Синхротрон зі змінним градієнтом [магнітного поля для фокусування пучків частинок] (Брукхевен, США)
- ALICE — англ. *A Large Ion Collider Experiment* — Експеримент на великому іонному колайдері — один з експериментальних проєктів на Великому адронному колайдері, присвячений дослідженню властивостей кварк-глюонної плазми
- CF — англ. *correlation function* — кореляційна функція
- FSI — англ. *final-state interaction* — взаємодія у кінцевому стані
- GLISSANDO — англ. *GLauber Initial-State Simulation AND mOre* — Симуляція початкового стану за Глаубером та інше
- HBT — англ. *Hanbury Brown and Twiss* — [метод інтерферометрії інтенсивностей] Хенбері Брауна і Твісса.
- iHKM — англ. *integrated hydrokinetic model* — інтегрована гідрокінетична модель [релятивістських ядро-ядерних зіткнень]
- JAM — англ. *Jet AA Microscopic transport model* — Мікроскопічна транспортна модель ядро-ядерних зіткнень з врахуванням струменів
- LHC — англ. *Large Hadron Collider* — Великий адронний колайдер (ЦЕРН, Швейцарія)
- RHIC — англ. *Relativistic Heavy Ion Collider* — Релятивістський колайдер важких іонів (Брукхевен, США)
- SMASH — англ. *Simulating Many Accelerated Strongly interacting Hadrons* — Симуляція багатьох прискорених сильно взаємодіючих адронів
- SPS — англ. *Super Proton Synchrotron* — Протонний суперсинхротрон (ЦЕРН, Женева)
- UrQMD — англ. *Ultra relativistic Quantum Molecular Dynamics* — Ультра-

релятивістська квантова молекулярна динаміка

- КГП — кварк-глюонна плазма
- КХД — квантова хромодинаміка
- ЦЕРН — фр. *CERN* (*Conseil Européen pour la Recherche Nucléaire*) — Європейська організація з ядерних досліджень.

ВСТУП

Обґрунтування та актуальність теми досліджень. Питання про будову матерії на фундаментальному рівні хвилювало людство з найдавніших часів (можна згадати роздуми ще давньогрецьких філософів Геракліта, Демокрита, Платона, Аристотеля на цю тему). В процесі розвитку науки вчені поступово прийшли до сучасних уявлень про те, що звичайні тіла і речовини, які нас оточують, складаються з атомів. Ядра атомів складаються з важких протонів і нейтронів, навколо яких рухаються легкі електрони. Теоретичні міркування та експерименти на прискорювачах частинок дозволили встановити, що протони та нейтрони, в свою чергу, теж не є точковими об'єктами — кожен з них має внутрішню структуру, яка включає три більш фундаментальних складових, що отримали назву *кварків*. Наразі відомо шість типів кварків (u, d, s, c, b, t), але до складу протонів та нейтронів входять лише найлегші з них (u і d), в той час як решта присутні у більш екзотичних частинках, що утворюються штучно на прискорювачах або потрапляють до нас разом з космічними променями. Кварки утримуються у складі протонів, нейтронів та інших подібних частинок (*адронів*, різновидів яких наразі відомо близько 400) за рахунок *сильної взаємодії*, яка реалізується шляхом обміну *глюонами* (від англійського *glue* — клей — глюони ніби склеюють кварки у адроні)¹. Окрім електричного, кварки мають також “кольоровий” заряд, асоційований із сильною взаємодією.

За звичайних умов вільні кварки не спостерігаються — вони завжди входять до складу певного адрона у безбарвній (нейтральній з точки зору кольорового заряду) комбінації з трьох кварків (утворюючи баріон) або з кварка та антикварка (утворюючи мезон). Це явище отримало назву “конфайнмент” (від англійського *color confinement* — утримання кольору). Сильна взаємодія між кварками влашто-

¹Самі ж *нуклони*, тобто протони та нейтрони, що входять до складу ядра, утримуються у ньому за рахунок так званої “залишкової” сильної взаємодії (на зразок ван-дер-ваальсової електромагнітної взаємодії між поляризованими молекулами), яка реалізується шляхом обміну мезонами.

вана таким чином, що спроби “відірвати” окремий кварк від адрона призводять лише до утворення нових адронів (явище адронних *струменів* у експериментах на прискорювачах), але не до відокремлення кварків. Разом з тим, зі зменшенням відстані r між кварками взаємодія між ними послаблюється і у граничному випадку $r \rightarrow 0$ має повністю зникати (явище *асимптотичної свободи* [1, 2, 3]).

Кваркова модель адронів була запропонована Гелл-Манном [4] та Цвейгом [5] у 1964 році, а перші експериментальні підтвердження було отримано в 1968 році у Стенфордському центрі лінійних прискорювачів [6]. В ході розвитку теорії сильної взаємодії між кварками і глюонами (квантової хромодинаміки, КХД) та подальших експериментів з адронами висловлювались гіпотези про можливість існування кваркової матерії — досі невідомого стану речовини, в якому кварки та глюони були б вільними і не зв’язаними в адрони [7]. Раніше вже припускалось утворення холодної кваркової матерії завдяки гігантському тиску всередині нейтронних зірок [8, 9], тепер же почали говорити про можливість отримання гарячої кваркової матерії в лабораторних умовах — у зіткненнях ядер при високих енергіях. Ці гіпотези певним чином узгоджувались і з теоретичним обмеженням на максимальну температуру адронної матерії у $T_H \approx 150 - 160$ MeV (або близько $2 \cdot 10^{12}$ K), отриманим у 1965 році Хагедорном [10]. Вище цієї “температури кипіння” адронна матерія мала б “випаруватись”, або перейти в інший, “кварковий” стан [11].

З кінця 70-х – початку 80-х років минулого століття теорія деконфайнменту і утворення нового стану сильновзаємодійної матерії, *кварк-глюонної плазми* (КГП), активно обговорюється у науковій спільноті. Згодом висувуються пропозиції і щодо експериментального підтвердження. Починаються експериментальні програми на прискорювачах SPS (*Super Proton Synchrotron* — Протонний суперсинхротрон) у ЦЕРНі (Європейська організація з ядерних досліджень, Женева, Швейцарія) та AGS (*Alternating Gradient Synchrotron* — Синхротрон зі змінним градієнтом [магнітного поля для фокусування пучків частинок]) у Брукхевенській національній лабораторії (Брукхевен, США). Головною метою цих програм було отримання нових форм сильновзаємодійної матерії та всебічне дослідження їх вла-

стивостей. У 2000 році в ЦЕРНі вперше було повідомлено про успішне отримання кварк-глюонної плазми в експерименті на SPS [12, 13]. На сьогодні накопичено величезний масив експериментальних даних, які практично не залишають сумнівів у тому, що новий стан матерії дійсно утворюється в зіткненнях ядер за високих енергій — про це свідчать різноманітні сигнали формування КГП, наявні у кінцевих спостережуваних (наприклад, прояви ефектів *радіального* та *еліптичного потоку* у спектрах поперечного імпульсу народжених частинок, характер імпульсної залежності *фемтоскопічних радіусів*, підвищення виходів дивних частинок (особливо з двома та трьома *s*-кварками у складі), “гасіння” адронних струменів, “плавлення” чармонію J/ψ ($c\bar{c}$) та боттомонію Υ ($b\bar{b}$), нелінійна залежність адронних спостережуваних від числа нуклонів — учасників зіткнення, натовість, масштабування цих спостережуваних за кількістю валентних кварків, термальні фотонні та дилептонні спектри тощо [13, 14]).

Властивості КГП активно досліджувалися і продовжують досліджуватися з не меншим інтересом в експериментах з релятивістських зіткнень важких іонів (ядер) на сучасних потужних прискорювачах RHIC (*Relativistic Heavy Ion Collider* — Релятивістський колайдер важких іонів) у Брукхевені та LHC (*Large Hadron Collider* — Великий адронний колайдер) у ЦЕРНі. Кварк-глюонна плазма характеризується надзвичайно високими температурою та густиною енергії і при цьому поводить себе як майже ідеальна рідина з надзвичайно малою в’язкістю, близькою до теоретичного мінімуму відношення зсувної в’язкості до густини ентропії, $\eta/s = 1/4\pi$. Вважається, що в стані кварк-глюонної плазми перебував увесь Всесвіт на одній з ранніх стадій своєї еволюції (приблизно в інтервалі $10^{-10} - 10^{-5}$ секунд після Великого Вибуху), тож експерименти з ядро-ядерними зіткненнями надають унікальну можливість відтворити і дослідити в лабораторії умови, в яких існувала матерія в перші мікросекунди після народження нашого світу. Оскільки всередині КГП завдяки глюон-глюонній взаємодії активно утворюються дивні *s*-кварки, що можуть поєднуватися також із більш важкими *c* і *b* кварками, відсутніми у звичайній речовині, але народжуваними завдяки величезній кінетичній енергії у релятивістських зіткненнях ядер, утворювані згустки сильновзаємодій-

ної матерії є щедрим джерелом екзотичних частинок, які містять у собі вказані типи кварків. Це відкриває нові широкі можливості для досліджень властивостей даних частинок та взаємодій між ними. Таким чином, дослідження КГП та процесів її утворення, еволюції і розпаду становлять один з найважливіших напрямків сучасної фундаментальної фізики, який бурхливо розвивається

Сама назва “кварк-глюонна плазма” обрана за аналогією зі звичайною плазмою, в якій електронейтральні атоми розпадаються на позитивно й негативно заряджені іони. У кварк-глюонній плазмі подібним чином “стають вільними” кольорові заряди кварків та глюонів, які за звичайних умов утримуються у складі безбарвних адронів.

Для отримання КГП на сучасних колайдерах зіштовхують важкі ядра (на RHIC використовують ядра золота, а на LHC — ядра свинцю), попередньо розігнані до ультрарелятивістських швидкостей. Енергія в розрахунку на пару нуклонів в системі центра мас сягає 200 GeV на RHIC та кількох 1000 GeV на LHC. Через сильне лоренц-стиснення (у сотні і тисячі разів) такі ядра мають вигляд надтонких “млинців” товщиною порядку 10^{-2} фм у випадку RHIC та 10^{-3} фм у випадку LHC (1 фм (*фемтометр*) = 10^{-15} м). Оскільки власний розмір нуклона становить порядку 1 фм, у таких вкрай стиснутих ядрах всі нуклони можна вважати “розчавленими” в єдину масу кварків та глюонів (*партонів*). Ядра проходять одне крізь одне і продовжують летіти кожне у своєму напрямку, відносячи з собою майже весь початковий баріонний заряд, а у просторі між ядрами, що розлітаються в протилежні боки, утворюється гаряча та густа система партонів з практично нульовим баріонним хімічним потенціалом. Ця система швидко розширюється, охолоджується і зрештою перетворюється на систему з кількох тисяч адронів, які ще деякий час рухаються як єдине ціле, інтенсивно взаємодіючи між собою; нестійкі резонансні стани поступово розпадаються. Нарешті, коли усі взаємодії припиняються, вільні частинки рухаються до детекторів. Весь складний процес еволюції утвореної системи, від першого контакту ядер до розльоту продуктів реакції, займає всього 10^{-22} секунди, або 10 фм/с (тут c — швидкість світла).

Кварк-глюонна плазма утворюється лише на проміжній стадії процесу зіткнення ядер і існує дуже короткий час, тож її властивості досліджують аналізуючи кінцеві спостережувані, які стосуються утворених в результаті зіткнення адронів, фотонів та лептонів. При цьому для надійної та коректної інтерпретації отриманих експериментальних даних, для ретельного аналізу динаміки процесу та механізмів формування спостережуваних незамінними стають теоретичні моделі ядро-ядерних зіткнень за високих енергій. Через складність формалізму квантової хромодинаміки такі моделі мають в значній мірі феноменологічний характер — з перших принципів наразі вдається обчислити лише окремі величини та характеристики, які описують в основному статистичні, рівноважні властивості сильновзаємодійної матерії (так, в рамках Монте-Карло симуляцій КХД на ґратках розраховують маси кварків та адронів, рівняння стану КГП тощо), але не динаміку такого складного процесу як релятивістське зіткнення важких ядер і її зв'язок з кінцевими спостережуваними, що стосуються масиву народжених частинок.

Перші успішні моделі багаточастинкового народження були запропоновані на початку 50-х років минулого століття Фермі [15], Померанчуком [16] та Ландау [17]. Дані моделі були *статистичними* — адрони народжувались у них відповідно до рівноважних розподілів. У моделі Фермі використовувалися розподіли, що відповідають високим температурам та густинам енергії, характерним для системи у перші моменти зіткнення. Померанчук зауважив, що має пройти деякий час, поки адрони перестануть взаємодіяти одне з одним і почнуть вивільнятися з системи — цього не можна очікувати одразу після зіткнення, температура має опуститися до значень порядку маси пі-мезона (140 — 150 MeV). Ландау ж висунув дуже важливу піонерську пропозицію про те, що перехід від початкового, гарячого та густого, до кінцевого, розрідженого та охолодженого, стану має відбуватися за гідродинамічним сценарієм. Даний підхід став основою для подальшого розвитку моделей ядро-ядерних зіткнень і залишається актуальним дотепер [18].

Важливість гіпотези Ландау полягає в тому, що, як переконливо свідчить вся сукупність результатів численних експериментів на прискорювачах, утворена у зіткненні система взаємодіючих партонів з часом термалізується та демонструє

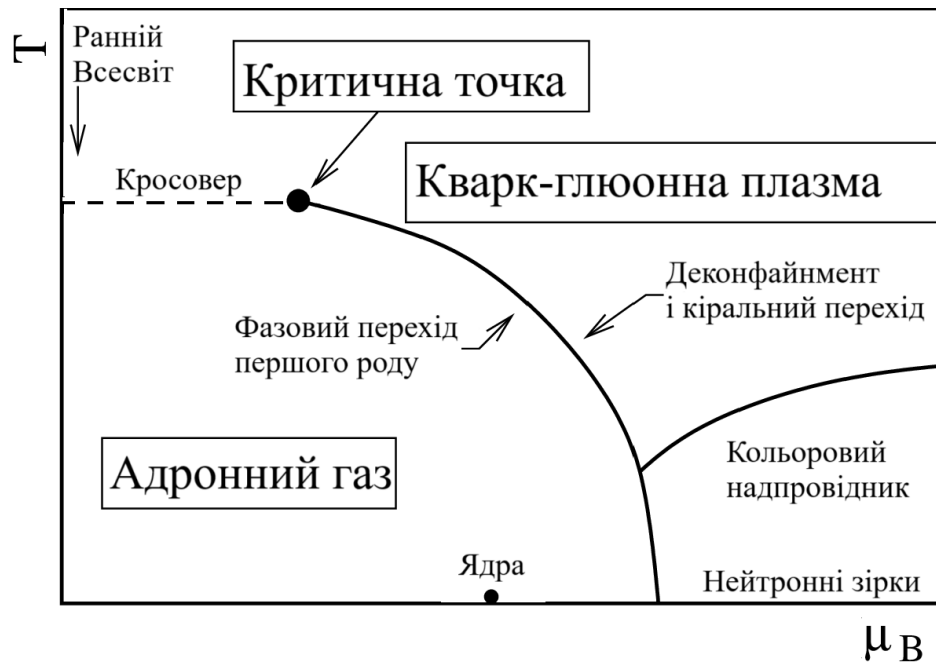


Рис. 1 Фазова діаграма сильновзаємодійної матерії КХД [28]. Вздовж осей T — температура, μ_B — баріонний хімічний потенціал.

колективні властивості. Точний механізм термалізації наразі залишається невідомим (обговорення даної проблеми можна знайти у роботах [19, 20, 21, 22, 23, 24, 25, 26, 27]), але сам факт того, що термалізація має місце, дає підставу для застосування наближення релятивістської гідродинаміки при описі еволюції системи на етапі, коли стан матерії можна вважати близьким до локальної рівноваги.

Однак в процесі розширення і охолодження системи локальна рівновага поступово втрачається, і кварк-глюонна матерія знову адронізується — відбувається так звана *партиклізація*, що вимагає переходу від гідродинамічного опису системи як неперервної рідини до опису в термінах частинок. Дана трансформація зазвичай пов’язується також із кіральним фазовим переходом і відбувається за температури $T_p \approx 150$ MeV. При цьому в ультрарелятивістських зіткненнях, де утворена гаряча система (або “файербол”, від англійського *fireball* — вогняна куля) характеризується близьким до нуля баріонним хімічним потенціалом, перехід між кварк-глюонною та адронною фазами може відбуватися шляхом “кросоверу”, коли термодинамічні величини та їх похідні змінюються хоча і швидко,

але гладко і неперервно, без явних стрибків (див. Рис. 1) ². Згідно з прескрипцією Купера-Фрая [29], в ході партиклізації частинки будуть народжуватися відповідно до локально рівноважних (з деякими поправками у випадку в'язкої гідродинаміки [30, 31]) функцій розподілу.

Подальша еволюція утвореної системи взаємодіючих між собою адронів може бути адекватно описана в рамках мікроскопічних транспортних (каскадних) моделей, таких як UrQMD (*Ultra relativistic Quantum Molecular Dynamics* — Ультрарелятивістська квантова молекулярна динаміка) [32, 33], JAM (*Jet AA Microscopic transport model* — Мікроскопічна транспортна модель ядро-ядерних зіткнень з врахуванням струменів) [34], SMASH (*Simulating Many Accelerated Strongly interacting Hadrons* — Симуляція багатьох прискорених сильно взаємодіючих адронів) [35].

Відповідно, найбільш розвинуті сучасні моделі високоенергетичних зіткнень ядер як правило реалізуються у вигляді програмного коду для комп'ютерних симуляцій, що складається з декількох модулів, кожен з яких відповідає за моделювання певної стадії процесу зіткнення (це так звані *гібридні* моделі [36, 37, 38, 39, 40, 41, 42, 43, 44, 45], в основі яких лежить поєднання гідродинамічної та каскадної частин). Обчислення у таких моделях різноманітних спостережуваних (множинності народжених частинок, їх відношення, імпульсні спектри, кореляційні функції тощо) для кожного конкретного типу зіткнень потребує попереднього калібрування (налаштування параметрів моделі для опису відповідних експериментальних даних) та об'ємних машинних розрахунків десятків і сотень тисяч подій зіткнення, набору достатньої статистики для отримання достовірних результатів. Подібні розрахунки зазвичай виконуються на обчислювальних кластерах (фермах), які об'єднують велику кількість обчислювальних ресурсів (процесорів

²Натомість, в рамках запланованих експериментальних програм Сканування за енергією зіткнень (Beam Energy Scan) на RHIC, а також експериментів HADES (*High Acceptance Di-Electron Spectrometer* — Диелектронний спектрометр з високою сприйнятливістю) та CBM (*Compressed Baryonic Matter* — Стиснута баріонна матерія) у Центрі дослідження важких іонів імені Гельмгольца GSI (Дармштадт, Німеччина) передбачається проведення зіткнень ядер за менших енергій, в яких утворені системи будуть мати відмінний від нуля баріонний хімічний потенціал, що дозволить дослідити різні області фазової діаграми КХД і, можливо, спостерігати фазовий перехід скінченного роду між КГП та адронною матерією, а також локалізувати відповідну критичну точку.

і пам'яті) і дозволяють у пакетному режимі паралельно запускати десятки і сотні копій програми для обчислення та аналізу результатів.

Дана дисертація присвячена моделюванню релятивістських ядро-ядерних зіткнень за енергій колайдерів RHIC та LHC в рамках *інтегрованої гідрокінетичної моделі (iHKM)* [46, 47] і дослідженню властивостей та характеру еволюції систем, що формуються у даних зіткненнях.

Інтегровану гідрокінетичну модель також можна віднести до числа гібридних, адже в ній використовуються рівняння в'язкої гідродинаміки Ізраеля-Стюарта [48, 49, 50], які чисельно розв'язуються за допомогою коду vHLLE [51], та адронний каскад UrQMD (партиклізація при цьому розраховується модулем Cornelius [52, 31]). Проте на відміну від інших моделей, iHKM додатково включає в себе опис *до-термальної (передрівноважної)* стадії зіткнення і таким чином вирішує відому теоретичну проблему *ранньої термалізації* системи. Остання полягає в тому, що, як показують усі теоретичні оцінки, термалізація матерії має наставати за часів близько 1 фм/с після контакту ядер, в той час як для успішного опису експериментальних даних у моделі гідродинамічну стадію необхідно починати як можна скоріше, в часи порядку 0.1 фм/с після зіткнення — інакше до партиклізації системи колективні ефекти радіального та еліптичного потоку у ній не встигнуть розвинути достатньо, щоб забезпечити відтворення результатів експерименту.

За майже два десятиліття пошуків вирішення проблеми ранньої термалізації були проаналізовані різноманітні можливі її механізми, в тому числі на основі AdS/CFT відповідності [22, 53] (формальна математична відповідність між гравітацією у просторі анти-де-Сіттера та рідиною у конформній теорії поля [54, 55]) та ефекту Унру [56, 57]. Проте переконливо обґрунтувати фактичний перехід до термалізованого стану на ранньому етапі еволюції системи так і не вдалося.

Крім того, початкові умови, що використовуються у моделях ядро-ядерних зіткнень (початковий розподіл густини енергії або ентропії, початковий імпульсний розподіл), відповідають стану, який має система одразу після зіткнення ядер (для моделювання цих початкових умов зазвичай використовуються розрахунки за методом Монте-Карло, наприклад, на основі моделі Глаубера, як у програмі GLI-

SSANDO (*GLauber Initial-State Simulation AND mOre* — Симуляція початкового стану за Глаубером та інше) [58], або на основі моделі Харзеєва-Левіна-Нарді [59]). Такий стан є нерівноважним і сильно анізотропним у імпульсному просторі в локальній системі спокою, а тому не може слугувати стартовою точкою для гідродинамічної еволюції [60]. Повна модель зіткнення має включати проміжну, дорівноважну стадію, впродовж якої відбувається перехід від початкового нерівноважного і анізотропного стану матерії до (майже) локально рівноважного, сумісного з гідродинамічним описом. Спроби обійти дану проблему серед іншого привели до створення підходу “анізотропної гідродинаміки” [61, 62], який однак є недостатньо обґрунтованим з фізичної точки зору та має свої обмеження в аспекті опису складної поперечної динаміки розширення системи, особливо у випадку аналізу на основі початкових умов, що змінюються від події до події і є вкрай поперечно неоднорідними [46].

Натомість, у інтегрованій гідрокінетичній моделі початковий нерівноважний стан (що включає поперечний профіль густини енергії, згенерований за допомогою коду GLISSANDO, та анізотропний імпульсний розподіл, мотивований моделлю конденсату кольорового скла (color glass condensate) [60, 63]) відповідає ранньому початковому часу $\tau_0 \sim 0.1$ фм/с. Після формування вказаного стану слідує дотермальна стадія еволюції системи, впродовж якої початковий нерівноважний тензор енергії-імпульсу $T_0^{\mu\nu}(x)$ поступово трансформується у тензор $T_{\text{hydro}}^{\mu\nu}(x)$, що відповідає формалізму Ізраеля-Стюарта для релятивістської в’язкої гідродинаміки. Трансформація відбувається шляхом поступового “підмішування” гідродинамічної складової $T_{\text{hydro}}^{\mu\nu}(x)$ до загального тензора енергії-імпульсу $T^{\mu\nu}(x)$ в рамках релаксаційної динаміки на основі закону збереження енергії-імпульсу та наближення часу релаксації для больцманівської кінетики. Перевагою такого підходу є те, що він не потребує додаткових припущень, на зразок “анізотропної рівноваги” у підході анізотропної гідродинаміки чи умов відповідності Ландау у чисто кінетичному підході, та дозволяє моделювати поперечну динаміку розширення системи у довільному випадку, навіть за наявності сильних неоднорідностей у початкових умовах, характерних для симуляцій в режимі “від події до події”. При цьому коле-

ктивні ефекти, на зразок радіального та еліптичного потоку, розвиваються за час життя системи в достатній мірі без раннього старту гідродинаміки, лише завдяки геометричній обмеженості та (у випадку нецентральных зіткнень) азимутальній асиметрії системи на початковому етапі її еволюції, як було показано в роботі [64].

Ще однією перевагою використання моделі iHKM є додаткова можливість проводити розрахунки на основі власне гідрокінетичного підходу [65, 66, 67, 68, 69, 70], в якому опис еволюції системи відбувається неперервно в термінах функцій вільноти, що характеризують ймовірність для частинок певного сорту бути вільними, перетинаючи певну гіперповерхню. Перехід на адронний каскад UrQMD при цьому може відбуватися на просторовоподібній гіперповерхні постійного власного часу τ_p , що відповідає повному завершенню партиклізації системи, яка відбувається поступово, від периферії до центру. Це, серед іншого, дозволяє уникнути можливого порушення закону збереження енергії-імпульсу при переключенні до адронного каскаду на ізотермічній гіперповерхні адронізації $T = T_p$, яка включає в себе непросторовоподібні елементи, — таке переключення характерне для розрахунків у звичайному гібридному режимі “гідродинаміка+каскад”. Однак, як було встановлено у роботі [69], внески від непросторовоподібних частин гіперповерхні адронізації є нехтовно малими (менше 2%), принаймні у випадку гладких початкових умов, усереднених за великою кількістю подій, коли ми не цікавимося результатами флуктуацій “від події до події”. Це можна пояснити малими розмірами відповідних елементів гіперповерхні при малих часах τ та великими колективними швидкостями при великих τ . З огляду на даний факт і на те, що гідрокінетичні симуляції вимагають суттєво більше машинного часу, у дисертаційному дослідженні використовувався стандартний гібридний режим моделі iHKM.

Для аналізу просторово-часової структури систем, що формуються у зіткненнях ядер за високих енергій, широко застосовується метод *кореляційної фемтоскопії* [71, 72, 73, 74], який базується на квантово-статистичних кореляціях між народженими в результаті зіткнення частинками. Математично такі кореляції певним чином подібні до кореляцій між інтенсивностями світлового випромінювання від зірки у двох просторово рознесених детекторах, вимірювання яких лежить

в основі астрофізичного методу інтерферометрії інтенсивностей Хенбері Брауна і Твісса (Hanbury Brown and Twiss, HBT) для визначення кутових розмірів зірок [75, 76]. *Фемтоскопічні*, або *інтерферометричні радіуси*, які одержують у фізиці зіткнень важких іонів шляхом фітування гаусіанами двочастинкових імпульсних кореляційних функцій, можна асоціювати з *довжинами однорідності* системи в різних напрямках, тобто з довжинами, на яких відповідні функції розподілу $f(x, p)$ змінюються не більше ніж вдвічі [77, 78, 79, 80, 81, 82, 83, 84]. Довжини однорідності, визначені для певного середнього імпульсу пари $\mathbf{k}$, відповідають розмірам частини системи, звідки частинки з середньою швидкістю, що відповідає даному значенню імпульсу, переважно випромінюються. Ці розміри залежать від локальних градієнтів швидкості колективного розширення та інтенсивності теплового руху, що характеризується локальною температурою. Для проведення фемтоскопічного аналізу спочатку необхідно відокремити квантово-статистичні кореляції від кореляцій, викликаних взаємодіями у кінцевому стані, а також врахувати ефект від розпадів довгоживучих резонансів [85]. Додаткову інформацію щодо просторово-часової картини зіткнення можна отримати за допомогою методу “імеджингу” (від англійського *source imaging* — отримання образу джерела). Цей метод дозволяє на основі виміряної в експерименті кореляційної функції відновити так звану *функцію джерела* — інтегрований за часом розподіл відстаней між точками випромінювання частинок у парі [86, 87, 88, 89]. В рамках стандартного методу кореляційної фемтоскопії функція джерела вважається гаусовою, тоді як насправді її форма може бути дещо іншою. Аналізуючи відновлені шляхом імеджингу реальні функції джерела випромінювання та опираючись на модельні розрахунки, можна досліджувати вплив різних факторів еволюції системи (таких як перерозсіювання і розпади резонансів на адронній стадії та інші) на результуючі спостережувані.

Незважаючи на велику увагу, приділену світовою науковою спільнотою дослідженням релятивістських зіткнень важких іонів та нових форм сильновзаємодійної матерії, що народжується у таких процесах, незважаючи на велику кількість вже отриманих цікавих результатів, залишається ще багато відкритих питань, і

дана галузь науки продовжує активно розвиватись, як в експериментальному, так і в теоретичному плані.

Зокрема, актуальним залишається питання про час життя утворюваних систем та роль адронної, постгідродинамічної стадії їх еволюції у формуванні кінцевих спектрів та інших спостережуваних. Наразі існує два поширених підходи до цієї проблеми. Перший з них передбачає, що одразу після адронізації (і, можливо, дуже короткої адронної стадії) відбувається так званий “фріз-аут” (від англійського *freeze-out*), тобто “замороження” хімічного складу, а невдовзі й імпульсних спектрів системи, після чого вони більше не змінюються в ході подальшого розльоту народжених частинок (за виключенням внесків від розпадів резонансів). Даний підхід бере початок від згаданих піонерських робіт Фермі, Померанчука та Ландау [15, 16, 17] і продовжує застосовуватися у термічних [90, 91, 92, 93, 94] та гідродинамічних [95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108] моделях зі *статистичною адронізацією*, а також у гідродинамічно мотивованих параметризаціях фріз-ауту, таких як Краківська модель (Cracow) [109, 110], модель Буда-Лунд (Buda-Lund) [111], або модель Blast Wave [112, 113, 114]. Існують різноманітні комп’ютерні коди для розрахунків статистичної адронізації у різних моделях, наприклад, SHARE [115, 116], THERMUS [117], що застосовуються для визначення термодинамічних величин шляхом фітування відношень чисел частинок, або THERMINATOR [118, 119], що використовується як генератор подій, в якому частинки продукуються відповідно до термодинамічних параметрів, заданих на певній гіперповерхні.

Другий підхід передбачає наявність довгої постгідродинамічної стадії зіткнення, яка триває ще приблизно $5 - 10$ фм/с, доки температура не впаде до $80 - 120$ MeV, і грає важливу роль у формуванні спостережуваних величин. Саме цей підхід реалізується у згаданих гібридних моделях ядро-ядерних зіткнень та у інтегрований гідрокінетичній моделі.

Експериментальні дані наразі не дозволяють надати чітку перевагу одному з названих підходів. Аналіз спектрів поперечного імпульсу дає дуже різні оцінки температури фріз-ауту (у діапазоні $100 - 140$ MeV [105]), в залежності від обра-

ного сорту частинок, інтервалу імпульсів, включення чи виключення внесків від розпадів резонансів тощо. Оцінки часу фріз-ауту на основі залежності поздовжніх фемтоскопічних масштабів від поперечної маси пари m_T , зроблені колаборацією ALICE для зіткнень ядер свинцю за енергії $\sqrt{s_{NN}} = 2.76$ TeV, дають результат $10 - 11$ фм/с [120], але при фітуванні не було враховано ефекти сильного поперечного колективного потоку, характерного для таких зіткнень. Зменшення виходів короткоживучих резонансів $K^*(892)$ та відношень виходів K^*/K у центральних зіткненнях на ЛНС порівняно з нецентральними (при відсутності подібного ефекту для довгоживучого резонансу $\phi(1020)$ та відношень $\phi(1020)/K$) свідчить на користь тривалої адронної стадії з інтенсивними зіткненнями між частинками [121, 122, 123]. Ці зіткнення призводять до перерозсіювання продуктів розпаду K^* і неможливості детектування певної частки народжених резонансів, яка є тим більшою, чим більша центральність зіткнення. Однак оцінки часу життя системи на основі вказаних результатів залежать від перерізів розсіювання та каналів реакцій у каскадній моделі, що використовується в дослідженні.

Таким чином, пошук розв'язання даної проблеми вимагає комплексного аналізу широкого набору адронних спостережуваних у подіях релятивістських ядроядерних зіткнень з різних класів центральності при варіативній енергії зіткнення. Окремої уваги потребує питання розпаду короткоживучих резонансів, на зразок K^* , на адронній стадії зіткнення, взаємодії продуктів їх розпаду з густим адронним середовищем і впливу цієї взаємодії на можливість детектування резонансів та їх кінцеві виходи. Актуальною у вказаному контексті є також проблема оцінки часової та просторової асиметрії між випромінюванням частинок різних сортів, наприклад, піонів і каонів, яка має відображати особливості еволюції системи на гідродинамічній та постгідродинамічній стадіях. Для більш точної оцінки даних асиметрій у випадку порівняння сортів частинок з помітно різними масами спокою корисно дослідити поведінку фемтоскопічних масштабів при найнижчих значеннях поперечного імпульсу пари.

В контексті розвитку інтегрованої гідрокінетичної моделі важливою задачею є калібрування моделі для опису усіх основних м'яких спостережуваних, включа-

ючи фемтоскопію, у зіткненнях ядер свинцю за енергії колайдера LHC $\sqrt{s_{NN}} = 5.02$ TeV і порівняння результатів, початкових умов і параметрів моделювання з обчисленнями у iHKM для інших типів зіткнень, як-от зіткнення ядер золота на колайдері RHIC за енергії $\sqrt{s_{NN}} = 200$ GeV, зіткнення ядер свинцю на LHC за нижчої енергії $\sqrt{s_{NN}} = 2.76$ TeV, зіткнення ядер ксенону за енергії $\sqrt{s_{NN}} = 5.44$ TeV. Також цікаво дослідити, як залежать результати моделювання від рівняння стану кварк-глюонної матерії, що використовується при описі гідродинамічної стадії розширення системи, і як застосування іншого рівняння стану вплине на значення решти параметрів моделі.

Ще одним перспективним напрямком досліджень є визначення параметрів, що описують сильну взаємодію між баріонами (довжини розсіювання, ефективні радіуси), за допомогою *кореляційного аналізу взаємодій у кінцевому стані* для частинок, що народжуються у високоенергетичних ядро-ядерних зіткненнях [124]. Адже колайдер LHC не дарма називають “фабрикою частинок” — в ультрарелятивістському зіткненні важких іонів народжуються тисячі адронів різних сортів, у тому числі екзотичних, які містять важкі s , c , b кварки. Аналіз кореляцій, що виникають між баріонами за рахунок сильної взаємодії, надає унікальну можливість визначати характеристики цієї взаємодії навіть у випадках, коли традиційні низькоенергетичні експерименти з розсіювання не дозволяють зробити це з достатньою точністю [125, 126]. Дослідження взаємодій у парах нуклон — гіперон та гіперон — гіперон є важливими з огляду на питання про можливу наявність Λ -гіперонів у ядрах нейтронних зірок [127], а також вдосконалення опису адрон-адронної взаємодії в рамках КХД та в каскадних моделях. Проведення відповідного аналізу на основі обчислень у реалістичній моделі зіткнення дозволить підвищити точність та надійність отриманих результатів за рахунок використання модельних функцій джерела та даних про чистоту пар.

Зв’язок роботи з науковими програмами, планами, темами. Дана дисертаційна робота була підготовлена за результатами досліджень, що проводились у відділі фізики високих густин енергії Інституту теоретичної фізики ім. М. М. Боголюбова Національної академії наук України в рамках наступних ві-

домчих державних програм:

“Дослідження сильновзаємодіючої матерії та структури адронів в релятивістських зіткненнях адронів та ядер” (2016–2017), номер державної реєстрації — 0113U001092, шифр — 1.4.1.

“Пошук нових форм сильновзаємодійної матерії в зіткненнях адронів та ядер за високих енергій” (2018–2022), номер державної реєстрації — 0118U003197, шифр — 1.4.1.

“Теорія та моделювання властивостей сильновзаємодійної матерії в релятивістських зіткненнях адронів та ядер” (2023–2024), номер державної реєстрації — 0123U100302, шифр — 1.4.1.

Дослідження також проводилися в Інституті теоретичної фізики ім. М. М. Боголюбова НАН України в рамках таких проєктів конкурсної та цільової тематики НАН України та ДФФД:

“Кореляційний аналіз сильної взаємодії екзотичних баріонів у релятивістських ядро-ядерних зіткненнях” (2016), конкурсний грант НАН України для молодих учених в рамках програми “Фундаментальні властивості матерії у широкому інтервалі масштабів простору і часу”, номер державної реєстрації — 0116U002745.

“Важкі іони з ультрарелятивістськими енергіями (гідрокінетичний опис).” (2016), відповідно до Угоди про співробітництво між НАН України та Національним центром наукових досліджень Франції (CNRS), номер державної реєстрації — 0116U005425.

“Кореляційний аналіз сильної взаємодії між баріонами у зіткненнях важких іонів” (2018), грант Ф75/219-2018 Президента України для підтримки наукових досліджень молодих учених у 2018 році, отриманий за конкурсом ДФФД, номер державної реєстрації — 0118U006969.

“EUREA: Європейська угода з ультрарелятивістських енергій” (2018–2019), відповідно до Угоди про співробітництво між НАН України та Національним центром наукових досліджень Франції (CNRS), номери державної реєстрації — 0118U002310, 0119U001775.

“Просторово-часова динаміка і властивості надщільної матерії в релятивіст-

ських зіткненнях ядер, та їх прояви в поточних експериментах на LHC, RHIC, і запланованих FAIR, NICA” (2020), за Цільовою програмою НАН України “Фундаментальні дослідження з фізики високих енергій та ядерної фізики (міжнародна співпраця)”, номер державної реєстрації — 0120U103135.

“Фундаментальні властивості матерії в релятивістських ядерних зіткненнях та у ранньому Всесвіті” (2020), за конкурсом НАН України “Підтримка розвитку пріоритетних напрямів наукових досліджень”, номер державної реєстрації — 0120U100935.

“Динамічні та статистичні властивості надщільної речовини, що формується в релятивістських ядро-ядерних та протон-протонних зіткненнях з високою множинністю при варіативних енергіях колайдерів RHIC та LHC” (2021, 2023), за Цільовою програмою НАН України “Участь в новітніх міжнародних проєктах з фізики високих енергій та ядерної фізики”, номери державної реєстрації — 0121U111762, 0123U101684.

Мета і задачі дослідження. Метою даної дисертаційної роботи є теоретичне дослідження просторово-часової структури та характеру еволюції систем, що народжуються в зіткненнях важких ядер за високих енергій, з’ясування властивостей нових видів сильновзаємодійної матерії, які утворюються на проміжних стадіях зіткнення, та аналіз механізмів формування м’яких адронних спостережуваних в процесі еволюції системи.

Для досягнення вказаної мети було поставлено наступні задачі:

1. Відкалібрувати інтегровану гідрокінетичну модель (іНКМ) для опису м’яких спостережуваних у зіткненнях ядер свинцю на Великому адронному колайдері (LHC) за енергії $\sqrt{s_{NN}} = 5.02$ TeV. Калібрування провести для двох рівнянь стану кварк-глюонної матерії на гідродинамічній стадії — рівняння Лейна-Шредера [128] та рівняння колаборації HotQCD [129] — з відповідними температурами партиклізації.
2. Розрахувати у моделі іНКМ всі основні адронні спостережувані в області м’якої фізики (виходи частинок різних сортів, їх відношення, спектри поперечного імпульсу, коефіцієнти імпульсної анізотропії, кореляційні функції

та відповідні фемтоскопічні масштаби). Порівняти результати обчислень при двох використаних рівняннях стану КГП з наявними експериментальними даними. Дослідити залежність результатів моделювання від параметрів моделі, зокрема від початкового профілю густини енергії. Порівняти результати моделювання для зіткнень ядер свинцю при $\sqrt{s_{NN}} = 5.02$ TeV з відповідними результатами для енергії $\sqrt{s_{NN}} = 2.76$ TeV, а також для зіткнень ядер ксенону за енергії LHC $\sqrt{s_{NN}} = 5.44$ TeV та зіткнень ядер золота на колайдері RHIC за енергії $\sqrt{s_{NN}} = 200$ GeV.

3. Дослідити у моделі iHKM вплив постгідродинамічної адронної стадії еволюції системи на результуючі відношення чисел частинок та фемтоскопічні масштаби.
4. Дослідити поведінку фемтоскопічних радіусів в області надмалих імпульсів ($m_T < 200$ MeV/c) у моделі iHKM. Порівняти отримані результати з розрахунками у моделі зі статистичною адронізацією LQTH.
5. В рамках інтегрованої гідрокінетичної моделі дослідити вплив взаємодії резонансів $K^*(892)$ і $\phi(1020)$ та продуктів їх розпаду з адронним середовищем на пізній стадії зіткнення на можливість детектування даних резонансів та їх кінцеві виходи.
6. На основі розрахованих у моделі iHKM спектрів та фемтоскопічних радіусів оцінити і порівняти значення ефективних часів емісії π - та K -мезонів. Дослідити залежність результатів від енергії та центральності зіткнення.
7. Оцінити тривалість гідродинамічної стадії еволюції системи в зіткненнях важких іонів при різних релятивістських енергіях, проаналізувати від чого вона залежить.
8. Розробити метод оцінки загального часу життя системи, утвореної у релятивістському зіткненні ядер, з урахуванням адронної стадії.
9. Вдосконалити метод кореляційного аналізу взаємодій у кінцевому стані для визначення довжини розсіяння сильної взаємодії та ефективного радіусу джерела емісії баріонів у парах $\bar{p}\Lambda$ і $p\bar{\Lambda}$ в зіткненнях ядер золота на колайдері RHIC. На основі результатів для RHIC змоделювати відповідні

кореляції у випадку енергій колайдера LHC. Розглянути також випадок одночасної наявності сильної та електромагнітної взаємодії між баріонами у парі.

Об'єктом дослідження дисертаційної роботи є системи, що народжуються у високоенергетичних зіткненнях важких ядер.

Предметом дослідження є просторово-часові масштаби систем, що утворюються в релятивістських ядро-ядерних зіткненнях, властивості кварк-глюонної матерії (такі як рівняння стану, густина енергії, температура, в'язкість, час формування, критерії адронізації тощо), властивості утворених систем на адронній стадії, характеристики сильної взаємодії між баріонами, а також зв'язок між всіма переліченими властивостями та кінцевими адронними спостережуваними в області м'якої фізики.

Методи дослідження. У дисертації переважно використовується метод комп'ютерного моделювання — релятивістські зіткнення важких ядер симулюються в рамках інтегрованої гідрокінетичної моделі, яка включає п'ять окремих модулів для реалістичного моделювання різних стадій еволюції систем, що утворюються в зіткненнях. Дана модель є однією з найбільш розвинутих сучасних моделей ядро-ядерних зіткнень за високих енергій, яка до того ж включає в себе стадію перед-рівноважної динаміки, що описує термалізацію системи та поступовий перехід від початкового нерівноважного стану до майже локально рівноважного, сумісного з подальшим гідродинамічним описом. До початку дисертаційного дослідження модель вже було успішно застосовано до опису широкого набору м'яких спостережуваних у зіткненнях ядер свинцю на Великому адронному колайдері при енергії на нуклонну пару $\sqrt{s_{NN}} = 2.76$ TeV, що підтверджує надійність моделі та дає підстави для її застосування і у даній роботі. Окрім комп'ютерного моделювання, в дисертації також застосовуються загальні математичні методи теоретичної фізики.

Наукова новизна отриманих результатів. Наукова новизна результатів дисертаційного дослідження забезпечується серед іншого тим, що кожне з розглянутих в рамках дисертації питань становить актуальну теоретичну проблему

фізики зіткнень важких іонів за високих енергій, тісно пов'язану з сучасними експериментальними дослідженнями в даній області та їх актуальними проблемами. Частину отриманих результатів становлять *передбачення* в рамках моделі певних адронних спостережуваних, щодо яких поки відсутні оприлюднені дані вимірювань.

Більшість результатів дослідження стосовно властивостей систем, які народжуються у релятивістських зіткненнях важких іонів (і зокрема, нової форми сильновзаємодійної матерії — кварк-глюонної плазми, що утворюється і існує впродовж дуже короткого часу на одній з проміжних стадій зіткнення), отримано здобувачем в рамках *нещодавно побудованої* найбільш повної моделі таких процесів, що включає в себе опис поступової термалізації системи перед початком її гідродинамічного розширення, — моделі іНКМ, — яку було *вперше* відкалібровано та застосовано до опису зіткнень ядер свинцю на колайдері ЛНС при енергії $\sqrt{s_{NN}} = 5.02$ TeV з використанням двох варіантів рівняння стану кварк-глюонної плазми на гідродинамічному етапі еволюції системи.

Новими оригінальними результатами, що виносяться на захист є наступні:

- Інтегровану гідрокінетичну модель ядро-ядерних зіткнень (іНКМ) вперше відкалібровано для опису широкого класу спостережуваних м'якої фізики (виходи частинок різних сортів, їх відношення, спектри поперечного імпульсу, коефіцієнти імпульсної анізотропії, кореляційні функції та відповідні фемтоскопічні масштаби) у зіткненнях ядер свинцю на Великому адронному колайдері (ЛНС) при енергії $\sqrt{s_{NN}} = 5.02$ TeV.
- Однаково гарний опис наявних експериментальних даних та передбачення для фемтоскопічних масштабів піонів і каонів вперше отримано у моделі іНКМ при двох різних рівняннях стану кварк-глюонної матерії на гідродинамічній стадії еволюції системи. Збереження узгодженості теоретичного опису з експериментом при переході до іншого рівняння стану досягається перенормуванням початкової густини енергії у центрі системи ϵ_0 . Таким чином, встановлено, що наявні результати експериментальних вимірювань м'яких адронних спостережуваних наразі не дозволяють надати

чітку перевагу одному з двох розглянутих рівнянь стану та визначити точні величини температури партиклізації і густини енергії на ранній стадії зіткнення.

- Встановлено, що адронна стадія зіткнення, яка у iHKM описується в рамках адронного каскаду UrQMD, компенсує різницю у значеннях м'яких спостережуваних, яка може мати місце в кінці гідродинамічної стадії внаслідок використання різних рівнянь стану КГП. Цей висновок говорить на користь концепції “поступового фріз-ауту”, тобто поступового формування імпульсних спектрів та виходів частинок в процесі еволюції системи, що включає в себе довгу адронну стадію. Зокрема, адронна стадія має ключове значення для успішного опису виходів резонансів $K^*(892)$ та відношень числа народжених протонів до числа пі-мезонів.
- В рамках моделі iHKM з'ясовано, що фемтоскопічні масштаби π - та K -мезонів у випадку енергії зіткнень $\sqrt{s_{NN}} = 5.02$ TeV близькі до відповідних значень при енергії $\sqrt{s_{NN}} = 2.76$ TeV. Ефект певного зростання множинності народжених частинок внаслідок збільшення енергії зіткнення частково компенсується більшими градієнтами колективної швидкості при енергії 5.02 TeV.
- В області надм'яких імпульсів ($m_T < 200$ MeV/c) у моделі iHKM виявлено відхилення поведінки фемтоскопічних піонних радіусів від степеневого закону виду am_T^{-b} . Незвичний характер залежності $R(m_T)$ для піонів з наднизькими імпульсами може свідчити про своєрідне відокремлення даних частинок від решти системи, їх триваліше утримання в її центрі та утворення своєрідної “пастки” для надм'яких піонів.
- У моделі HKM з'ясовано, що на адронній стадії зіткнення відбувається інтенсивна взаємодія короткоживучих резонансів $K^*(892)$ (час життя яких становить близько 5 фм/c) та продуктів їх розпаду з густим адронним середовищем. Для центральних зіткнень така взаємодія призводить спочатку до “втрати” близько 70% первинно утворених K^* , а потім до утворення шляхом рекомбінації додаткових 50% від початкової кількості K^* .

В результаті спостерігається недостача числа ідентифікованих K^* у 20%. Ефект зменшується при переході від центральних до периферичних зіткнень. Виходи ж довгоживучих резонансів $\phi(1020)$ (з часом життя порядку 50 фм/с), навпаки, не залежать від центральності зіткнення.

- У інтегрований гідрокінетичній моделі отримано оцінки на часи випромінювання піонів і K -мезонів у зіткненнях важких ядер на колайдерах LHC та RHIC. Несподіваним результатом з точки зору гідродинамічних міркувань є одержане в роботі більше значення ефективного часу емісії каонів у порівнянні з піонами. Даний результат можна пояснити сильним впливом взаємодій між адронами на фінальній стадії зіткнення, зокрема перерозсіянь і рекомбінації продуктів розпаду резонансів $K^*(892) \leftrightarrow K\pi$, на результуючі каонні спостережувані. Для більш периферичних зіткнень та при зниженні їх енергії часи емісії частинок обох сортів зменшуються, разом з інтенсивністю поперечного колективного потоку та ефективною температурою спектрів.
- В рамках моделі iHKM встановлено, що тривалість гідродинамічної стадії для зіткнень ядер золота при енергії $\sqrt{s_{NN}} = 200$ GeV і ядер свинцю при енергіях $\sqrt{s_{NN}} = 2.76$ TeV та $\sqrt{s_{NN}} = 5.02$ TeV майже однакова і становить 8 – 10 фм/с. Головними причинами близькості отриманих значень, попри різницю в енергії зіткнення, є інтенсивне, суттєво тривимірне гідродинамічне розширення системи, яке починається з певного моменту власного часу, та схожі співвідношення між розмірами системи та середньою величиною швидкості звуку на гідродинамічній стадії в усіх вказаних випадках.
- Запропоновано метод оцінки загального часу життя системи, що формується у релятивістському зіткненні ядер, на основі фітування спектрів та фемтоскопічних радіусів піонів і каонів в області малих поперечних імпульсів. Для зіткнень ядер свинцю на Великому адронному колайдері отримано значення часу життя системи порядку 17 фм/с, що говорить про наявність у таких зіткненнях довгої адронної стадії, яка грає важливу роль у фор-

муванні кінцевих спектрів та фемтоскопічних радіусів.

- На основі формалізму Куніна-Пратта та Ледніцкі-Любошица, а також розрахунків у гідрокінетичній моделі запропоновано просту аналітичну модель для опису баріон-баріонних та баріон-антибаріонних кореляційних функцій з урахуванням ефекту залишкових кореляцій. В рамках запропонованого підходу вдалося вирішити проблему видимої двократної відмінності між радіусами джерел емісії пар $p\Lambda$ і $p\bar{\Lambda}$ в експериментальній роботі колаборації STAR [125] щодо зіткнень ядер золота за найвищої енергії прискорювача RHIC. Визначено довжину розсіювання сильної взаємодії у парах $p\bar{\Lambda}$ та $\bar{p}\Lambda$. На основі отриманих результатів зроблено передбачення кореляційних функцій для зіткнень при енергії колайдера LHC з урахуванням сильної та кулонівської взаємодії між баріонами.

Практичне значення отриманих результатів. Отримані результати мають теоретичний характер і є внеском у дослідження властивостей сильновзаємодійної матерії, що утворюється у високоенергетичних ядро-ядерних зіткненнях, та поглиблення розуміння динаміки даних процесів.

Результати дисертації можуть бути використані при аналізі та інтерпретації експериментальних даних, а також можуть стимулювати нові експериментальні дослідження на сучасних прискорювачах частинок, таких як LHC та RHIC.

Визначена у дисертації довжина розсіювання $p\bar{\Lambda}$ може бути використана для вдосконалення існуючих моделей адрон-адронної взаємодії та каскадних моделей.

Результати роботи також можуть слугувати підґрунтям для подальшого розвитку теоретичного опису еволюції матерії в ході зіткнень важких іонів.

Особистий внесок здобувача. У роботах, виконаних зі співавторами, здобувачеві належить:

- постановка задачі, ідея вдосконалення методу кореляційного аналізу за рахунок обчислення функції джерела у гідрокінетичній моделі та ефективного врахування залишкових кореляцій, розрахунки функцій джерела у гідрокінетичній моделі, фітування експериментальних кореляційних функцій [1^a];

- постановка задачі, оцінка невизначеностей у передбачених кореляційних функціях через невизначеності у значеннях окремих параметрів, оцінка чистоти пар на основі розрахунків у гідрокінетичній моделі, розрахунок баріон-баріонної кореляційної функції, підготовка тексту та рисунків для рукопису статті [2^a];
- пояснення одержаного більш пізнього часу емісії K -мезонів порівняно з π -мезонами; аналіз залежності отримуваної величини фемтоскопічних радіусів від діапазону відносного імпульсу, в якому проводиться фітування кореляційної функції; підготовка тексту та рисунків для рукопису статті [3^a];
- розрахунок функцій джерела та чистоти пар у гідрокінетичній моделі, розрахунок кореляційних функцій $p\Xi^-$ з урахуванням кулонівської взаємодії [4^a];
- розрахунки у гідрокінетичній моделі виходів резонансів K^* і ϕ та їх відношень до виходів каонів, розрахунок відповідних спектрів поперечного імпульсу; розрахунок функцій емісії K^* та ϕ і чисел резонансів, що можуть бути детектовані за інваріантною масою продуктів їх розпаду на різних стадіях зіткнення; аналіз впливу адронної стадії на виходи резонансів та інтерпретація результатів [5^a];
- розрахунок відношень чисел частинок у гідрокінетичній моделі, аналіз результатів, пояснення значних відмінностей між результатами обчислень у моделі з вимкненою адронною стадією та у термальній моделі для відношень K^*/K^{ch} [6^a];
- постановка задачі, перевірка коректності комп'ютерних розрахунків, підготовка тексту та рисунків для рукопису статті [7^a];
- калібрування інтегрованої гідрокінетичної моделі при двох різних рівняннях стану для КГП з відповідними температурами партиклізації, розрахунки у моделі виходів частинок, їх відношень, імпульсних спектрів, v_n -коефіцієнтів, кореляційних функцій; порівняння швидкості звуку та швидкості поперечного колективного потоку для двох рівнянь стану КГП; ана-

ліз ролі адронної стадії еволюції системи у формуванні спостережуваних; підготовка тексту та рисунків для рукопису статті [8^a];

- перевірка коректності комп'ютерних розрахунків, участь в аналізі результатів, підготовка тексту та рисунків для рукопису статті [9^a];
- аналіз залежності параметра α , що регулює частки внесків моделей бінарних зіткнень та пораних нуклонів до початкового поперечного профілю густини енергії, від енергії зіткнення та геометрії ядер, що зіштовхуються; перевірка коректності комп'ютерних розрахунків м'яких спостережуваних, підготовка тексту та рисунків для рукопису статті [10^a];
- постановка задачі, перевірка розрахунків фемтоскопічних масштабів, аналіз та інтерпретація результатів, підготовка тексту та рисунків для рукопису статті [11^a];
- постановка задачі, обчислення імпульсних спектрів, фемтоскопічних масштабів та функцій емісії у інтегрований гідрокінетичній моделі, визначення часів емісії піонів та каонів для зіткнень різної центральності, аналіз та інтерпретація результатів, підготовка тексту та рисунків для рукопису статті [12^a];
- ідея методу оцінки повного часу життя системи, оцінка тривалості гідродинамічної стадії зіткнення та аналіз причин близькості її значень для різних типів ультрарелятивістських зіткнень; розрахунки у інтегрований гідрокінетичній моделі функцій джерела, розподілів часів випромінювання частинок, еволюції густини енергії з часом; підготовка тексту та рисунків для рукопису статті [13^a];
- розрахунки фемтоскопічних радіусів у інтегрований гідрокінетичній моделі, участь в аналізі та інтерпретації результатів, наочна демонстрація утримання надм'яких піонів у центрі системи та більшої тривалості їх випромінювання за допомогою розрахунку відповідних функцій емісії [14^a].

Частина результатів робіт [2^a, 3^a] увійшла до кандидатської дисертації В. Ю. Набоки “Термалізація та еволюція сильновзаємодійної матерії в інтегрований гідрокінетичній моделі ядро-ядерних зіткнень” (2016), а частина результатів робіт [7^a,

9^a, 10^a] увійшла до дисертації на здобуття наукового ступеня доктора філософії М. Д. Аджимамбетова “Спектри та кореляції частинок в ядро-ядерних і протон-протонних зіткненнях з високою енергією та множинністю” (2022). Вказані результати, захищені співавторами, не виносяться на захист у даній докторській дисертації. Результати наукових досліджень, за якими захищена кандидатська дисертація В. М. Шаповала “Кореляційна фемтоскопія в динамічних моделях ядро-ядерних та протон-протонних зіткнень” (2014), також не виносяться на захист у представленій докторській дисертації здобувача.

Апробація результатів дисертації. Усі результати, висвітлені у даній дисертації, були представлені на наукових зібраннях в Інституті теоретичної фізики ім. М. М. Боголюбова Національної академії наук України, зокрема на Боголюбовських читаннях 2022 (14 жовтня 2022 р.), Боголюбовських читаннях 2023 (17 жовтня 2023 р.), Наукових зборах (3 квітня 2025 р.), а також на семінарах відділу фізики високих густин енергії Інституту.

Результати дисертації доповідалися та обговорювалися на таких міжнародних та всеукраїнських конференціях:

- XVII GDRE Workshop “Heavy Ions at Relativistic Energies”, 28 June – 4 July 2015, Nantes, France
- VII Young Scientists Conference “Problems of Theoretical Physics”, dedicated to 50-th anniversary of Bogolyubov Institute for Theoretical Physics, December 13–15, 2016, Kyiv, Ukraine
- All-Ukrainian scientific conference “Principal Problems of Quantum Field Theory and Gravitation” dedicated to the jubilee of M. Korkina and V. Vanyashin, 4 November 2016, Dnipro, Ukraine
- XIX GDRE Workshop “Heavy Ions at Relativistic Energies”, 3–7 July 2017, Nantes, France
- XII Workshop on Particle Correlations and Femtoscopy, 12–16 June 2017, Amsterdam, Netherlands
- Nuclear, Particle physics and Cosmology Workshop, Ferry Oslo–Copenhagen–Oslo, 16–18 May 2017

- II NICA Days 2017 Conference associated with the II Slow Control Warsaw 2017, Warsaw, Poland, 6–10 November 2017
- XX GDRE Workshop “Heavy Ions at Relativistic Energies”, 8–14 July 2018, Nantes, France
- XIII Workshop on Particle Correlations and Femtoscopy, Krakow, Poland, 22–26 May 2018
- GDRI 2019 International Research Network Meeting, 14–20 July 2019, Nantes, France
- XIV Workshop on Particle Correlations and Femtoscopy, 3–7 June 2019, Dubna
- II International Workshop on Theory of Hadronic Matter Under Extreme Conditions, 16–19 September 2019, Dubna
- XI conference of young scientists “Problems of theoretical physics”, 21–23 December 2020, Kyiv, Ukraine
- XII Young Scientists Conference “Problems of Theoretical Physics”, Kyiv, 21–22 December, 2021
- XXIX щорічна наукова конференція ІЯД НАН України, Київ, 26–30 вересня 2022
- EMMI Rapid Reaction Task Force (RRTF) “Direct-Photon Puzzle”, 24–27 July 2023, Heidelberg, Germany
- XVI Workshop on Particle Correlations and Femtoscopy, 6–10 November 2023, Catania, Italy
- XVII Workshop on Particle Correlations and Femtoscopy, 4–8 November 2024, Toulouse, France.

Публікації. Основні наукові результати дисертаційної роботи представлені у 14 статтях [1^a–14^a], що опубліковані у провідних фізичних журналах, індексованих у базах Scopus та/або Web of Science Core Collection. Серед даних статей 13 відносяться до кuartилів Q1–Q2, а 1 стаття відноситься до кuartилію Q3 за даними SCImago Journal and Country Rank. Крім того, результати дисертації відображено в 9 публікаціях у збірках матеріалів і тез наукових конференцій [1^b–9^b]. Списки праць за темою дисертації наведено після анотації та в Додатку А.

Структура та обсяг дисертації. Дисертація включає титульний аркуш, анотацію українською та англійською мовами, список публікацій за темою дисертації, зміст, перелік умовних позначень, вступ, три розділи, поділені на підрозділи, висновки, подяки, список використаних джерел, який містить 129 посилань, та додаток з даними про опублікування та апробацію результатів. Роботу виконано у вигляді наукової доповіді за сукупністю статей, які є підрозділами дисертації. Дисертація включає 131 рисунок. Загальний обсяг роботи становить 243 сторінки.

РОЗДІЛ 1

ФОРМУВАННЯ М'ЯКИХ СПОСТЕРЕЖУВАНИХ НА КОЛАЙДЕРАХ LHC ТА RHIC

1.1. Виходи частинок та їх відношення у зіткненнях ядер свинцю на LHC

Particle production at energies available at the CERN Large Hadron Collider within an evolutionary model

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The particle yields and particle number ratios in Pb+Pb collisions at the CERN Large Hadron Collider (LHC) energy $\sqrt{s_{NN}} = 2.76$ TeV are described within the integrated hydrokinetic model (iHKM) at two different equations of state (EoS) for quark-gluon matter and the two corresponding hadronization temperatures $T = 165$ MeV and $T = 156$ MeV. The role of particle interactions at the final afterburner stage of the collision in the particle production is investigated by means of comparison of the results of full iHKM simulations with those where the annihilation and other inelastic processes (except for resonance decays) are switched off after hadronization/particlization, similarly as in the thermal models. An analysis supports the picture of continuous chemical freeze-out in the sense that the corrections to the sudden chemical freeze-out results, which arise because of the inelastic reactions at the subsequent evolution times, are noticeable and improve the description of particle number ratios. An important observation is that, although the particle number ratios with switched-off inelastic reactions are quite different at different particlization temperatures which are adopted for different equations of state to reproduce experimental data, the complete iHKM calculations bring very close results in both cases.

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I. INTRODUCTION

The analysis of the particle number ratios is carried out successfully in thermal models for different energies of $A + A$ collisions, from BNL Alternating Gradient Synchrotron (AGS) to CERN Large Hadron Collider (LHC) energies [1–7]. The thermal models suppose that, at some hypersurface characterized by uniform temperature and baryon chemical potential, the chemical composition of the hadron matter is frozen out, and in subsequent evolution of the hadron matter the particle yield is changed only because of the resonance decays. At LHC energies the afterburner “post-freeze-out” stage is the longest, and so there is a special interest to check the chemical freeze-out hypotheses within the dynamical models for these energies. The ALICE Collaboration has already noted [8,9] that annihilation processes at the afterburner stage, which are taken into account in the hydrokinetic model (HKM) [10], noticeably improve agreement with (anti)proton spectra/yield at the LHC. The analysis of the role of inelastic processes at the post-hydrodynamic stage in formation of the particle yield is ongoing (see, e.g., [11]).

It seems that continuous chemical freeze-out as well as kinetic freeze-out is an inevitable feature of the dynamical models of $A + A$ collisions since sudden chemical freeze-

out means instant transition from extremely fast chemically equilibrated expansion (presupposing a very intensive inelastic reactions) to evolution with totally forbidden inelastic reactions. Sudden kinetic freeze-out means an instant change of hadron cross section from a very large one (typical for near perfect hydrodynamics) to zero cross section (free streaming particles). Such sudden transitions are not typical for realistic dynamical models.¹ In our very recent note [12] we found, using a $K^*(892)$ probe, that at LHC energies a good agreement with the experimental data for these resonances requires a relatively long kinetic freeze-out, near 5 fm/c after particlization/hadronization. It is worth noting that continuous thermal freeze-out means not only successive freeze-out for different hadrons (as in, e.g., Ref. [13]), but continuous particle emission for each species; see new important details in Ref. [12].

In this study we calculate the particle number ratios in the integrated hydrokinetic model (iHKM) and compare the results with the ones obtained in thermal models. Also we calculate the particle p_T spectra in iHKM. We analyze the situation with different equations of state for quark-gluon and hadron matter and, correspondingly, at different temperatures of the so-called chemical freeze-out.

II. MODEL DESCRIPTION

The current study is carried out within the integrated hydrokinetic model (iHKM) [14] of relativistic nuclear collisions. This model includes the five stages of matter evolution and

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¹Note also that neither the first-order phase transition, nor the crossover are sudden in time in the process of system expansion.

observable formation in $A + A$ collisions: the initial state formation, the prethermal matter evolution, the hydrodynamic stage, the particlization stage, and the hadronic cascade stage.

The initial energy-density profile in iHKM is associated with a quite early proper time, $\tau_0 \approx 0.1$ fm/c. According to a combined method, described in [14], one presents the generally nonequilibrium boost-invariant (in the central region of rapidity) parton/gluon distribution function on the initial hypersurface σ_0 : $\tau = \tau_0$ in the factorized form

$$f(t_{\sigma_0}, \mathbf{r}_{\sigma_0}, \mathbf{p}) = \epsilon(b; \tau_0, \mathbf{r}_T) f_0(p), \quad (1)$$

where $\epsilon(b; \tau_0, \mathbf{r}_T)$, the initial energy density profile, is calculated in a hybrid approach, including both the wounded nucleon model and the binary collision approach. The proportion between the contributions of these two models to $\epsilon(b; \tau_0, \mathbf{r}_T)$ is regulated by the parameter $0 \leq \alpha \leq 1$.

In iHKM simulations we obtain the distributions of numbers of wounded nucleons and binary collisions at τ_0 with the help of the GLISSANDO code [15]. The weighed sum of such distributions (with the coefficients α and $1 - \alpha$) is then multiplied by a normalizing factor ϵ_0 , the energy density at τ_0 in the center of the system in central collisions. The value of ϵ_0 is the main free parameter of the model, defined, together with the parameter α , by means of fitting the observed mean charged particle multiplicity $dN_{ch}/d\eta$ dependence on centrality at a given collision energy. So, both ϵ_0 and α parameters do not depend on collision centrality. However, changing the equation of state together with the corresponding particlization temperature will require a modification of ϵ_0 and τ_0 parameters.² They are fixed in iHKM based on the measured multiplicity vs centrality distribution and the measured slope of the pion transverse momentum spectrum. As for the possible momentum anisotropy of the parton/gluon initial distribution, typical for the color-glass-condensate-based approaches, it is taken into account by the function $f_0(p)$ in Eq. (1), which is described in more detail in previous papers [14,16]:

$$f_0(p) = g \exp \left(-\sqrt{\frac{(p \cdot U)^2 - (p \cdot V)^2}{\lambda_\perp^2} + \frac{(p \cdot V)^2}{\lambda_\parallel^2}} \right), \quad (2)$$

where $U^\mu = (\cosh \eta, 0, 0, \sinh \eta)$ and $V^\mu = (\sinh \eta, 0, 0, \cosh \eta)$. In the rest frame of the fluid element one has $\eta = 0$, $(p \cdot U)^2 - (p \cdot V)^2 = p_\perp^2$, and $(p \cdot V)^2 = p_\parallel^2$, so that $\lambda_\parallel^2$ and $\lambda_\perp^2$ can be associated with the two effective temperatures: one along the beam axis and another along the axis orthogonal to it. In such a case the parameter $\Lambda = \lambda_\perp/\lambda_\parallel$ defines the momentum anisotropy of the initial state.

Once we have defined the initial conditions in the form of a nonthermal energy-momentum tensor, obtained from the distribution (1), we can proceed to the description of the prethermal matter dynamics, using the relaxation time approximation [14,16,17]. This pre-thermal stage starts in iHKM at the initial time $\tau_0 \approx 0.1$ fm/c, when the initial state is formed, and lasts till the thermalization time $\tau_{th} = 1$ fm/c,

when an approximate local thermal equilibrium is supposed to be reached by the initially nonequilibrated system.

The subsequent matter evolution is described within a relativistic viscous hydrodynamics formalism, with the relativistic current and energy-momentum tensor in the Israel-Stewart form. We neglect there the bulk viscosity and heat conductivity terms. Since at the LHC energies the baryonic chemical potential in the spatiotemporal region, where the midrapidity observables are formed, is negligibly small, we put it to be just zero. According to the iHKM results [14] for identified hadron multiplicities, spectra, elliptic flow, and femtoscopy data, we put the minimal possible ratio of the shear viscosity coefficient to the entropy density, $\frac{\eta}{s} = \frac{1}{4\pi}$ for the quark-gluon matter. The hydrodynamic approximation is justified as long as the matter can be considered remaining close to local chemical and thermal equilibrium. But at some temperature T_p both such quasi-equilibrium descriptions get destroyed, and the system's further evolution should be described in terms of particles. A switching to such a description can be done either gradually or suddenly at the T_p isotherm hypersurface. In this paper we utilize the latter mode of sharp particlization, comparing the simulation results in the cases of two different T_p values.³ The construction and treatment of the particlization hypersurface in iHKM is realized through the Cornelius routine [18].

The last stage of the system's evolution within iHKM is a hadronic cascade stage, described with the help of the UrQMD model [19]. At this stage all the particles, previously produced at the particlization stage collide and interact with each other; that includes both elastic scatterings and inelastic processes, such as baryon-antibaryon annihilation. The unstable particles and resonance states decay (and recombine) at this stage as well.

III. RESULTS AND DISCUSSION

In the current paper we present the results for different particle number ratios and spectra, calculated in iHKM at two different particlization temperatures, $T_p = 165$ MeV and $T_p = 156$ MeV, with two corresponding equations of state (EoS) for quark-gluon matter: the Laine-Schröder EoS [20] and the HotQCD Collaboration (HotQCD) EoS [21]. Using the two equations of state we investigate also whether the form of the EoS is significant for the description of particle number ratios in the evolutionary model with initial energy density $\epsilon(\tau_0)$ as a free parameter. Such a study is important since the extremely high rate of the fireball expansion, much larger than in the early Universe, would lead to modification of the effective equation of state as compared to the lattice QCD calculations for static system.

The ratios are calculated for central ($c = 0-10\%$) Pb+Pb collisions at the LHC energy $\sqrt{s_{NN}} = 2.76$ TeV. The results for particle p_T spectra are demonstrated for collisions with $c = 0-5\%$ and serve as an additional justification of the choice of model parameters (which do not depend on centrality).

²As for the parameter α , in the current analysis it is not changed when switching to another EoS.

³In fact, we suppose that the particlization temperature T_p coincides with the temperature when the hadronization process is (almost) completed.

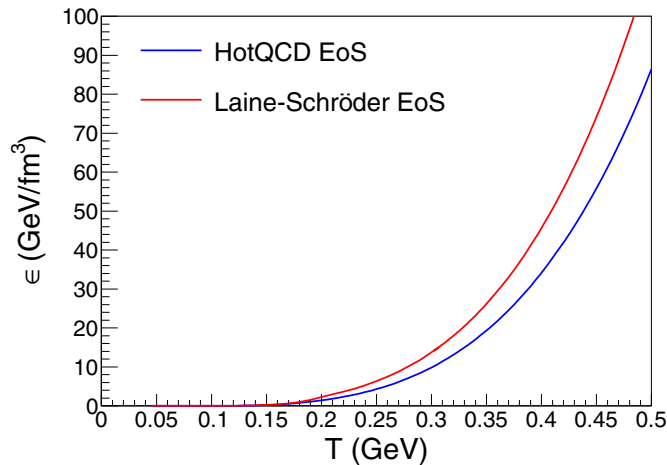


FIG. 1. A comparison of two equations of state for quark-gluon matter: the Laine-Schröder EoS [20], corresponding to the particlization temperature $T_p = 165$ MeV, and the HotQCD Collaboration “HotQCD” EoS [21], corresponding to the particlization temperature $T_p = 156$ MeV.

The Laine-Schröder equation of state was previously used in HKM, the predecessor of iHKM, as the lattice QCD inspired EoS, ensuring that the description of dense quark-gluon matter and its crossover-type transition to the hadron resonance gas pass without gaps in pressure and energy density. The corresponding particlization temperature $T_p = 165$ MeV was used in HKM calculations, that resulted in a successful simultaneous description of a variety of observables in heavy ion collision experiments at RHIC and LHC (spectra, interferometry radii, v_2 coefficients, source functions, etc. [10,22–25]). The “HotQCD” EoS corresponds to the recent HotQCD Collaboration results from lattice QCD simulations devoted to the quark-gluon matter state description. The respective particlization temperature, $T_p = 156$ MeV, is in agreement with the most recent estimates of the chemical freeze-out temperature obtained in the thermal model, $T_{ch} = 156 \pm 1.5$ MeV [26]. In Fig. 1 one can see a comparison of the two EoS on the plot in the coordinates (ϵ, T) . The Laine-Schröder EoS corresponds to more rapidly growing energy density at high temperatures.

The iHKM parameter values, used in the analysis in the case of $T_p = 165$ MeV, are chosen to be the same as those that have provided the optimal description of multiple LHC bulk observables [16]: $\tau_0 = 0.1$ fm/c, $\tau_{th} = 1$ fm/c, the relaxation time at the pre-thermal stage $\tau_{rel} = 0.25$ fm/c, $\epsilon_0 = 680$ GeV/fm³, $\alpha = 0.24$, and the momentum anisotropy of the initial state $\Lambda = 100$. For the new particlization temperature $T_p = 156$ MeV most parameter values remain the same, except for $\epsilon_0 = 495$ GeV/fm³ and $\tau_0 = 0.15$ fm/c, which are changed in order to ensure the correct charged particle multiplicity and pion p_T spectrum slope.

In Fig. 2 one can see a comparison of transverse momentum spectra calculated in iHKM for $c = 0$ –5% Pb+Pb collisions in the two mentioned regimes ($T_p = 165$ MeV and $T_p = 156$ MeV) together with the experimental points. At both particlization temperatures the model gives a sufficiently

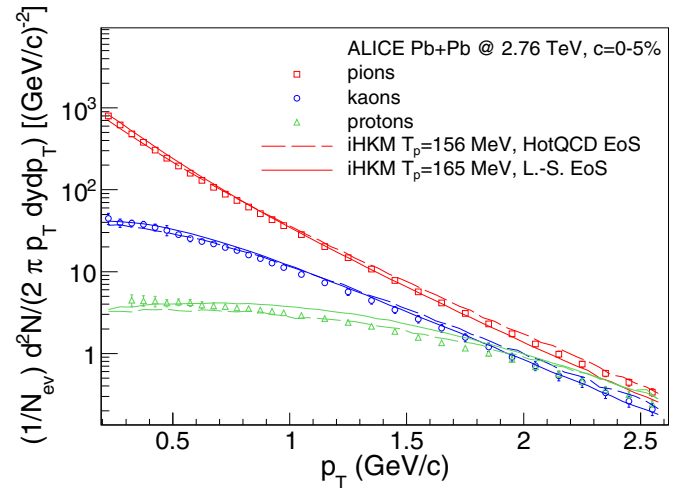


FIG. 2. The pion, kaon, and proton p_T spectra calculated in iHKM at two particlization temperatures, $T_p = 165$ MeV and $T_p = 156$ MeV and corresponding equations of state [20] and [21], compared with the ALICE experimental data [9] for central ($c = 0$ –5%) Pb+Pb collisions at the LHC energy $\sqrt{s_{NN}} = 2.76$ TeV.

good description of the data, which confirms that the model parameters are chosen correctly.

In Figs. 3 and 4 we demonstrate the iHKM results for a set of particle number ratios and compare them with the experimental results and those obtained from the thermal model [26,27]. Here the iHKM simulations are performed in two regimes: full calculation and the mode with the inelastic processes switched off (except for resonance decays). It is worth noting that the calculations without inelastic reactions, but with the initial conditions adjusted to provide the right description

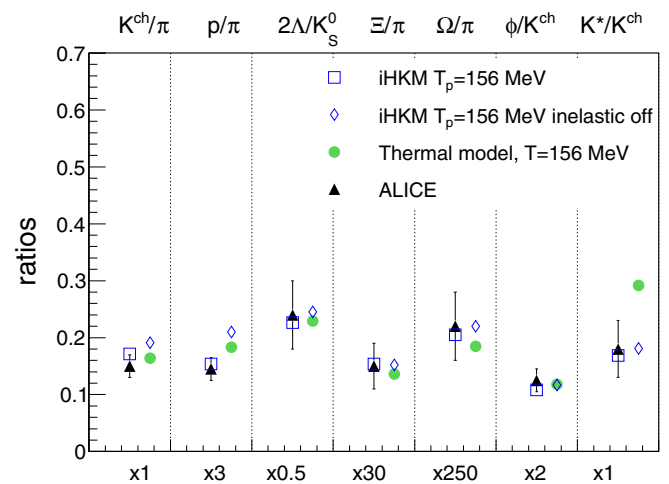


FIG. 3. A comparison of particle number ratios, calculated in iHKM (blue markers) at the particlization temperature $T_p = 156$ MeV and the HotQCD Collaboration equation of state, with the ALICE experimental data [28] and the thermal model results at $T = 156$ MeV [26]. The iHKM simulations are performed in two regimes: full calculation and the mode with inelastic reactions (except for resonance decays) switched off. The χ^2 values for these two regimes are 2.2 and 14.9 respectively.

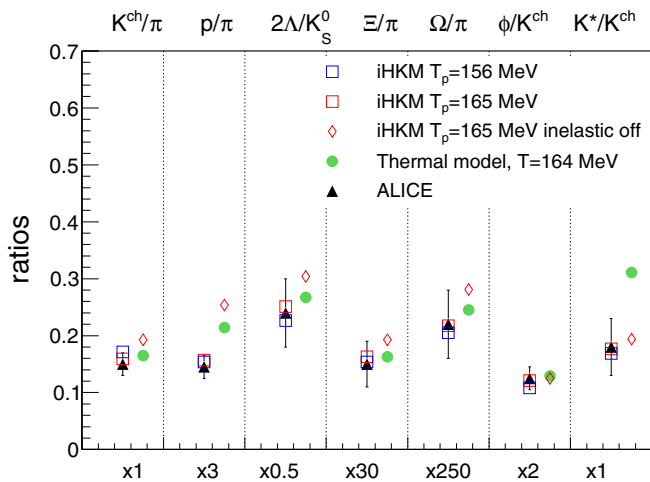


FIG. 4. The same as in Fig. 3, but the results for the iHKM calculation at $T_p = 165$ MeV and the Laine-Schröder equation of state are shown. The thermal model results are demonstrated for $T = 164$ MeV [26,27]. The χ^2 values for the full and “switched-off-inelastic” iHKM simulations are 0.7 and 37.7 respectively.

of the charged hadron multiplicities, give the same particle number ratios as without such an adjustment. This effect is clear: when we switch off the inelastic reactions (except for the resonance decays), then all the particle numbers on the hypersurface of the chemical freeze-out are proportional to the effective volume, $N_i = n_i(T, \mu) V_{\text{eff}}$, [29], which absorbs the hydro-velocities and space-time characteristics at the chemical freeze-out: $V_{\text{eff}} = \int_{\sigma_{\text{ch}}} u^\mu(x) d\sigma_\mu(x)$. The same happens with the similarly defined effective volume, related to the unity of rapidity in the case of boost invariance in the midrapidity region [29]. Therefore, when one fits the initial energy density (and the related initial time) in order to adjust multiplicity distribution at the artificially truncated “switched-off-inelastic” dynamics at the afterburner stage, the only common factor V_{eff} will be modified (EoS and corresponding particlization temperature are fixed). So, the particle number ratios will not change, no matter whether the initial conditions are retuned or not.

As one can see from the figures, the thermal model and the iHKM results, related to the case when the inelastic scatterings are switched off,⁴ are modified noticeably when the temperature T_p (or T_{ch} in the thermal models) is changing,

⁴Note that some deviation of iHKM results in this truncated case from those of the thermal model should be connected with the number of resonances taken into account. In the iHKM case we consider 329 types of resonances. As for the large deviation in the case of the K^*/K^{ch} ratio, it can be explained by different definitions of the K^*/K^{ch} ratio in the experiment and iHKM from the one side and the thermal model calculations from the other side. As follows from the experimental papers, e.g., [30], the $K^{*0}(892)$ resonances are reconstructed via the products of their decay into $K^+\pi^-$ pairs with branching ratio 0.66 (while the K^{*0} 's decaying through a channel $K^{*0} \rightarrow K^0\pi^0$ are excluded from the analysis). The same reconstruction procedure is applied in the iHKM study. Hence, the number of K^{*} 's identified in such a way is about 2/3 of the full K^{*}

and describe the data worse than the full iHKM calculations. As for the latter, they give very close results at both particlization temperatures and equations of state.

In a very recent paper [12] an essential influence of the particle rescatterings at the afterburner stage of the collision on the $K^{*}(892)$ resonance observability was shown. It means that the so-called thermal freeze-out is not sharp or sudden, but continuous. Our current study points out the dynamical continuous character of the so-called chemical freeze-out in relativistic heavy ion collisions. It demonstrates that accounting for inelastic processes at the afterburner stage of the collision plays a more important role in the correct description of experimental observables than the specific choice of the supposed particlization/hadronization temperature.

IV. CONCLUSIONS

The particle p_T spectra and particle number ratios calculated in iHKM with two different thermodynamic equations of state and corresponding particlization/hadronization temperatures demonstrate that a satisfactory description of the experimental data can be achieved at both T_p values if the initial energy density $\epsilon(\tau_0)$ is the free parameter. In this sense the results practically do not depend on the equation of state in complete dynamics of rapidly expanding fireballs formed in $A + A$ collision. However, the situation is different when one truncates the post-hydrodynamic stage: the description is better for lower temperature of chemical freeze-out, $T = 156$ MeV. But even in this case—when annihilation and other inelastic processes (except for the resonance decays) at the afterburner stage are neglected—the theoretical results get worse as compared to the full calculations. One can conclude that neither thermal nor chemical freeze-out can be considered to be sudden at some corresponding temperatures. Our analysis shows that even at the minimal hadronization temperature near 155 MeV, the annihilation and other nonelastic scattering reactions still play noticeable roles in the formation of particle number ratios, especially those where protons and pions are participating.

The fact that the results of the iHKM evolutionary model for small and relatively large particlization temperatures are quite similar means that inelastic processes (other than the resonance decays), which occur during the matter evolution below the corresponding temperature, play the role of a compensatory mechanism in formation of the particle number ratios.

Thus, the current analysis supports the picture of continuous chemical freeze-out at the LHC in the sense that the corrections to the sudden chemical freeze-out results, accounting for the inelastic reactions at the subsequent times, are important and improve the description of the experimental data.

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number. In contrast, the thermal model describes the full K^{*} number and therefore gives a higher K^{*}/K^{ch} ratio.

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1.2. М'які спостережувані та фемтоскопія у зіткненнях при енергії LHC $\sqrt{s_{NN}} = 5.02$ TeV

Bulk observables in Pb + Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV at the CERN Large Hadron Collider within the integrated hydrokinetic model

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The paper is devoted to the description and prediction of various bulk observables in the Pb + Pb collisions at the Large Hadron Collider energy $\sqrt{s_{NN}} = 5.02$ TeV within the integrated hydrokinetic model (iHKM). Sensitivity of the results to the choice of the appropriate model parameter values is also investigated. It is found that changing of the relaxation time and the rate of thermalization, which characterize the prethermal stage of the matter evolution, as well as switching to another equation of state at the hydrodynamic stage and the corresponding hadronization temperature, does not destroy the results if simultaneously one provides an appropriate adjusting of the initial time for the superdense matter formation and related maximal initial energy density.

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I. INTRODUCTION

The realistic simulation of a relativistic heavy-ion collision, which allows to describe or predict a wide variety of measured bulk observables, requires a complicated model consisting of several components, each of them describing a certain stage of the collision in the most appropriate approach.

According to this requirement, the well-known integrated hydrokinetic model (iHKM) model [1,2] consists of a set of modules/stages, corresponding to a series of successive phases of system's evolution process, including formation of superdense matter, its gradual thermalization, viscous relativistic hydrodynamic expansion as continuous medium, particlization, and the hadronic cascade stage. The “development” of the system at each stage is regulated by specific model parameters, and the aggregated effect of the matter's evolving during all the stages defines the final state of the system and the behavior of various observables, measured in the experiments.

Naturally, a researcher is interested in finding the connection between different parameter values and the simulation results in order to have the possibility to better understand the nature of the investigated processes and to discover the properties and characteristics of hot dense matter, basing on the experimental data.

In Refs. [1,2] the influence of different iHKM parameters (such as initial energy-density profile shape and its

momentum anisotropy, shear viscosity at the hydrodynamical stage etc.) on the obtained results for spectra, v_2 coefficients, interferometry radii, etc., was already studied in application to Pb + Pb collisions at the Large Hadron Collider (LHC) energy $\sqrt{s_{NN}} = 2.76$ TeV.

In the present paper we are aiming to investigate the dependence of our simulation results on such parameters as thermalization and relaxation times at the prethermal stage, τ_{th} and τ_{rel} , at the LHC energies, and the particlization/hadronization temperatures T_p , associated with the different equations of state, giving ourselves a task to predict/describe a set of bulk observables in the LHC Pb + Pb collisions at the higher energy $\sqrt{s_{NN}} = 5.02$ TeV.

II. RELAXATION RATE AND THERMALIZATION TIME IN THE LHC COLLISIONS WITHIN IHKM

The expansion of thermalized continuous medium near local thermal and chemical equilibrium is described in iHKM on the basis of the relativistic viscous hydrodynamics approximation in the Israel-Stewart formalism. This stage of the system's evolution is characterized by the shear viscosity parameter, η/s . As it was found in Ref. [1], this parameter for quark-gluon matter should have the value close to the minimal possible one, $1/4\pi \approx 0.08$. The initial conditions for hydrodynamical evolution of the system are formed during the prethermal stage, which models the transformation of initially nonthermalized system into thermalized one [2]. The dynamics of the system's expansion at the prethermal stage is described within the energy-momentum transport approach in relaxation time approximation [1].

At the initial proper time τ_0 the system's energy-momentum tensor $T_0^{\mu\nu}(x)$ has the nonhydrodynamic and non-thermal structure which is formed as discussed below. A gradual hydrodynamization of the system at the prethermal

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evolution stage is accompanied by the corresponding smooth transformation of the energy-momentum tensor, so that at the thermalization time τ_{th} it reaches the Israel-Stewart viscous hydrodynamics form $T_{\text{hydro}}^{\mu\nu}(x)$. At intermediate times, between τ_0 and τ_{th} , the system's energy-momentum tensor can be presented within the relaxation-time approximation for Boltzmann equation [3] as

$$T^{\mu\nu}(x) = T_{0,\text{free-evolving}}^{\mu\nu}(x)\mathcal{P}(\tau) + T_{\text{hydro}}^{\mu\nu}(x)[1 - \mathcal{P}(\tau)]. \quad (1)$$

The matter evolution is determined by the following equations:

$$\partial_\mu \{ [1 - \mathcal{P}(\tau)] T_{\text{hydro}}^{\mu\nu}(x) \} = -T_{0,\text{free-evolving}}^{\mu\nu}(x) \partial_\mu \mathcal{P}(\tau). \quad (2)$$

To arrive at the hydrodynamical tensor $T_{\text{hydro}}^{\mu\nu}(x)$ at $\tau = \tau_{\text{th}}$ one should require

$$\mathcal{P}(\tau_0) = 1, \quad \mathcal{P}(\tau_{\text{th}}) = 0, \quad \text{and} \quad \partial_\mu \mathcal{P}(\tau_{\text{th}}) = 0. \quad (3)$$

Equations (2) can be considered the equations of relativistic hydrodynamics for the modified energy-momentum tensor $\tilde{T}_{\text{hydro}}^{\mu\nu}(x) = [1 - \mathcal{P}(\tau)] T_{\text{hydro}}^{\mu\nu}(x)$ with a “source” term on the right-hand side and rescaled energy density $\tilde{\epsilon} = [1 - \mathcal{P}(\tau)]\epsilon$ and pressure $\tilde{p} = [1 - \mathcal{P}(\tau)]p$. Here we also assume that the relaxation time $\tau_{\text{rel}}(x, p)$ for each fluid element is very close to that in its rest frame, $\tau_{\text{rel}}^*(x, p)$, and depends only on the proper time τ , so that the weight function $\mathcal{P}(\tau, \mathbf{r}, p) = \exp[-\int_{\tau_0}^{\tau} 1/\tau_{\text{rel}}(\tau', \mathbf{r}, p) d\tau']$ also depends only on τ :

$$\mathcal{P}(\tau) = \exp \left[-\int_{\tau_0}^{\tau} \frac{1}{\tau_{\text{rel}}(\tau')} d\tau' \right]. \quad (4)$$

Such an approximation corresponds to the Bjorken picture [4], in which the processes of matter thermalization are synchronous in proper time of the fluid elements, and the system's thermalization completes simultaneously with the beginning of its hydrodynamic expansion at the proper time τ_{th} .

Assuming further $\tau_{\text{rel}}(\tau') = \tau_{\text{rel}}(\tau_0) \frac{\tau_{\text{th}} - \tau'}{\tau_{\text{th}} - \tau_0}$, and performing the integration in Eq. (4), one readily obtains $\mathcal{P}(\tau) = \left(\frac{\tau_{\text{th}} - \tau}{\tau_{\text{th}} - \tau_0} \right)^{\frac{\tau_{\text{th}} - \tau_0}{\tau_{\text{rel}}(\tau_0)}}$. The value of $\tau_{\text{rel}}(\tau_0) \equiv \tau_{\text{rel}}$ here is one of the iHKM parameters. From (3) it follows that one should have $(\tau_{\text{th}} - \tau_0)/\tau_{\text{rel}} > 1$.

Thus, the time parameters, determining the dynamics of the prethermal stage in iHKM, are the initial time τ_0 , relaxation time τ_{rel} , and thermalization time τ_{th} . The first one defines the proper time, when the initial energy-density profile of strongly interacting matter is formed after overlapping of the colliding nuclei, the second one is related to the rate of transformation of the system from the initial nonequilibrium state to the hydrodynamical one, and the third parameter defines the time when the system becomes nearly thermal.

The initial prethermal states are constructed with the help of MC Glauber GLISSANDO code [5] and are regulated by the following model parameters: $\epsilon_0(\tau_0)$ (the initial maximal energy density in the center of the system at the starting proper time τ_0), α (the parameter defining the proportion between the contributions from “binary collisions” and “wounded nucleons” models into the initial energy-density profile), and Λ [regulating the momentum anisotropy of the initial parton distribution function $f_0(p)$]. The latter function in iHKM is

chosen to have the following form, consistent with color glass condensate (CGC) approach:

$$f_0(p) = g \exp \left[-\sqrt{\frac{(p \cdot U)^2 - (p \cdot V)^2}{\lambda_\perp^2} + \frac{(p \cdot V)^2}{\lambda_\parallel^2}} \right], \quad (5)$$

where $U^\mu = (\cosh \eta, 0, 0, \sinh \eta)$, $V^\mu = (\sinh \eta, 0, 0, \cosh \eta)$. In the rest frame of the fluid element one has $\eta = 0$, $(p \cdot U)^2 - (p \cdot V)^2 = p_\perp^2$ and $(p \cdot V)^2 = p_\parallel^2$, so the parameters $\lambda_\parallel^2$ and $\lambda_\perp^2$ can be associated with the two temperatures—one along the beam axis and another, orthogonal to it, correspondingly. The parameter $\Lambda = \lambda_\perp/\lambda_\parallel$ thus defines the momentum anisotropy of the initial state.

The values of these initial state parameters are defined from the experimental data on final particle multiplicities and pion spectra. It turns out that α and Λ at the LHC remain the same at different energies of Pb + Pb collisions and various sets of other model parameters. The value $\Lambda = 100$, utilized in our study, corresponds to a very large momentum anisotropy of the initial state, typical for the CGC-based models.

After the prethermal and hydrodynamical stages, the particlization stage follows, since eventually the matter loses thermal and chemical equilibrium, the continuous medium description becomes inappropriate, and the system must be then considered as the set of hadrons. This stage is characterized by the particlization temperature T_p (close to the temperature of hadronization) which is determined by the QCD equation of state and defined as the temperature when the quark-gluon matter mostly transforms into the hadron-resonance gas.

At the last, hadronic cascade stage, the particles created during the particlization collide with each other, experiencing elastic and inelastic scatterings, and the resonance decays take place. This stage is simulated within UrQMD model [6]. Here one can allow or forbid certain decays (e.g., in order to reproduce the experimental feed-down treatment, etc.) and switch off some processes, like baryon-antibaryon annihilation, and so on.

In the article [1] it was determined that the behavior of bulk observables in iHKM depends strongly on the time of the initial state formation, τ_0 . As for the relaxation time τ_{rel} and thermalization time τ_{th} characterizing the intensity of the thermalization process, this issue remained investigated insufficiently. To clarify this dependence in the current study we compare the iHKM results on the particle p_T spectra at different τ_{rel} and τ_{th} , having fixed the initial time $\tau_0 = 0.1$ fm/c and simultaneously changing the maximal initial energy density $\epsilon_0(\tau_0)$.

First, we fix the relaxation time $\tau_{\text{rel}} = 0.25$ fm/c and compare p_T spectra for main particle species, obtained in the model at the two thermalization times $\tau_{\text{th}} = 1.0$ fm/c and $\tau_{\text{th}} = 1.5$ fm/c for Pb + Pb collisions at the LHC energy $\sqrt{s_{NN}} = 2.76$ TeV. To ensure the right reproduction of the mean charged-particle multiplicity in the model for all centralities in both cases, we use two different ϵ_0 values: $\epsilon_0 = 834$ GeV/fm³ for $\tau_{\text{th}} = 1.0$ fm/c and $\epsilon_0 = 681$ GeV/fm³ for $\tau_{\text{th}} = 1.5$ fm/c. The corresponding results almost coincide: The comparison can be seen in Fig. 1, where the iHKM pion, kaon, and proton transverse-momentum spectra are presented for the case of $c = 0$ –5% events together with the ALICE

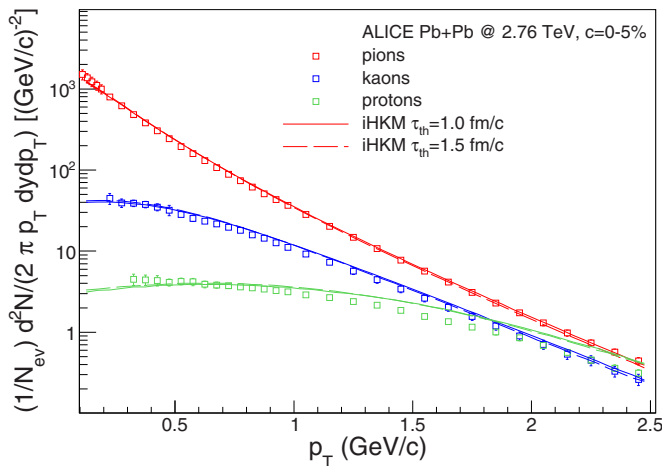


FIG. 1. The iHKM results on pion, kaon, and proton spectra in the Pb + Pb collisions with $c = 0-5\%$ at the LHC energy $\sqrt{s_{NN}} = 2.76$ TeV. The two different thermalization time values, $\tau_{th} = 1.0$ fm/c (solid lines) and $\tau_{th} = 1.5$ fm/c (dashed lines) were utilized. The experimental data from the ALICE Collaboration [7] are shown as square markers.

Collaboration [7] experimental data. One can notice that the slope of proton spectra obtained in iHKM is underestimated, which can be explained by strong collective flow in the model. It increases the effective temperature (inverse of the slope) of the spectra for all particle species; however, the spectra of massive particles are most sensitive to this effect, since the increase of effective temperature is roughly proportional to $mv^2/2$, where m is the particle mass and v is the root-mean-square flow velocity. Therefore the relatively high effective temperature for soft protons is, probably, due to contribution from slightly overestimated velocity gradient in the central part of the system. However, such a strong flow is necessary to correctly describe the interferometry radii in the model, as we found in the study devoted to the LHC energy 2.76 A TeV [1].

In the next step, we, on the contrary, fix the value of the thermalization time at $\tau_{th} = 1.5$ fm/c and calculate the spectra at the two values of the relaxation time, $\tau_{rel} = 0.25$ fm/c and $\tau_{rel} = 0.6$ fm/c (here we choose the second τ_{rel} value to be noticeably larger than the first one, but smaller than the time of thermalization). Again, we retune also the value of

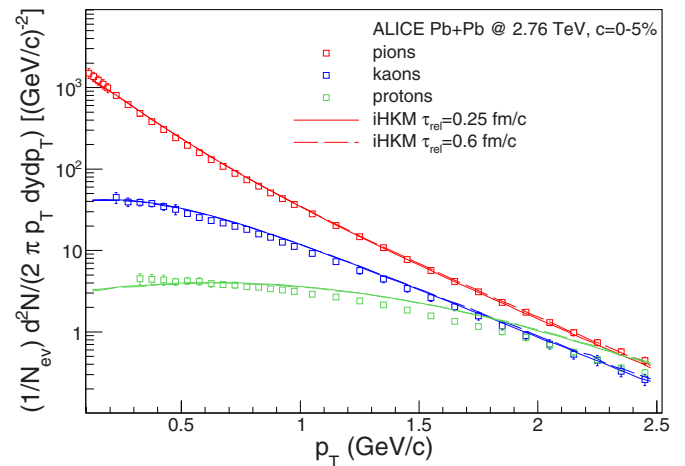


FIG. 2. The comparison of the iHKM results on pion, kaon, and proton spectra in the Pb + Pb collisions with $c = 0-5\%$ at the LHC energy $\sqrt{s_{NN}} = 2.76$ TeV. The two different relaxation times, $\tau_{rel} = 0.25$ fm/c (solid lines) and $\tau_{rel} = 0.60$ fm/c (dashed lines), were used. The thermalization time value in both cases was set to $\tau_{th} = 1.5$ fm/c. The experimental data measured by the ALICE Collaboration [7] are shown as square markers.

the maximal initial energy density, setting it to 630 GeV/fm³ for the case of $\tau_{rel} = 0.6$ fm/c. Figure 2 shows the compared results for the two τ_{rel} , again together with the experimental points.

Both comparisons of the results in Figs. 1 and 2 demonstrate that while the maximal initial energy density ϵ_0 remains a free parameter, the experimentally measured p_T spectra can be successfully described at different thermalization and relaxation times, characterizing the rate of the matter's thermalization process. This fact complicates the experimental study of the process of thermalization in heavy-ion collisions.

III. RESULTS FOR $\sqrt{s_{NN}} = 5.02$ TeV AND DISCUSSION

In this section we provide the systematic study of the wide class of bulk observables at the LHC energy $\sqrt{s_{NN}} = 5.02$ TeV within iHKM using the two equations of state (EoS) for quark-gluon phase with the two corresponding particleization temperatures T_p . In Ref. [8] for the LHC energy $\sqrt{s_{NN}} =$

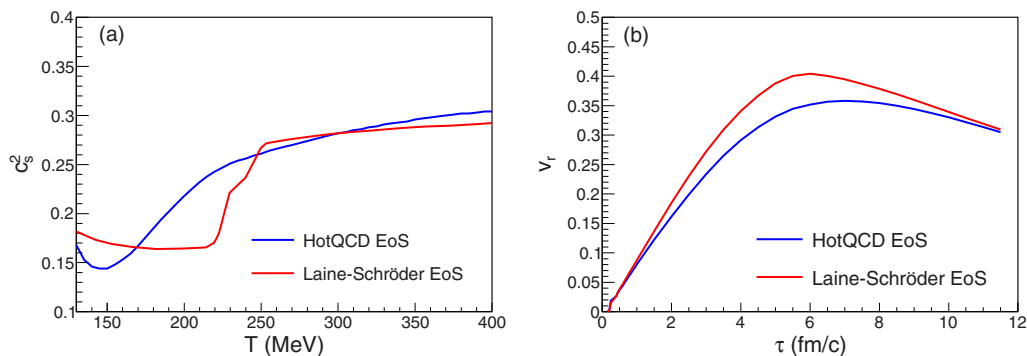


FIG. 3. The dependence of square of the speed of sound c_s^2 on the temperature T (a) and radial flow's v_r dependence on τ for $r_T = 3$ fm in iHKM (b) for Laine-Schroeder [9] and HotQCD Collaboration equations of state [10].

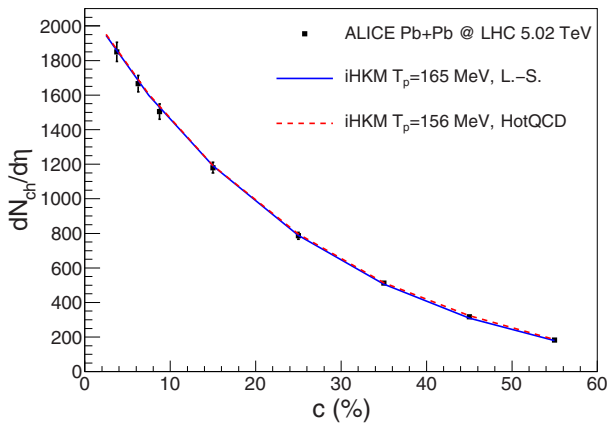


FIG. 4. The mean charged-particle multiplicity $dN_{ch}/d\eta$ dependence on collision centrality for Pb + Pb collisions at the LHC energy $\sqrt{s_{NN}} = 5.02$ TeV. The best iHKM descriptions at $T_p = 165$ MeV (with Laine-Schroeder EoS) and $T_p = 156$ MeV (with HotQCD Collaboration EoS) of the ALICE experimental data [11] are used to fix the main model parameters ϵ_0 and α in both cases.

2.76 TeV, a similar analysis was done only for particle number ratios. Here we are going to investigate whether the crucial observation made in Ref. [8], as for the possibility to get equally good data description at both EoS/ T_p 's if the $\epsilon_0(\tau_0)$ parameter is correspondingly adjusted, can be confirmed at higher energy and for all the observables, including not only particle yields, but also spectra, elliptic flow, femtoscopia radii, etc.

Thus, in this paper iHKM is for the first time applied to the study of collisions at the recently achieved highest LHC energy, 5.02 A TeV. At first, in order to adjust the model parameters to the description of Pb + Pb collisions at this new energy, we fitted the mean charged-particle multiplicity dependence on centrality and the slope of the pion p_T spectrum at the two chosen particlization temperatures and corresponding equations of state for the quark-gluon phase. We considered two cases: the particlization temperature $T_p = 165$ MeV with the Laine-Schroeder equation of state [9] and $T_p = 156$ MeV with the HotQCD Collaboration EoS [10]. For $T_p = 165$ MeV the best fit to multiplicity dependence corresponds to the value $\epsilon_0 = 1067$ GeV/fm³ at the initial time for the system formation $\tau_0 = 0.1$ fm/c, and for $T_p = 156$ MeV the values $\epsilon_0 = 870$ GeV/fm³ and $\tau_0 = 0.12$ fm/c are obtained from the fit. The different initial times τ_0 were used to optimize the description of pion transverse-momentum spectrum at relatively high $p_T > 1$ GeV/c. The parameter regulating the proportion between wounded nucleons and binary collisions model contributions to the initial energy density profile from GLISSANDO code [5] for both particlization temperatures is $\alpha = 0.24$ —the same as for the energy $\sqrt{s_{NN}} = 2.76$ TeV.

The rest of the model parameters—the thermalization time $\tau_{th} = 1$ fm/c, the relaxation time at the prethermal stage $\tau_{rel} = 0.25$ fm/c, and the momentum anisotropy of the initial state $\Lambda = 100$ —also have the values equal to those provided a successful description of Pb + Pb collision observables at $\sqrt{s_{NN}} = 2.76$ TeV [1,2].

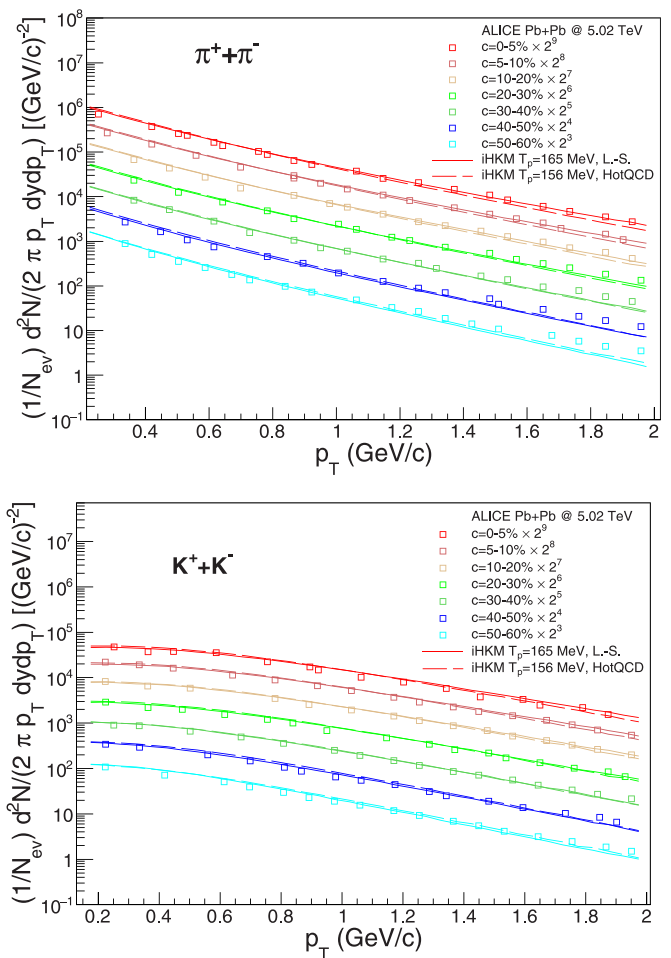


FIG. 5. Pion and kaon transverse-momentum spectra for different centrality classes obtained from iHKM at the two particlization temperatures and equations of state for quark-gluon phase in comparison with the preliminary experimental data from the ALICE Collaboration [12]. For better visibility the spectra points for different centralities are scaled by different powers of 2.

As one can see, the adjustment of iHKM to the description of collisions at new energy, 5.02 A TeV, is reached by natural changing of only one (!) parameter—the maximal initial energy density $\epsilon_0(\tau_0)$ —as compared to that used at the previous LHC energy, 2.76 A TeV. This fact looks natural in case of similarity of the space-time evolution and properties of the matter at different LHC energies, and in such a case it indicates the adequacy of the model for the theoretical description of these processes.

In Fig. 3 one can see the plots, demonstrating the square of the speed of sound dependencies on temperature for both considered equations of state and the corresponding radial flow $v_r(\tau)$ evolution curves, obtained in iHKM for $r_T = 3$ fm. One can see that despite the fact that the two EoS's lead to different values of c_s^2 near the phase transition region, and hence to different matter acceleration during the hydrodynamic stage, the radial flow magnitudes, developed to system's particlization time ($\tau \approx 10$ fm/c) are quite close in both cases. This is partially due to a larger flow, developed during the early evolution stage in case of Laine-Schroeder

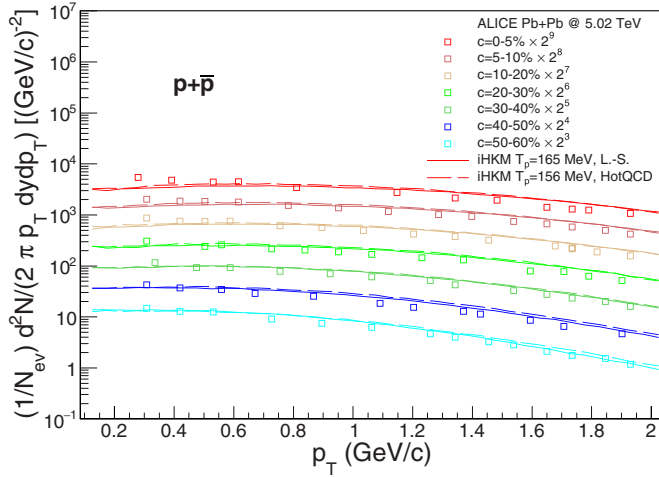


FIG. 6. The same as in Fig. 5 for protons.

EoS, that can be connected with earlier start and essentially larger gradient of pressure in this case. At the later times, around the temperature region $170 < T < 250$ MeV, much stiffer character of HotQCD Collaboration EoS compensates the tendencies of the earlier stage.

The mean charged-particle multiplicities at different collision centralities, obtained in iHKM at chosen parameter values for $\sqrt{s_{NN}} = 5.02$ TeV Pb + Pb collisions, in comparison to the experimental data are presented in Fig. 4. At both EoS/ T_p 's the model reproduces the experimental dependency quite well, so that the two model curves almost coincide.

The iHKM results on transverse-momentum spectra of pions, kaons, and protons for different centralities are shown in Figs. 5 and 6. The model points are compared to the preliminary experimental data on p_T spectra measurement, presented at Quark Matter 2017 [12]. As one can see, iHKM gives a satisfactory description of the particle production for the $\sqrt{s_{NN}} = 5.02$ TeV Pb + Pb collisions in both T_p /EoS modes. It is worth noting that the pion spectra in iHKM model are described within the experimental errors in a wide- p_T region including the soft momentum interval. Previously, a similar result was observed and emphasized in Ref. [2] for the LHC energy $\sqrt{s_{NN}} = 2.76$ TeV. Therefore, in iHKM there is no necessity to include a specific mechanism for soft pion radiation, for example, through the Bose-Einstein condensation [13], in order to describe the soft pion emission in Pb + Pb collisions at the LHC.

In addition, in Fig. 7 we demonstrate the iHKM results for mean transverse momentum of pions, kaons, and protons for seven centrality classes at the two collision energies, $\sqrt{s_{NN}} = 2.76$ TeV and $\sqrt{s_{NN}} = 5.02$ TeV, compared with the ALICE Collaboration experimental points [7,12,14]. We see that in the main iHKM points agree with the data, in particular the model reproduces a certain (up to 8–10%) increase of mean p_T values at 5.02 A TeV against 2.76 A TeV, although at 2.76 A TeV the model slightly overestimates pion $\langle p_T \rangle$ and proton $\langle p_T \rangle$ in central events, while kaon mean p_T are slightly underestimated at 5.02 A TeV in peripheral events.

In Fig. 8 one can see the p_T -integrated particle yields dN/dy for different species, calculated in the model in com-

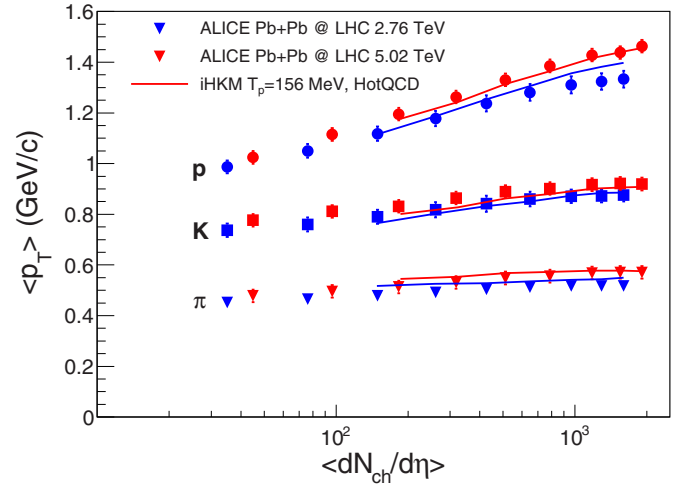


FIG. 7. The iHKM results (for $T_p = 156$ MeV and HotQCD Collaboration EoS) on pion, kaon, and proton mean p_T (lines) in Pb + Pb collisions at the two LHC energies— $\sqrt{s_{NN}} = 2.76$ TeV (blue) and $\sqrt{s_{NN}} = 5.02$ TeV (red)—for different centrality classes in comparison with the ALICE Collaboration experimental data [7,12,14] (markers).

parison with the preliminary experimental data, provided by the ALICE Collaboration and reported in the talk at Quark Matter 2018 [15].

In Fig. 9 we demonstrate the results of iHKM simulations for the various particle number ratios in central Pb + Pb collisions ($c = 0$ –10%) at the considered LHC energy again together with the preliminary experimental results from the ALICE Collaboration [15], obtained by division of the corresponding experimental particle yields shown in Fig. 8. Here the model ratios obtained from the full iHKM calculations are presented along with those calculated in the mode with inelastic reactions turned off. As one can see, while the results

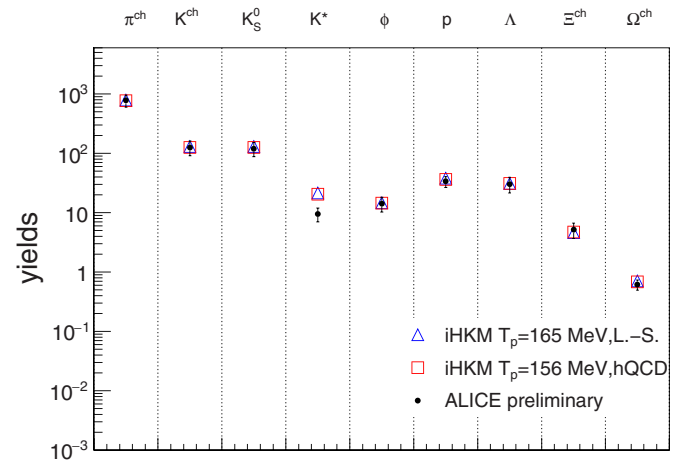


FIG. 8. Particle yields dN/dy in 5.02-A TeV events with $c = 0$ –10% calculated in iHKM at the two particleization temperatures and equations of state ($T_p = 165$ MeV with Laine-Schroeder EoS and $T_p = 156$ MeV with HotQCD Collaboration EoS) compared to the preliminary ALICE data [15].

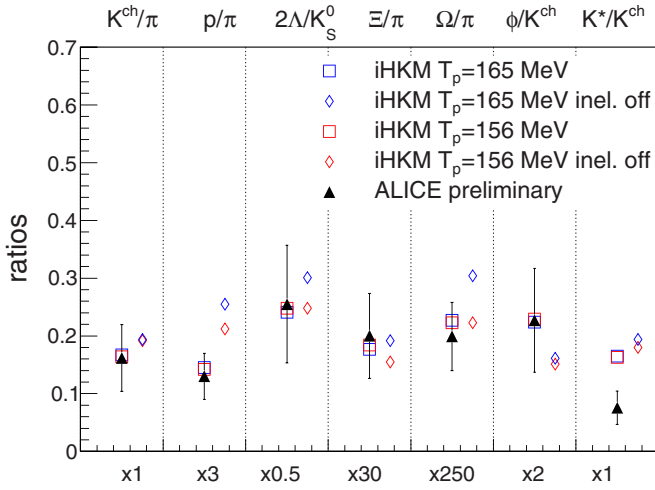


FIG. 9. The iHKM results for the particle number ratios calculated at the two particlization temperatures and quark-gluon matter equations of state ($T_p = 165$ MeV with Laine-Schroeder EoS and $T_p = 156$ MeV with HotQCD Collaboration EoS) for 5.02 A TeV collisions with $c = 0$ –10% compared to the preliminary ALICE data [15]. The simulations are performed in two regimes: the full one and the one with inelastic processes turned off.

for certain ratios in the latter “reduced” mode can noticeably differ for the two particlization temperatures, the full calculation always gives very close values in both cases. This peculiarity in results is in accordance with that observed and discussed in our previous work [8]. The compensatory mechanism, eliminating in the “full” mode the difference, observed between the ratios at the two EoS/ T_p ’s in the “reduced” mode, is associated with inelastic reactions in the hadronic cascade, which last longer at a higher particlization temperature. The full mode iHKM results are in good agreement with the data, except for the K^*/K ratio. The latter detail is unexpected and needs specific discussion.

Turning back to the Fig. 8, one sees that the latter discrepancy is due to overestimation of the K^* yield in the model—its value is approximately twice higher in iHKM than in the experiment. This fact looks strange, since in our

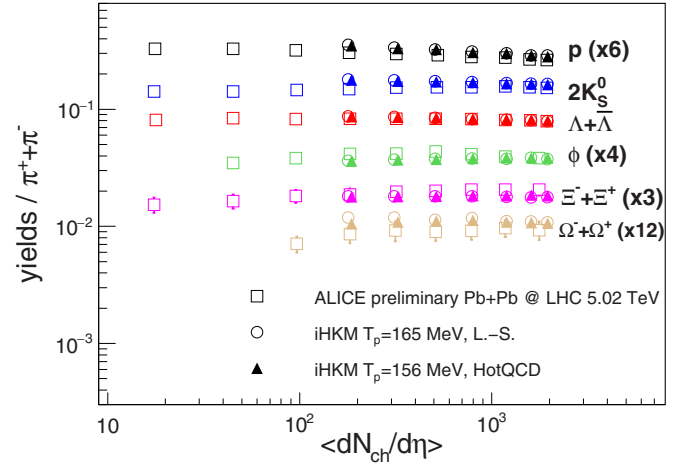


FIG. 11. The iHKM results on the ratios of different particle yields to the pion yield at different collision centralities for the LHC Pb + Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV together with the preliminary ALICE Collaboration data [17].

recent study [8], focused on the particle production in the LHC Pb + Pb collisions at lower energy, $\sqrt{s_{NN}} = 2.76$ TeV, we obtained a good agreement between model K^* yield (and K^*/K ratio) description and the corresponding ALICE data. Note that in our simulations we accounted also for possible problems with the K^* identification through the products of its decay into kaons and pions, which could be caused by the interaction of these daughter particles with the surrounding hadronic medium. This issue was analyzed in detail in another our paper [16]. There we considered the case of Pb + Pb collisions at the LHC at $\sqrt{s_{NN}} = 2.76$ TeV and performed the simulations in iHKM, applying the experimental procedure of K^* identification, i.e., selecting the $K\pi$ pairs, emitted from the close space points and having the specific invariant mass, corresponding to $K^*(892)$ resonance. Then we compared in such a way obtained K^* yields with the full numbers of K^* , generated in the model during the hadronization of continuous quark-gluon medium. We found that the observed number of K^* resonances, reconstructed via the products of their decays, for the most central events can be up to 20% less

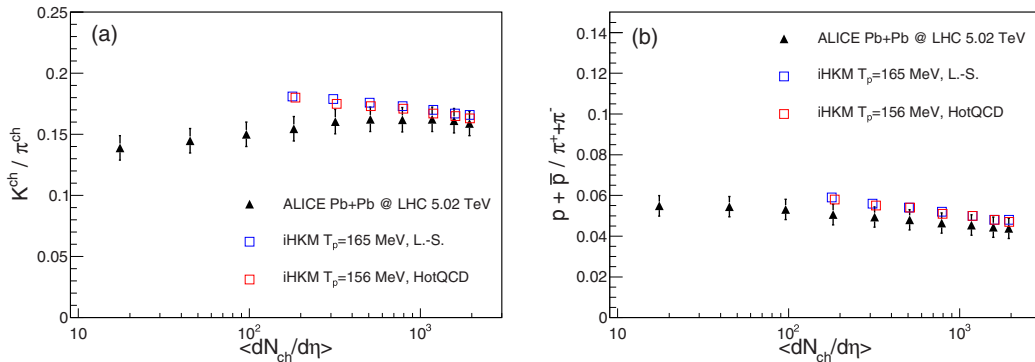


FIG. 10. The iHKM results for the K/π (a) and p/π (b) particle number ratios’ dependence on mean charged-particle multiplicity calculated at the two particlization temperatures and equations of state ($T_p = 165$ MeV with Laine-Schroeder EoS and $T_p = 156$ MeV with HotQCD Collaboration EoS) and the corresponding experimental data [12] for $\sqrt{s_{NN}} = 5.02$ TeV.

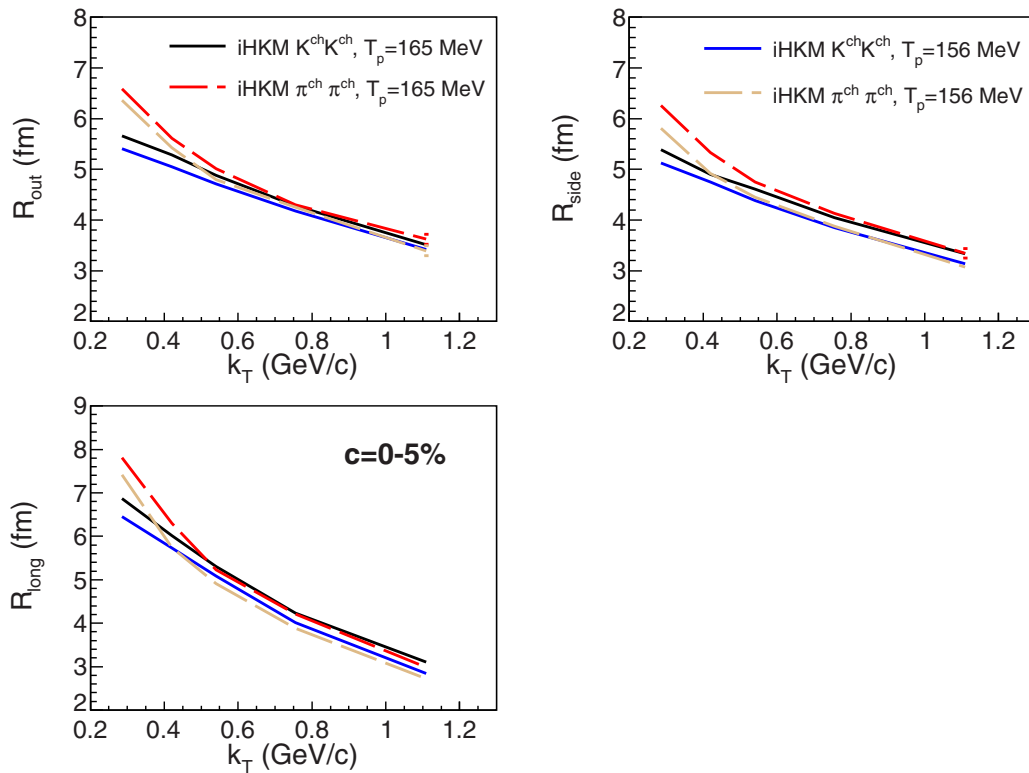


FIG. 12. The iHKM prediction for the charged pion and kaon interferometry radii k_T dependence in the LHC Pb + Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV, $c = 0-5\%$. The calculations were performed at the two particlization temperatures and equations of state for quark-gluon matter ($T_p = 165$ MeV with Laine-Schroeder EoS and $T_p = 156$ MeV with HotQCD Collaboration EoS).

than the actual number of “primary” K^* ’s. But it is worth noting that the mentioned reduction is a result of interplay of two opposite processes—the scattering of daughter K and π , resulting in an impossibility to detect the respective parent K^* ’s (this affects up to 70% of direct resonances), and the recombination of $K\pi$ pairs, producing additional K^* resonances (this mechanism can give up to 50% of the initial number of direct K^* ’s). Both these processes in iHKM are simulated at its final “afterburner” stage within the UrQMD model.

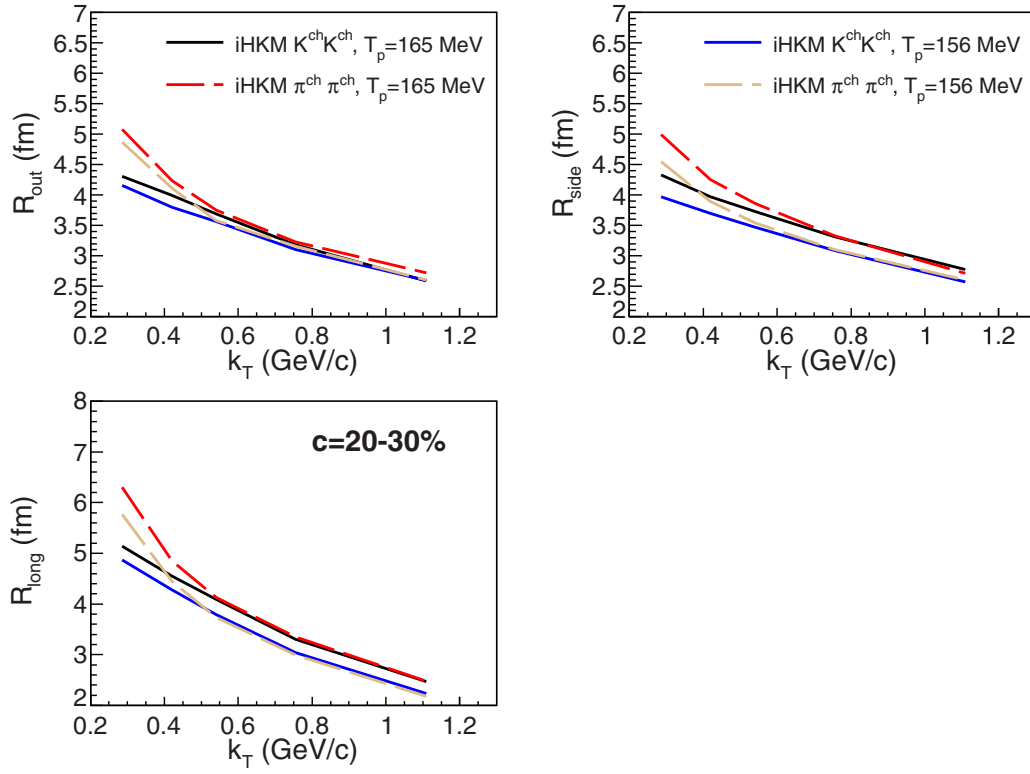
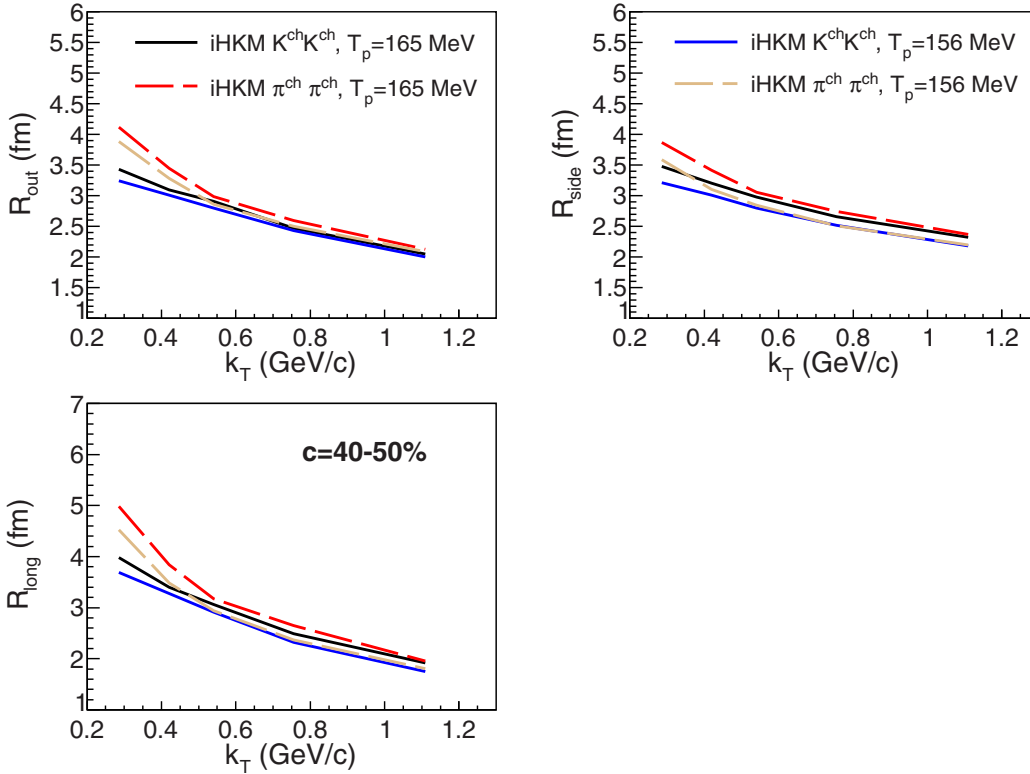
What is interesting, in the present study we apply the same K^* restoration procedure, but this time obtain a higher dN/dy , than in the data (note also that this preliminary experimental dN/dy value for K^* at $\sqrt{s_{NN}} = 5.02$ TeV is noticeably smaller than that at $\sqrt{s_{NN}} = 2.76$ TeV, while the iHKM results at both collision energies are close). The reason for a low value of K^* yield in the preliminary experimental results is unclear and needs further investigation. It can be possibly explained by the identification problems in the experiment and one cannot exclude that the dN/dy value for K^* can be corrected in the final version of ALICE data.

For the K/π and p/π ratios we have also studied within iHKM their dependence on the mean charged-particle multiplicity for different centrality classes and compared the simulation results to the preliminary experimental data, presented in Ref. [12] (see Fig. 10). In the plot one can see that the model describes well both ratios’ behavior in not-far-from-central

events at both EoS/ T_p ’s.¹ An equally good agreement of iHKM results with the data can be observed in Fig. 11, where the model calculations of the ratios of various particle species yields to the pion yield are compared to the preliminary experimental points from the ALICE Collaboration [17].

In Figs. 12–14 we demonstrate our predictions for the 5.02 A TeV pion and kaon interferometry radii R_{out} , R_{side} , and R_{long} measured at different mean pair transverse momentum k_T . The predictions are provided for the three centrality classes: $c = 0-5\%$, $c = 20-30\%$, and $c = 40-50\%$. The radii behavior for each particle species looks regular and quite similar to that observed at lower LHC energy, 2.76 A TeV (see comparison for $c = 0-5\%$ in Fig. 15). The radii’s absolute values are close at both energies (some values at 5.02 A TeV are slightly higher, about 3–5%) and, as expected, show a decrease when going from central to noncentral events. The similarity of the radii at both LHC energies, observed despite the different geometric sizes of the systems at freeze-out stage, is conditioned by higher velocity gradients at larger energies that compensate the geometric effects and lead to closeness of the homogeneity lengths in both cases [18].

¹Here the “full regime” calculations are implied. The theoretical values obtained in the “reduced regime” without inelastic scatterings are not expected to describe the measured data. Their calculated values are fairly larger than the measured ones and for this reason are not shown on the plots in Figs. 10 and 11.

FIG. 13. The same as in Fig. 12 for the centrality $c = 20-30\%$.FIG. 14. The same as in previous two figures for the centrality $c = 40-50\%$.

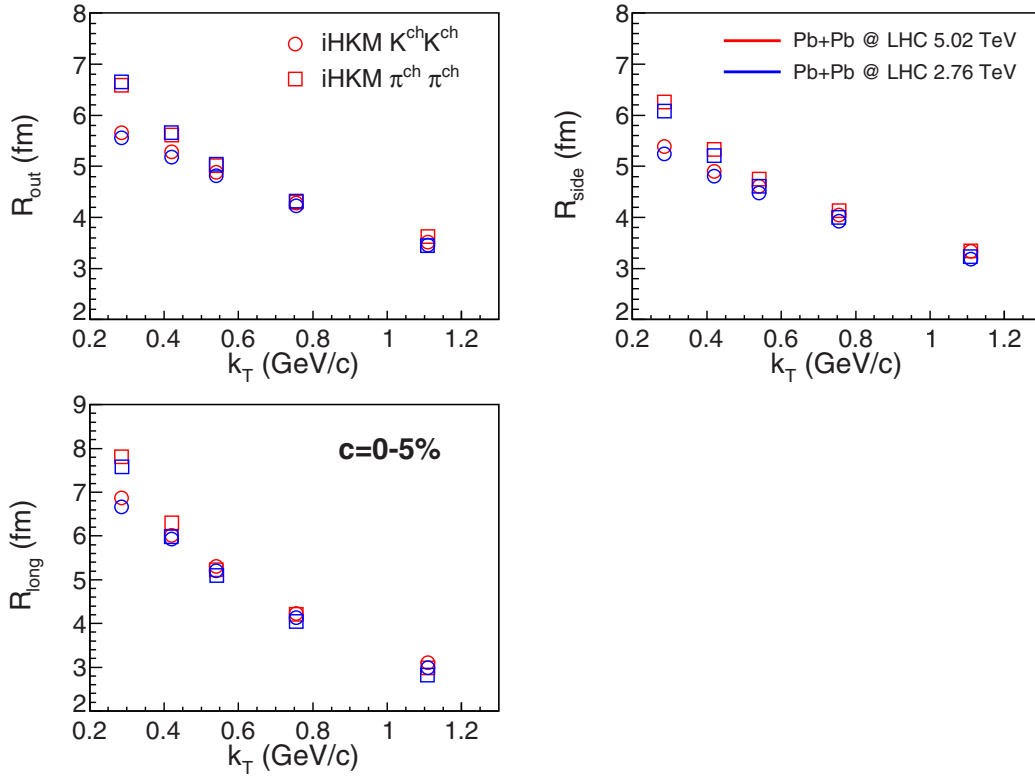


FIG. 15. The comparison of iHKM results ($T_p = 165$ MeV, Laine-Schroeder EoS) for pion and kaon interferometry radii at the two LHC energies, $\sqrt{s_{NN}} = 2.76$ TeV and $\sqrt{s_{NN}} = 5.02$ TeV, centrality $c = 0-5\%$.

Comparing kaon and pion points, one can see the approximate k_T scaling between two dependencies at $k_T > 0.4$ GeV/c, repeating the behavior observed for the collisions at $\sqrt{s_{NN}} = 2.76$ TeV and noticed in iHKM simulations [19] prior to the experimental confirmation in Ref. [20].

In Figs. 16 and 17 for the two centrality classes one can see the comparison of iHKM results on the all charged particles flow harmonics $v_2(p_T)$, $v_3(p_T)$, and $v_4(p_T)$ with the ALICE experimental data for $\sqrt{s_{NN}} = 5.02$ TeV [21]. Note, however, that while in the ALICE paper $v_n(p_T)$ were calculated using the two-particle cumulant method, in our analysis we use the standard event-plane method (see, e.g., Refs. [22,23]).

Following the event-plane method procedure, we at first calculate for each analyzed event the event-plane angles Ψ_n , using their relations with the event flow vectors $\mathbf{Q}_n$:

$$Q_n \cos(n\Psi_n) = \sum_i w_i \cos(n\phi_i), \quad (6)$$

$$Q_n \sin(n\Psi_n) = \sum_i w_i \sin(n\phi_i), \quad (7)$$

which give us

$$\Psi_n = \frac{1}{n} \tan^{-1} \frac{\sum_i w_i \sin(n\phi_i)}{\sum_i w_i \cos(n\phi_i)}. \quad (8)$$

Here i numerates the analyzed particles (we select particles with p_T from the range $0.2 < p_T < 2.4$ GeV/c for the analysis), and w_i is the weight of i th particle. In our study w_i are chosen to equal the particle transverse momentum $p_{T,i}$.

The flow harmonics $v_n(p_T)$ are calculated as follows:

$$v_n(p_T) = \langle \cos[n(\phi_i - \Psi_n)] \rangle / R_n, \quad (9)$$

where angle brackets mean the averaging over all the analyzed particles in considered p_T bin and $R_n = \langle \cos[n(\Psi_n - \Psi_{RP})] \rangle$

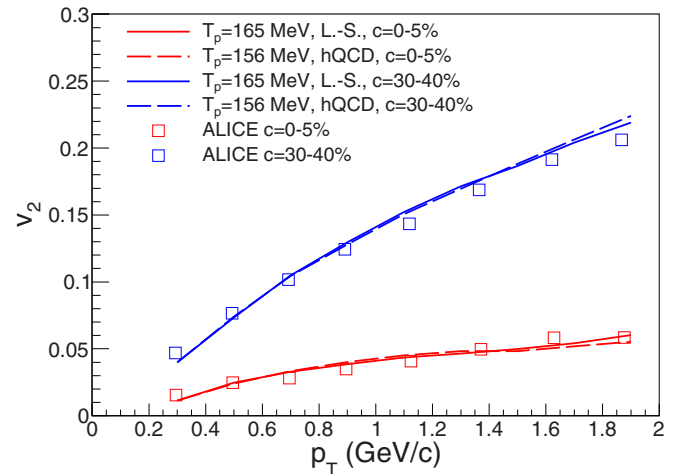
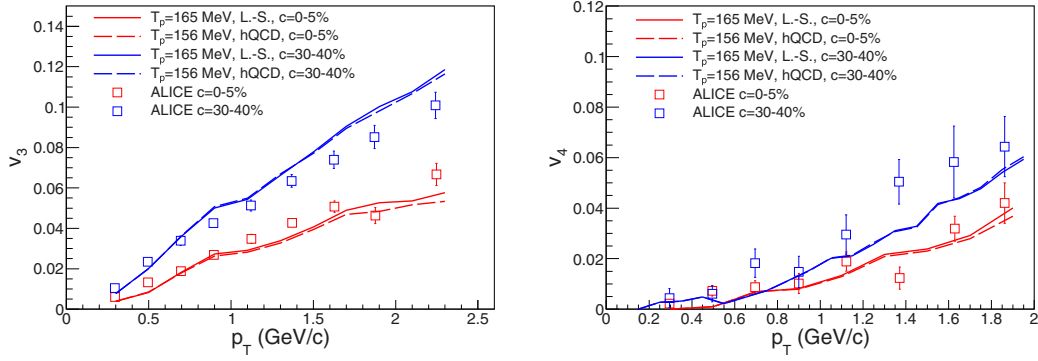


FIG. 16. The iHKM results on the elliptic flow v_2 dependence on p_T for all charged particles together with the corresponding ALICE data [21] for the centrality classes $c = 0-5\%$ and $c = 30-40\%$. The model curves for the two particleization temperatures and the corresponding equations of state ($T_p = 165$ MeV with Laine-Schroeder EoS and $T_p = 156$ MeV with HotQCD Collaboration EoS) are presented.

FIG. 17. The same as in Fig. 16 for v_3 and v_4 flow harmonics.

is the event-plane resolution, with Ψ_{RP} being the reaction plane angle. The event-plane resolution can be expressed through the resolution parameter χ_n as [22]

$$R_n(\chi_n) = \frac{\sqrt{\pi}}{2\sqrt{2}} \chi_n \exp\left(-\chi_n^2/4\right) [I_0(\chi_n^2/4) + I_1(\chi_n^2/4)]. \quad (10)$$

Here I_k denotes the modified Bessel function of order k and $\chi_n = v_n/\sigma$, where σ characterizes the variance of $n(\Psi_n - \Psi_{RP})$ distribution. To estimate the event-plane resolution we divide each event into two subevents with equal multiplicities and calculate the subevent value $R_n^{\text{sub}} = \sqrt{\langle \cos[n(\Psi_n^A - \Psi_n^B)] \rangle}$. After that we numerically solve (10) with respect to χ_n and obtain the subevent resolution parameter χ_n^{sub} . The full event-plane resolution R_n is then calculated using (10) as $R_n(\sqrt{2}\chi_n^{\text{sub}})$, since the particle number in full event is twice bigger than that in each subevent and χ_n is proportional to the square root of particle number.

Examining the iHKM results, presented in Figs. 16 and 17, one can notice that the model curves for the two equations of state and particlization temperatures almost coincide. The v_2 iHKM points are in good agreement with data for both centralities. As for v_3 and v_4 , the iHKM describes well the central collision measurements, and for the $c = 30\text{--}40\%$ model gives a satisfactory data description only at $p_T < 1$ GeV/c. The statistical errors for all the presented theoretical $v_n(p_T)$ points are less than 2%. The results for the LHC energy $\sqrt{s_{NN}} = 2.76$ TeV are quite close to those for $\sqrt{s_{NN}} = 5.02$ TeV, especially in the “soft” p_T region, $p_T < 2.5$ GeV/c, both in the experiment and in the model calculations, so we demonstrate here only the 5.02-A TeV results.

As one can see, the iHKM results on all the considered bulk observables at the two particlization temperatures and corresponding equations of state are close to each other, which means that when constructing the model for a high-energy collision process, one has a certain freedom in choosing the equation of state, particlization temperature T_p , and maximal initial energy density $\epsilon_0(\tau_0)$, since different combinations of these parameter choices give similar results. This also means that experimentally one is hardly able to strictly define each of these parameters separately basing on the considered observables.

Thus, the obtained results for Pb + Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV allow us to confirm and extend an impor-

tant conclusion, made in Ref. [8] basing only on the analysis of particle number ratios at $\sqrt{s_{NN}} = 2.76$ TeV—the multiple hadronic rescatterings at the final stage of the collision play a vital role in formation of all the bulk observables and compensate the changes introduced to the system’s evolution by switching to another EoS for (quasi)thermalized matter, used at the hydrodynamic stage, and another hadronization temperature.

IV. CONCLUSIONS

The theoretical predictions and the description of the preliminary experimental results for bulk observables in Pb + Pb collisions at the LHC energy $\sqrt{s_{NN}} = 5.02$ TeV within iHKM model are presented. The model parameters are the same as at simulations of 2.76-A TeV Pb + Pb collisions, except for the maximal initial energy density $\epsilon_0(\tau_0)$, appearing to be the only parameter that needed retuning. This justifies the utilization of iHKM for the description of high-energy heavy-ion collisions and confirms that it provides an adequate approach to the simulation of such processes.

The systematic study of a large set of bulk observables (particle yields, particle number ratios, p_T spectra, v_n coefficients, femtoscopy radii) was performed using the two different particlization temperatures associated with two corresponding equations of state for quark-gluon phase. The comparison of the obtained results in these cases shows that both T_p /EoS can provide an equally good description of measured data and quite close predictions for not-yet-measured observables, if having changed the T_p and EoS, one simultaneously readjusts the maximal initial energy density parameter $\epsilon_0(\tau_0)$.

This result confirms our previous observation [8] for particle number ratios in Pb + Pb collisions at lower LHC energy $\sqrt{s_{NN}} = 2.76$ TeV and supports the conclusion about the great importance of the final afterburner stage of the collision and intensive hadron interactions, taking place at this stage, for the formation of the observed results. It means, however, that the matter EoS at the hydrodynamic stage and the particlization temperature cannot be strictly constrained by the experimental data.

Among the obtained results one can highlight an unexpectedly low value of $K^*(892)$ yield in the preliminary experimental data, as compared to the iHKM result and the

previous ALICE result for $\sqrt{s_{NN}} = 2.76$ TeV collisions. We associate this discrepancy with imperfection of preliminary experimental data analysis and believe that eventually it can likely be eliminated in the final version of ALICE data.

Also, the simulation results' dependence on such characteristics of the prethermal stage of matter evolution as the thermalization time τ_{th} and relaxation time τ_{rel} is investigated for the LHC energy 2.76 A TeV. In this article we demonstrate that varying these parameters in quite wide ranges (1–1.5) fm/c for τ_{th} and (0.25–0.6) fm/c for τ_{rel} accompanied again by the corresponding change of $\epsilon_0(\tau_0)$ does not influence the final particle p_T spectra. Thus, one may assume that some important characteristics of the matter and its evolution in A + A collisions cannot be unambiguously extracted from the experimental results if the maximal initial energy density (or—in other than iHKM kinds of approaches—a combination of some other model parameters that set the initial energy)

can be considered an (effective) free parameter. The situation could become more clear when the dynamical mechanism of loosely bound objects formation [24] in nuclear collisions becomes clarified and incorporated into evolutionary models like iHKM.

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1.3. Фемтоскопія надм'яких піонів у зіткненнях на LHC

Femtoscopic analysis of ultrasoft pion trap at energies available at the CERN Large Hadron Collider

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Femtoscopic studies of pion radiation in heavy-ion collisions have been conducted extensively at all available collider energies, both theoretically and experimentally. In all these studies special attention is given to m_T dependency of pion femtoscopic radii, usually approximated by a power-law function at transverse momenta above 200 MeV/c. However, the radii behavior has been much less explored for the ultrasoft pions, possessing transverse momentum comparable to or lower than the pion mass. For many experimental setups this region is difficult to measure. In this work we present theoretical calculations of pion emission in the ultrasoft region in the two hybrid models iHKM and LHYQUID+THERMINATOR2. Along with the particle transverse momentum spectra, we present the calculated femtoscopic radii, in both one-dimensional and three-dimensional representations. We investigate the radii dependence on pair m_T and observe, in particular, a departure from the power-law behavior at ultrasoft momenta, potentially reflecting a decoupling of such slow pions from the rest of collectively expanding system. We provide the theoretical interpretation of this result and discuss its significance, in particular, for the ongoing nonidentical particle femtoscopic analysis for pairs consisting of a pion and a baryon (or of a pion and a charmed meson).

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I. INTRODUCTION

The interferometry of particles emitted from small radiating source enables one to study the evolving geometry of that source. The particle interferometry originated from the experimental paper [1] and theoretical works [2,3]. In the latter it was also found that the results for the quantum statistics correlation effect for two close sources of identical particle emission in collision physics are formally similar to those for classical intensity correlations of the light radiation coming from different parts of a star, known in astronomy as the Hanbury-Brown–Twiss (HBT) effect [4–6]. The similarity, as well as the difference between the quantum correlations in identical-particle pairs and the classical HBT effect are analyzed, e.g., in [7]. Note that the correlation femtoscopic method, initially developed for finding the overall *size* of identical quantum particles' source, now is considered to measure the *homogeneity lengths* [8] of such femtometer -small sources. This general interpretation is especially important for the studies of the space-time structure of expanding sources, such as those created in proton/antiproton and heavy-ion collisions [1–3,9–14].

The homogeneity lengths, defined as a result of correlation femtoscopic analysis, reflect the spatial dimensions of the

region within the entire strongly interacting system formed in the collision, from which particles are emitted with similar velocities (having close, nearly coinciding values and directions) [8,15]. Gaussian parametrizations for the particle emission source function (SF) and the two-particle correlation function (CF) are usually used in femtoscopic studies, although the realistic source shape differs from a perfect Gaussian; e.g., resonance decay contributions cause exponential behavior near the peak of the correlation function. The conventional use of the Gaussian shape allows one to standardize the description of experimental data and easily compare the results of different femtoscopic measurements, as well as to interpret the obtained radii as the homogeneity lengths. It is also well motivated experimentally in heavy-ion collisions, where correlation shapes for pions in three-dimensional (3D) analyses are universally observed to be Gaussian in a wide range of centralities and pair transverse momenta. The study of the dependence of the radii on the particle species, collision type, and collision energy is the main objective of the femtoscopic analysis of heavy-ion collisions.

In experimental studies, one clearly observes certain types of universal scaling behavior for the measured femtoscopic scales. For a given colliding system and collision energy a linear scaling of the femtoscopic radii is universally observed versus the cube root of the final state mean particle multiplicity, $\langle dN_{ch}/d\eta \rangle^{1/3}$ [16–22]. A similar scaling across collision energies and colliding systems is only approximate [23,24]. The radii versus the pair transverse momentum exhibit a power-law-like scaling in pair transverse mass m_T ($m_T = \sqrt{k_T^2 + m^2}$, where k_T is pair mean transverse momentum, $k_T = |\mathbf{p}_{T1} + \mathbf{p}_{T2}|/2$, and m is particle mass) [16,17,25–30].

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In heavy-ion collisions both these scalings are predicted by hydrodynamic models [31,32]. Specifically, the m_T scaling is explained as a direct consequence and one of the main signatures of the collective radial flow of the system.

The observed m_T power-law scaling means a decrease of the radii with growing m_T that is a signature of hydrodynamic collectivity, typical for all particle species affected by the same flow field. Such dependence can be helpful for the prediction of the source size at certain m_T through the interpolation between radii measured at different m_T values. Nonetheless, the m_T range of experimental radii measurement depends on the acceptance of the detectors used in the study, and the very-low- m_T region has not been reached yet via such measurements. Therefore, the femtoscopy -scale predictions in that region can come only from extrapolation, which, following the commonly used power-law function, shows a very rapid growth of the radii with decreasing m_T . The theoretical studies, by contrast, do not face such problems, and so simulations within a realistic collision model can help estimate the possible character of the femtoscopy radii behavior in the ultrasoft momentum region.

The knowledge of femtosopic radii for ultrasoft pions is becoming increasingly important in the context of rapidly developing experimental analyses of femtosopic correlations of nonidentical particles [33,34]. In the case of such pairs the correlation occurs between two particles with similar velocity. If the pair in question contains a pion and a relatively massive particle (e.g., a deuteron or a charmed meson), then the pion in the pair needs to be ultrasoft in order to be correlated. The femtosopic radius of the pair is directly related to the size of emission regions of both particles in the pair. Therefore, in order to study the non-identical correlations between pions and massive particles, the knowledge of femtosopic radii of ultrasoft pions becomes essential.

Several previous theoretical studies of ultralow momentum pions were motivated by the experimental evidence of enhanced production [35,36]. These observations could be addressed by nonequilibrium models [37]. In addition to the semiclassical models, pure quantum effects such as pion condensation and Bose-Einstein enhancement were also explored [38]. However, none of these studies addressed the pion femtosopic radii nor their scaling at very low transverse momenta.

Thus, in this paper, we focus on studying the pion femtosopic radii dependence on pair m_T , including the region of very low transverse masses, in Pb-Pb collisions at the Large Hadron Collider (LHC) energy $\sqrt{s_{NN}} = 5.02$ TeV simulated within the integrated hydrokinetic Model (iHKM) [31,39,40] and the hydrodynamics model [41] coupled to the statistical hadronization code THERMINATOR2 (LHYQUID + THERMINATOR2) [42]; LQTH.

II. MODELS' DESCRIPTION

The collisions of heavy ions at ultrarelativistic energies allow to create quark-gluon plasma (QGP): a new state of strongly interacting matter, where quarks and gluons are no longer confined within individual nucleons. The evolution of such matter can be successfully described in complicated

models realized in the form of codes for labor-consuming computer simulations. Such codes typically include modules describing the initial state formation, relativistic hydrodynamics expansion of liquid-like QGP at the intermediate stage of the system's evolution, its subsequent particlization (turning into a set of hadrons), and, finally, the hadron-resonance gas expansion. As a result of typical calculation, one obtains from the model a set of created hadrons, characterized by the space-time points of their last collision and four-momenta. Based on these data one can construct different observables, like spectra, correlation functions, etc.

The data analyzed in this study were generated using two such models, iHKM and LQTH, each having its own characteristics.

The iHKM is one of the most complete models, describing all the essential phases of the matter evolution in the course of a relativistic $A + A$ collision (see [31,39] for details). The initial conditions (ICs) for each simulation include the energy density spatial distribution at the starting time τ_0 , usually close to 0.1 fm/c (for the high-energy collisions we use GLISSANDO code [43] to generate it), and anisotropic momentum distribution, inspired by the color glass condensate model. These ICs correspond to the very initial nonequilibrium partonic state right after the two nuclei collision. At the next stage of the system's evolution, it gradually thermalizes and approaches a nearly hydrodynamical state. Then (starting from the thermalization time $\tau_{th} \approx 1$ fm/c) there follows a continuous medium expansion described in the Israel-Stewart viscous hydrodynamics formalism. As the matter expands and cools down, it eventually reaches the "particlization temperature" $T_p \approx 160$ MeV (depending on the QGP equation of state used at the hydrodynamics stage), when one switches to the description of the system in terms of hadrons (here the Cornelius routine [44] is applied). Produced hadrons are then fed to the UrQMD hadron cascade code [45,46], performing multiple hadronic rescatterings and resonance decays taking place at this final "afterburner" phase of the collision. The model is calibrated based on the experimental mean charged particle multiplicity and pion p_T spectrum slope in the most central collisions of a given type. In this paper, we use the iHKM tuning, used for the LHC 5.02 ATeV Pb-Pb collisions simulation described in detail in [40].

The LQTH is similarly properly calibrated to describe the $\sqrt{s_{NN}} = 5.02$ TeV Pb-Pb collisions at the LHC. The model assumes a second order viscous (3+1)D hydrodynamics evolution of the created fireball including shear and bulk viscosity as well as the Israel-Stewart stress corrections. The hydrodynamic expansion starts at 0.6 fm/c and lasts until the single (both chemical and kinetic) freeze-out at a selected temperature ($T_p \approx 140$ MeV), where statistical hadronization takes place. The system's chemical composition and momentum spectra no longer change, except for the resonance propagation and decays, which are simulated by the THERMINATOR2 package. At the freeze-out temperature, particles are generated according to statistical rules from the freeze-out hypersurface following the Cooper-Frye formula. The model reproduces the heavy-ion collisions results including flow and femtosopic measurements [32,41,47]. All details about the

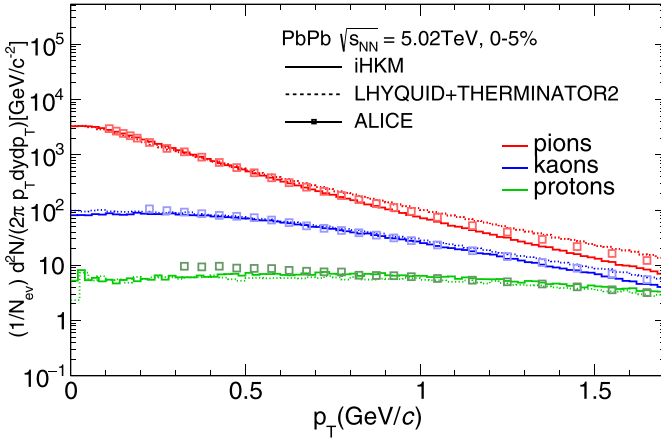


FIG. 1. Pion, kaon, and proton transverse momentum spectra calculated in the iHKM (solid lines) and in the LQTH models (dashed lines) for the most central $\sqrt{s_{NN}} = 5.02$ TeV Pb-Pb collisions at the LHC [48].

model implementation and initial conditions can be found in [41].

Both models describe well the experimental results for pion, kaon, and proton transverse momentum spectra, including the region of very soft momenta (see Fig. 1). Note, that such a description is achieved without the implementation of additional specific mechanisms of the soft pion emission, like the Bose-Einstein condensation, discussed, e.g., in [38]. Also, the two utilized models were previously used to successfully describe the LHC data on 2.76 ATeV Pb-Pb collisions [32,39,40,47].

III. CONSTRUCTION OF THE CORRELATION FUNCTION

The femtoscopic radii constituting the main subject of this study are typically defined from the Gaussian fits to the correlation functions depending on pair relative momentum $q = p_1 - p_2$. Here we consider the correlations in pairs of identical charged pions (π^+ and π^-), performing 1D and 3D analysis. The 1D correlation of identical particles is calculated as a function of $q_{inv} = \sqrt{q_0^2 - \mathbf{q}^2}$ in the pair rest frame (PRF) and requires minimal statistics in contrast to the 3D study. However, the radius R_{inv} , in this case, is the only length extracted, so that the emission source is assumed to be a spherically symmetric Gaussian one. The 3D study is based on relative momentum components q_{out} , q_{side} , and q_{long} calculated in the longitudinally comoving system (LCMS), where the longitudinal direction is along the beam axis, the outward direction is along the pair transverse momentum and the side-ward direction is perpendicular to the other two. Accordingly, the three radii R_{out} , R_{side} , and R_{long} are extracted from the CF fit in this case.

The correlation function is generally defined as

$$C(p_1, p_2) = \frac{P_{12}(p_1, p_2)}{P_1(p_1)P_2(p_2)}, \quad (1)$$

and can be understood as the ratio of conditional probability to detect a pair of particles with the specific momentum

values, $P_{12}(p_1, p_2)$, to the probability of finding them with such momenta independently, $P_1(p_1)P_2(p_2)$.

In experimental analysis, the procedure of obtaining the correlation function is based on making pairs of detected particles. One uses particles coming from the same event to build the correlated distribution (numerator), while for the background distribution (denominator) pairs are created using particles coming from different events (meaning that they retain all aspects of experimental acceptance, while not being correlated due to the mutual interaction).

In simulation studies, one usually fills two histograms (1D or 3D, depending on the analysis type) representing the corresponding particle pairs' relative momentum q distribution—one histogram for the numerator and another one for the denominator in Eq. (1)—based on the generated model output (set of hadrons). However, since in all currently available heavy-ion collision models the quantum statistical effects (including the Bose-Einstein symmetrization of the two identical bosons' wave function) are not implemented directly, on a microscopic level, one usually introduces a weight factor

$$w = 1 + \cos(qr), \quad (2)$$

coming from the Bose-Einstein interference at the so-called “afterburner” stage (to simplify the presentation, Coulomb and strong final-state interactions are not considered here). This factor w is taken as a weight for the pairs entering the correlated distribution histogram (a numerator one). For the background distribution (denominator) the weight is equal to 1. In this study, we also use the rapidity cut, $|y| < 1$, during the identical charged pion pairs selection to reproduce the acceptance of the ALICE detector.

To obtain a fitting formula for the CF, which will allow us to define the desired femtoscopic radii, one can consider the 7D (if particles are on the mass shell) particle emission functions $S_i(x, p)$ for each particle species i and derive the approximate expressions for single-particle and two-particle momentum spectra, and then using Eq. (1) obtain the well known Bertsch-Pratt [49,50] representation for the correlation function of two identical bosons in LCMS:

$$C(\mathbf{k}, \mathbf{q}) = 1 + \lambda_{3D}(\mathbf{k}) \exp \left(-R_{out}^2(\mathbf{k})q_{out}^2 - R_{side}^2(\mathbf{k})q_{side}^2 - R_{long}^2(\mathbf{k})q_{long}^2 \right), \quad (3)$$

where femtoscopic radii R_i depend on mean pair momenta $\mathbf{k}$, and λ_{3D} are referred to as correlation strength factors.

The invariant one-dimensional form of the correlation function Gaussian parametrization is

$$C(k, q_{inv}) = 1 + \lambda_{1D}(k) \exp \left(-R_{inv}^2(k)q_{inv}^2 \right). \quad (4)$$

IV. PION FEMTOSCOPY SCALES

This section presents the results on pion femtoscopic radii obtained as a result of correlation function fitting for identical pions in 1D and 3D studies using the two models (iHKM and LQTH). The correlation functions were calculated separately for the six centrality classes (0–5%, 5–10%, 10–20%, 20–30%, 30–40% and 40–50%) and the ten k_T ranges (0.00–0.05, 0.05–0.10, 0.10–0.15, 0.15–0.20,

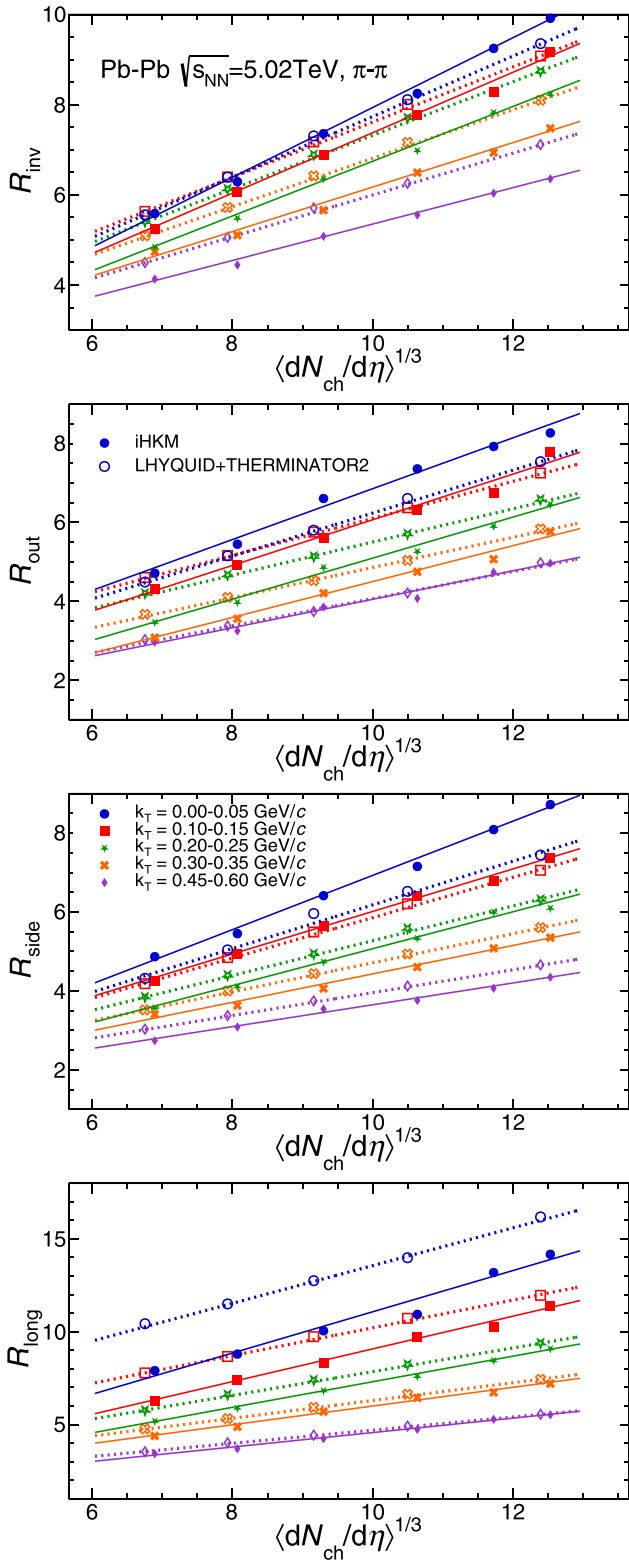


FIG. 2. Femtoscopic radii of charged identical pions as functions of $\langle dN_{ch}/d\eta \rangle^{1/3}$ calculated using iHKM (full markers) and LQTH (empty markers) models, for five k_T ranges (from top to bottom: 0.00–0.05, 0.10–0.15, 0.20–0.25, 0.30–0.35, 0.45–0.60 GeV/c). Different panels from top to bottom correspond to 1D R_{inv} and 3D R_{out} , R_{side} , and R_{long} radii respectively. Lines correspond to linear fits to the radii dependencies. Symbols are slightly shifted in the x direction for visibility.

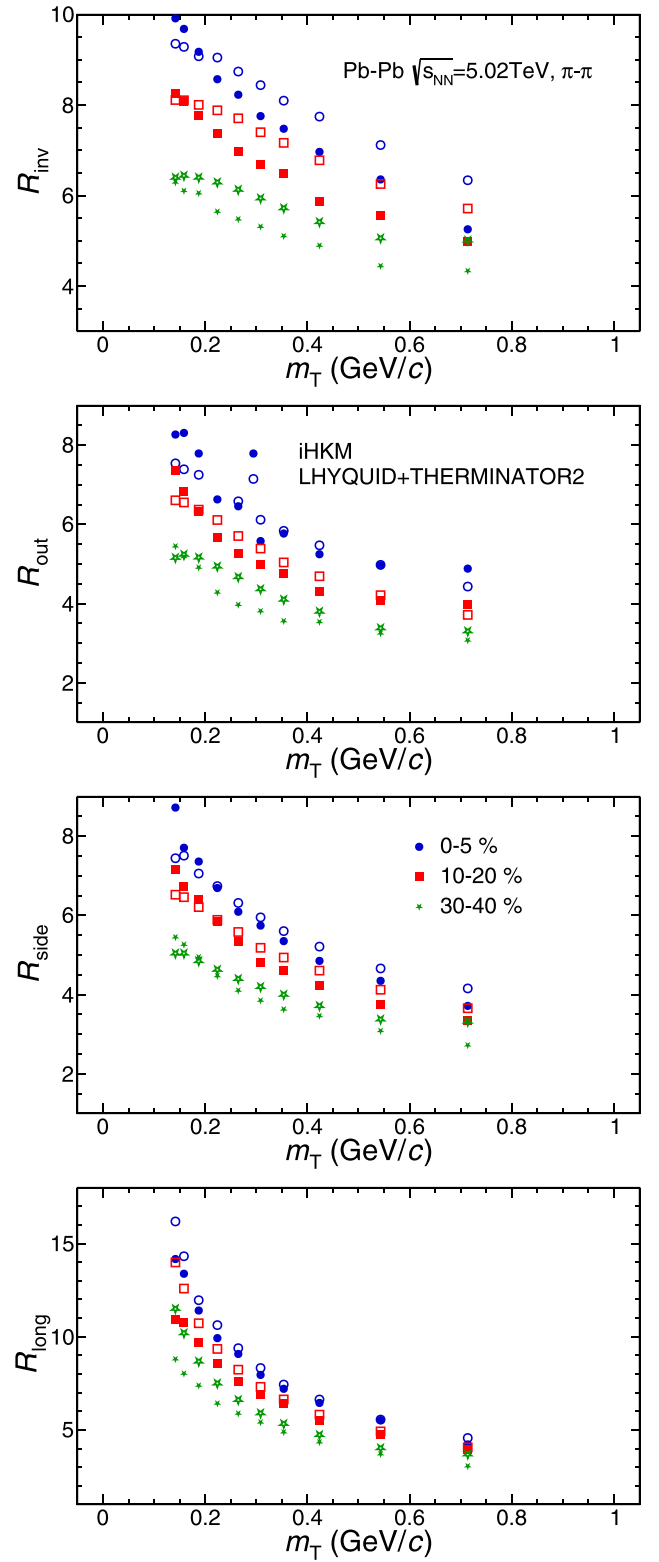


FIG. 3. Femtoscopic radii of charged identical pions as functions of m_T calculated using iHKM (full markers) and LQTH (empty markers) models, for the three centrality classes (0–5% blue, 10–20% red, 30–40% green). Different panels from top to bottom correspond to 1D R_{inv} and 3D R_{out} , R_{side} , and R_{long} radii respectively.

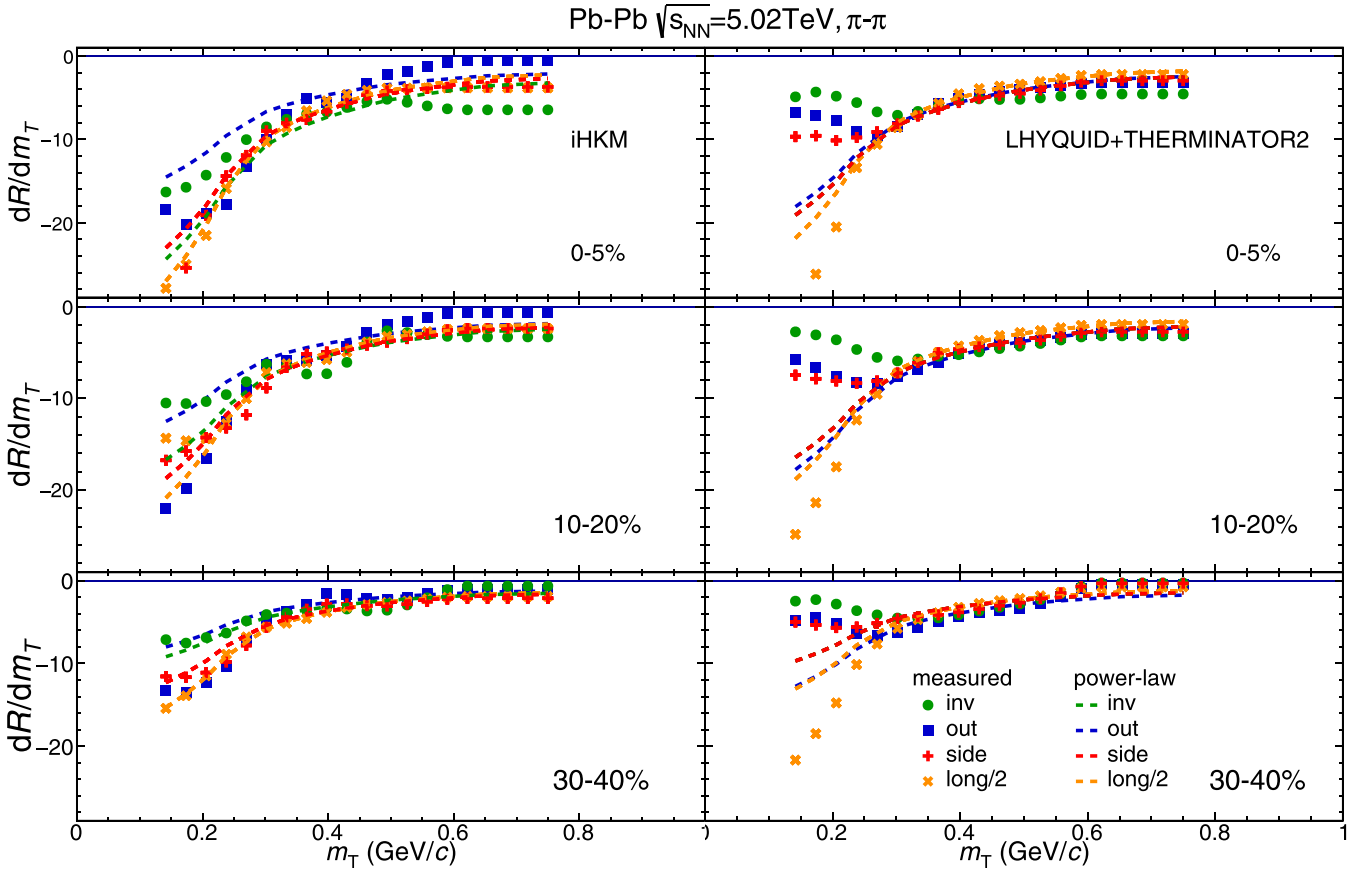


FIG. 4. First-order derivative of the charged identical pions' femtoscopic radii m_T dependency, dR/dm_T , in the iHKM (left) and the LQTH (right) models, for the three collision centralities (from top to bottom: 0–5%, 10–20%, 30–40%). Each panel contains derivatives of the three radii from the 3D study (R_{out} blue, R_{side} red, R_{long} orange) and R_{inv} from the 1D study (green). Markers represent results obtained from simulations and the dashed lines are obtained for the expected power-law trend based on fits to simulation points above 250 MeV/c.

0.20–0.25, 0.25–0.30, 0.30–0.35, 0.35–0.45, 0.45–0.60, 0.60–0.80) GeV/c. The results from both models were based on approximately 2×10^5 events, and the statistical uncertainty of each correlation function was much smaller than the systematic effects.

In Fig. 2 all the radii for identical pion pairs, corresponding to 1D and 3D calculations in the two models are demonstrated as functions of $\langle dN_{ch}/d\eta \rangle^{1/3}$. The results for five of all ten m_T bins are shown for better visibility since the radii in all bins follow a monotonic trend. The radii values vary in the range from about 15 fm (R_{long} for the lowest m_T and the highest multiplicity in LQTH) to about 2 fm (R_{side} for high m_T and low multiplicity in iHKM). All the radii universally grow linearly with the cube root of final-state multiplicity. The multiplicity dependencies for various m_T ranges generally show m_T ordering, except for 1D and transverse 3D results from the LQTH model, where the first bins of m_T overlap. This is because, in the LQTH model, radii of the first m_T bins take similar values. Figure 3, where femtoscopic radii are shown as functions of m_T presents it more clearly. All the measured radii decrease with increasing m_T . However, in the case of the LQTH model, we can observe a small plateau for the 1D and the transverse 3D radii at low m_T . The iHKM results, however, mostly fall monotonically in the lowest m_T region. Figure 3 also exhibits an ex-

pected ordering with respect to analysed centrality: the more peripheral the collision, the smaller the radii. In both Figs. 2 and 3, the difference in femtoscopic scales between the two models is also evident.

One can see from Fig. 3 that the femtoscopic radii in the iHKM model decrease faster than in the LQTH one. This is likely due to a very early, at $\tau_0 = 0.1$ fm/c, expansion start in the iHKM, leading to stronger velocity gradients at the final stage. This is the main reason for the reduction of the femtoscopic radii: the homogeneity lengths are reduced with the growing flow intensity. So, at the intermediate and high k_T the iHKM radii are smaller than those in the LQTH model with a later start of the expansion ($\tau_0 \approx 0.6$ fm/c). For the radii values at ultrasoft transverse momenta, the situation becomes even more complicated. At very small k_T the transverse radii are larger in the iHKM approach (the mentioned gradient could be the reason), while the longitudinal ones are larger in the LQTH model. The physics of pion production in the very-low- k_T region appears to be nontrivial and worthy of further study and better understanding.

To better investigate the character of the radii m_T dependencies, they were differentiated, and the obtained derivatives are presented in Fig. 4 (for the three centrality classes). The derivative has been extracted in the range ± 0.07 GeV/c around each point in the figure. Most of the functions show

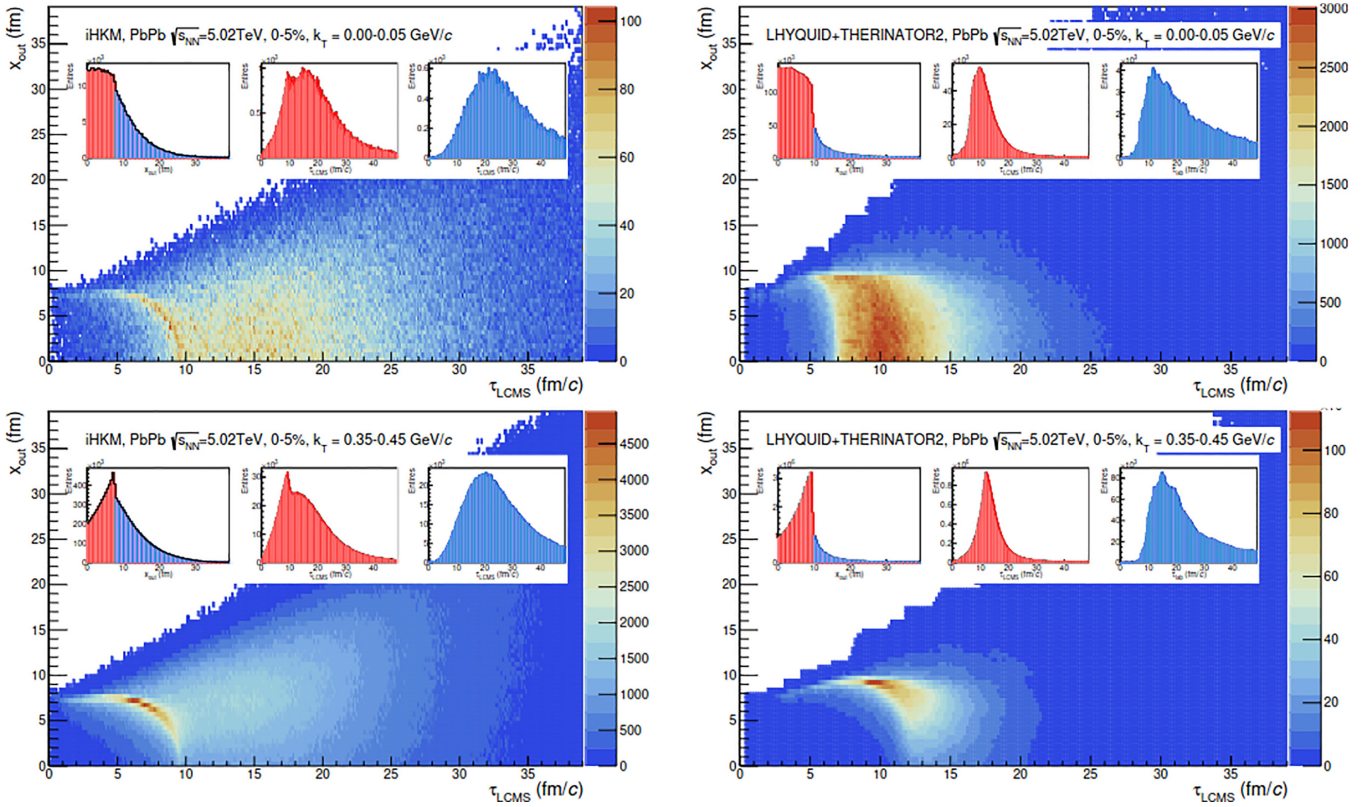


FIG. 5. The pion last interaction time and transverse coordinate in the out direction of the LCMS system for iHKM (left) and LQTH (right) simulations of Pb-Pb $\sqrt{s_{NN}} = 5.02$ TeV collisions. The upper panels show distributions for the lowest k_T interval, and the bottom ones show distributions for the middle k_T interval considered in this study. Subfigures show projection on the x_{out} direction (left) and projection on the time axis for the two x_{out} ranges (center and right).

similar behavior, with a clear change in the derivative form at certain momenta. Namely, the derivative behavior at low momenta ($m_T < 0.25$ GeV/c), especially for the R_{out} and R_{inv} radii in LQTH model, is noticeably different from that of a power-law function derivative, typical for the higher momentum region and approximately followed even at low m_T by the dR_{long}/dm_T dependencies. In more detail, in the iHKM one observes that the radius derivatives for the 1D case go flatter and are smaller in absolute value than the R_{long} derivatives for all the collision centralities, while the *out* and the *side* curves go either slightly higher or slightly lower than the *long* one, depending on the collision type. In the LQTH model we see a somewhat different situation, but one rather similar for all the centrality classes: all the derivatives, except for the *long* one, at low m_T have much smaller absolute values than the power-law function and demonstrate nearly flat behavior with small falls and rises, so that the radii dependency in this region can be sufficiently well approximated by a linear function.

To interpret above mentioned results, one can use a very recent iHKM femtoscopy analysis [51] as follows. When the system has just formed (at times near 0.1 fm/c), huge gradients of density in the transverse direction take place, since the system is essentially finite and borders with vacuum. The gradient is not equal locally along the radial directions: it is stronger at the periphery and less strong in the center, where soft hadrons mostly come from. In the vicinity of the geomet-

rical center of the system, its decay into free particles happens at significantly later proper times than for most other parts of the system. Such a difference in proper times of spectra formation can be up to 5 fm/c [51]. Thus, in the context of the present study, one can expect that maximal formation times (about 15 fm/c) should be typical for the ultrasoft pions with transverse momenta less than 0.3 GeV/c, that are emitted from the central region of the fireball. For the pions radiated from other parts of the decaying expanding system and having higher transverse momenta, 0.45–2 GeV/c, the (proper) time of maximal emission is close to 10 fm/c [51]. The situation looks like a pion trap formed in the center of the created quark-gluon-hadron system. The hadrons stay together (cannot leave the system) for a longer time in the system's center because of the following:

- (i) very low (close to zero) transverse collective velocity;
- (ii) smaller density gradient in the center during almost the entire duration of the evolution, as compared to noncentral and periphery parts;
- (iii) the initially highest density in the geometrical center in central and semicentral nucleus-nucleus collisions. Therefore, for this high-mass-density region, it is difficult to expand because of the relatively small transverse pressure gradient in the center.

Note that we see the mentioned effects of non-power-law transverse radii behavior in the ultrasoft momentum region in

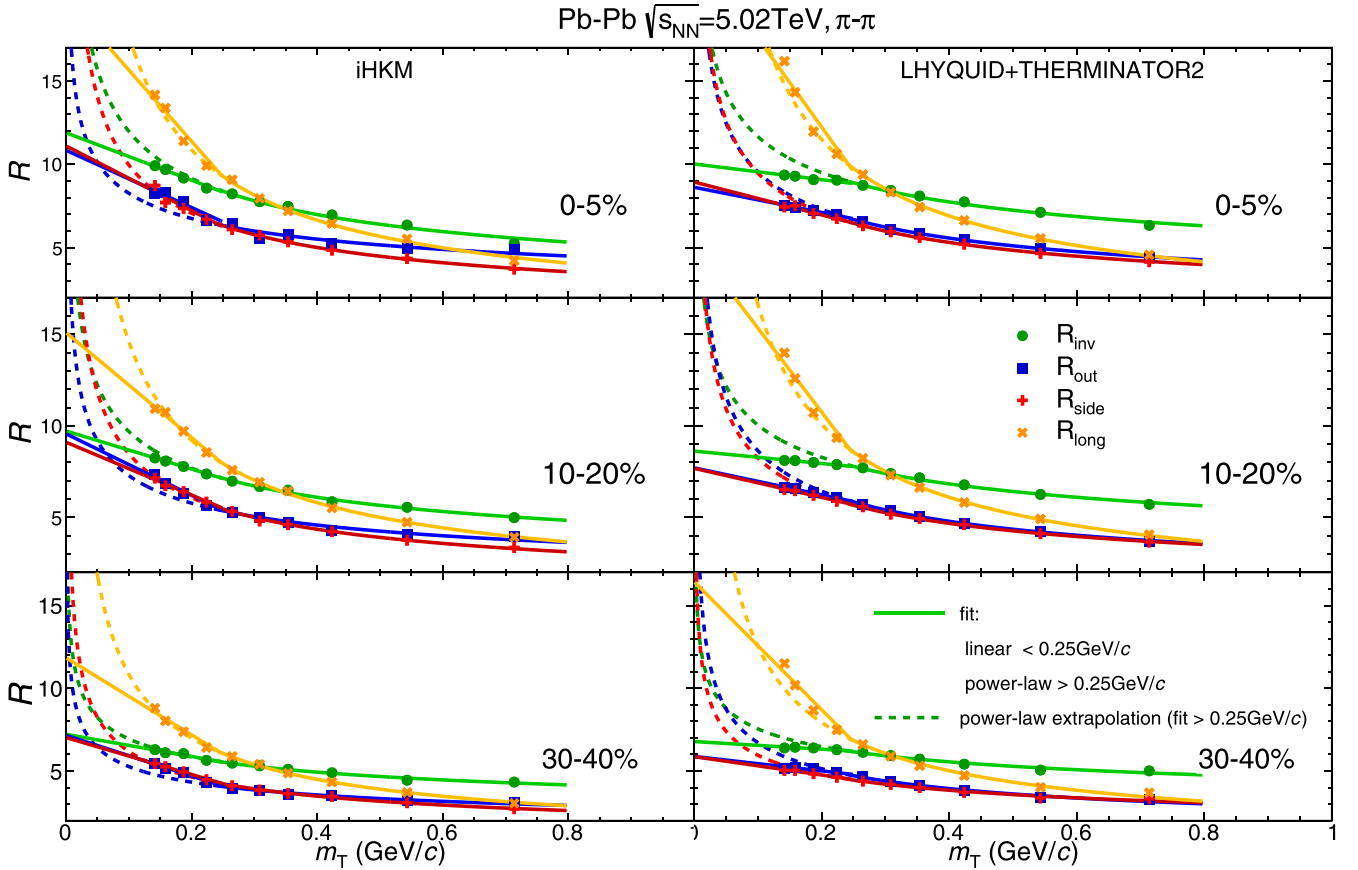


FIG. 6. Pion femtoscopic radii dependencies on m_T calculated in iHKM (left) and LQTH (right) models, for the three centrality classes (from top to bottom: 0–5%, 10–20%, 30–40%). Each panel contains results for the three radii from the 3D study (R_{out} squares, R_{side} crosses, R_{long} tilted crosses) and for R_{inv} from the 1D study (circles). Lines correspond to power-law fits am_T^{-b} to radii dependencies in the range above 0.25 GeV/c.

“hydro plus hadronic cascade” models, where the dynamics is built in a quasiclassical approximation. The Bose-Einstein quantum statistics effect is taken into account in the very final stage only, but even this way, using the femtoscopy method, one can see specific (delayed freeze-out) features of emission in the ultra-low-momentum region (see also Fig. 1 from [51]). In particular, as we already mentioned, the femtoscopy analysis brings us the signal about formation of a long-lived ultrasoft pion trap in the central part of the system created in ultrarelativistic heavy ion collision. To further check this signal, we built the distributions of the transverse *out* coordinate of pion last collision points vs its time in the LCMS system; see Fig. 5. It can be easily noticed that in both models the origin of particles forming low and high k_T pairs is qualitatively different. Low k_T pairs are mainly created of particles with low momenta most intensively emitted from the decaying system close to its center. To see this, one can compare the red part of the x_{out} distributions shown in the panels of Fig. 5 for low (top panels) and high (bottom panels) particle momenta, respectively. The red parts correspond to the emission coming from the hadronizing hydrodynamic tube itself, while the blue parts correspond to the emission at larger distances from the expanding hadron-resonance gas. The blue parts are rather similar at low and high momenta, while the red ones have a maximum in the center of the system for low momenta and at

the periphery of the system for higher momenta. The red time distributions at low momenta have two maxima and larger mean emission time than those at high momenta having only one maximum near the system’s particlization time. The blue time distributions are similar for the low and high momentum cases.

The dependencies of the radii on m_T , together with the power-law fits, are presented for the three centrality classes in Fig. 6. The fitting was performed outside the “non-power-law” region observed in Fig. 4 (above 0.25 GeV/c). As one can see from Fig. 6, the power-law in the form am_T^{-b} works well for all the radii at not very low momenta, and in case of the *long* direction it describes even the ultrasoft region below 0.25 GeV/c. Other radii, especially the 1D ones, at very low momenta go below (in the iHKM sometimes above) the power-law fitting curve.

In Fig. 6 the fall of 1D radii is less prominent than the fall for radii from 3D studies. This can be connected with the shape of the respective correlation functions. The 1D functions are less Gaussian than the 3D functions, and therefore the fitting has to compensate for this by reducing the correlation strength parameter λ . The m_T dependencies of λ in the two applied models are shown in Fig. 7. In the low m_T region λ behaves similarly in both cases, whereas with growing m_T it tends to go down in the 1D case.

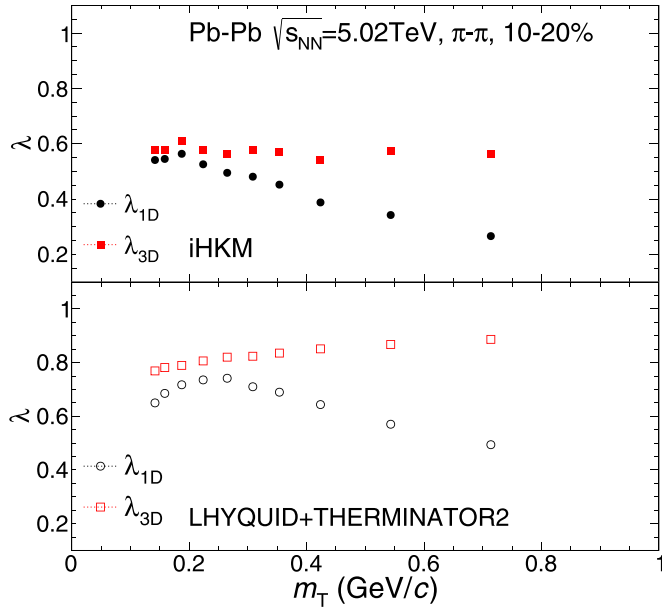


FIG. 7. The correlation strength λ parameters from 1D (black) and 3D (red) fits to the two-pion CF functions for the ten m_T bins. The iHKM (top) and the LQTH (bottom) results are shown for $c = 10\text{--}20\%$ Pb-Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV.

Finally, we analyzed the femtoscopic-radii dependence on pair velocity; see Fig. 8 for different centrality intervals and for both used model approaches. In all cases, we notice a slow radii falling at low velocities. All the distributions were fitted with the analytic formula derived for the purpose of this study,

$$R(\beta) = \frac{a}{e^{(\beta-b)/c} + 1}, \quad (5)$$

where $\beta = k_T/m_T$ is pair velocity, and a, b, c are free parameters of the fit. As particle velocity decreases, the radii seem to reach a saturated value, similar in a very broad range of β . Therefore such a value could be used to determine an extrapolated value of the radii for ultrasoft pions, instead of the historically used dependence based on power-law m_T scaling. It is particularly important for non-identical femtoscopy studies, where one considers pairs of particles with different masses moving at the same velocity, thus having different momenta.

V. SUMMARY

This paper presents a study of femtoscopic radii obtained from simulated data using iHKM and LQTH. The results point out that the power-law character of the femtoscopic m_T -scaling is no longer valid for 1D studies in the low m_T region (below 250 MeV/c). In the case of 3D studies, the radii are better described by the power-law dependence than the 1D radii; however, the final conclusions are model dependent. The iHKM model results are closer to a power-law behavior, especially in the 3D analysis case. On the other hand, for the LQTH model one sees noticeable deviations from the power-law description everywhere, except for the longitudinal direction. Both models show that in low- m_T region radii distribution tends to be more linear than power law or even flat

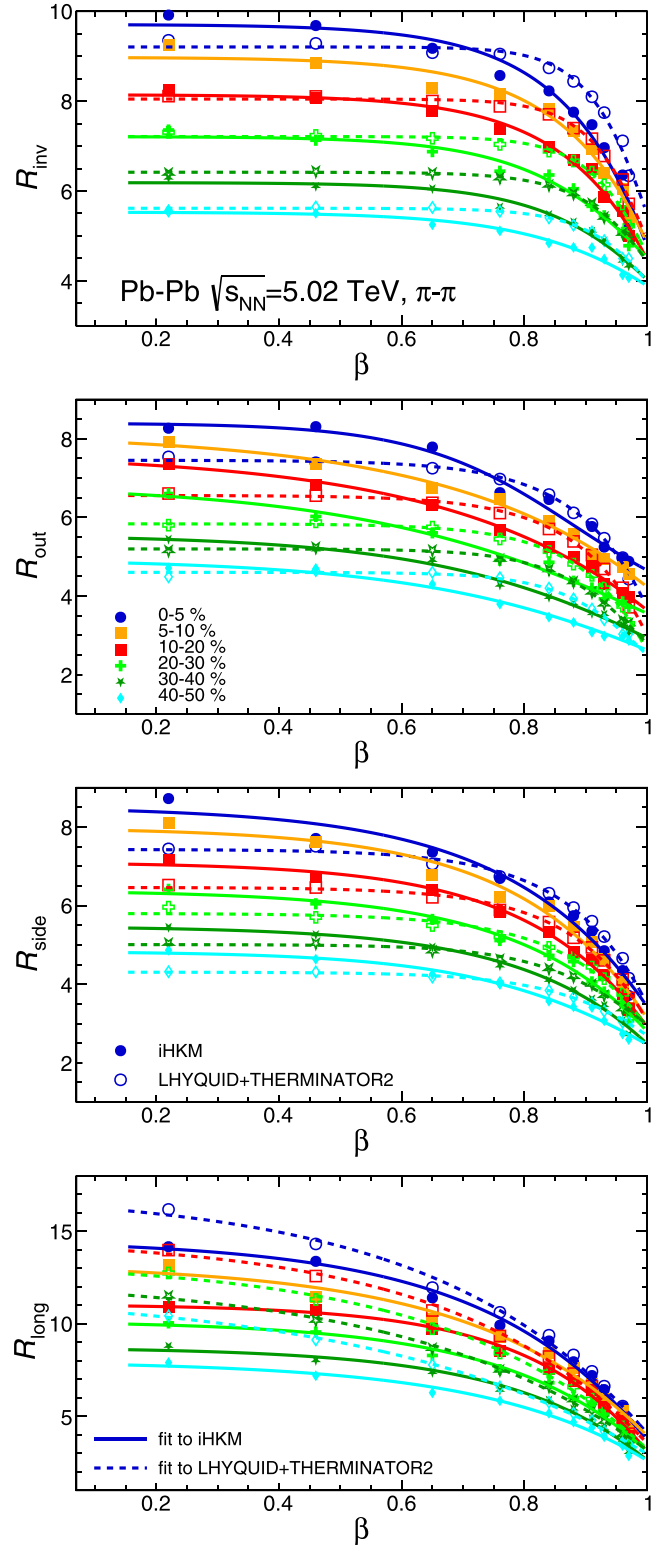


FIG. 8. The identical pion femtoscopy radii dependencies on pair velocity β in the iHKM (full markers) and the LQTH (empty markers) models. The results for the six centrality classes are shown. The lines represent fits to the radii by the formula (5).

in some cases. This results in smaller radii than expected from the extrapolation of the power-law scaling from intermediate and high m_T .

This modification of the behavior is motivated by the capacity of the ultrasoft pions to decouple from the energetic and dense core of the expanding system, resulting in longer emission times but from the center of the created system. This can be interpreted as both spatial and kinematic trapping of the ultrasoft pions by the more dense part of the created system. Such pions cannot escape this phase-space region until this confining mechanism is released. This finding is fundamental for addressing experimentally the expected pion radii in nonidentical femtoscopic studies when the second particle has a large mass difference with the pion mass, e.g., pion-deuteron, pion-omega, or pion-charm hadron pairs.

The observations of the non-monotonic behavior of ultrasoft pions obtained from semiclassical models should also be explored from the quantum perspective. The role of

confinement and its connection to the direct photon puzzle [52–56] might as well influence the pion homogeneity lengths and should be studied further.

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1.4. Вплив початкових умов еволюції системи на результуючі фемтоскопічні масштаби

Correlation analysis of high-energy heavy-ion collisions within the integrated hydrokinetic model

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Abstract. The interferometry radii, obtained from the Gaussian fits to the two-particle momentum correlation functions for different high-energy heavy-ion collision experiments at the LHC and the top RHIC energies, are described in the integrated hydrokinetic model. The role of the different geometrical sizes of the colliding ions in the formation of the observed femtoscopy scales is investigated by comparing the results for Au+Au, Pb+Pb and Xe+Xe collisions at close values of the mean charged particle multiplicity.

1 Introduction

The correlation femtoscopy method [1–5] is based on the quantum statistics correlations between particles produced in high-energy nuclear collisions and is widely applied for estimating the space-time structure of hot and dense systems formed in such collisions. The femtoscopy, or interferometry radii, which one extracts from the Gaussian fits to the two-particle momentum correlation functions, measured in heavy-ion collision experiments, can be associated with the homogeneity lengths of the system in different directions, i.e. with the lengths, at which the corresponding distribution functions $f(x, p)$ do not change much (no more than twice) [6–8].

To perform a correlation femtoscopy analysis one at first needs to separate informative quantum statistics correlations from those caused by the final state interactions (FSI) between particles and to take into account the effect of long-lived resonances' decays, which also make contribution to the observed correlation functions [9]. When the baryon-(anti)baryon correlations are studied, the FSI analysis allows also to extract the quantities characterizing the strong interaction between the corresponding baryons, such as scattering length, effective range, etc. [10–16]. With the FSI technique such an extraction can be successfully carried out even in cases, when the traditional scattering experiments do not give the result due to insufficient beam densities.

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The source imaging method [17–19], also based on the correlation measurements in heavy-ion collisions, can be used to obtain even more information about the collision spatiotemporal picture. This technique allows to restore the emission source function (i.e., the time-integrated distribution of the distance between the points of particle emission in the pair rest frame) from the experimental correlation function. While the standard correlation femtoscopy approach assumes the source function to be Gaussian, the source imaging reveals its actual shape, which often deviates from the Gaussian one (e.g. the source function can have power-law tails in certain directions [20]). Analyzing the actual source functions, one can better understand the dynamics of the collision process and investigate the role of different factors (such as intensive particle scatterings at the late stage of the collision, long-lived resonances decays, space-momentum correlations, and other ones) in the formation of the observed emission picture.

A reliable interpretation of the experimental results, obtained in correlation femtoscopy studies, can hardly be reached without parallel simulation of the investigated collision processes within a realistic model and the subsequent analysis of the model results as for the measured observables.

In this paper we use the integrated hydrokinetic model (iHKM) [21,22] to analyze the femtoscopic structure of relativistic Au+Au collisions at the top RHIC energy 200A GeV, Pb+Pb collisions at the two LHC energies, 2.76A TeV and 5.02A TeV, and the LHC Xe+Xe collisions at 5.44A TeV.

2 Results and discussion

In this section we firstly give a short description of the iHKM model utilized in our analysis and then present and discuss the obtained results. For a detailed description of iHKM please see references [21,22].

The integrated hydrokinetic model allows to simulate the full process of the evolution of hot and dense matter, formed in a relativistic nuclear collision. We consider this evolution to pass in several successive stages, each described in the iHKM using a separate, most suitable approach.

The first model stage simulates the prethermal dynamics of the system just after the collision in terms of an energy-momentum transport approach in the relaxation time approximation. During this stage the system gradually thermalizes and eventually reaches a state of local chemical and thermal equilibrium.

At the second stage of matter evolution the system undergoes hydrodynamical expansion and behaves as a continuous medium. This stage is simulated within the Israel-Stewart relativistic viscous hydrodynamics formalism.

As time goes by, the system loses local chemical and thermal equilibrium and decouples into particles. This stage of the evolution can be called “particlization”. We assume particlization to take place at the isotherm hypersurface, corresponding to the temperature T_p , close to 160 MeV and depending on the equation of state (EoS), utilized for quark-gluon matter description at the hydrodynamical stage. At the particlization hypersurface the hadrons of all the species considered in iHKM (we consider 329 well established hadronic states) with their space-time coordinates and 4-momenta are generated according to the distribution functions defined during the hydrodynamics stage.

The created particles then pass the last, “afterburner” phase of the system’s evolution. In iHKM it is simulated using the UrQMD model [25,26]. During this phase particles experience intensive elastic and inelastic collisions with each other, and also the resonance decays take place.

The initial energy-density distribution for the prethermal stage of system’s evolution is generated within the Monte Carlo Glauber approach using the GLISSANDO

Table 1. The values of the iHKM parameters α and ϵ_0 , adjusted for the simulation of different nuclear collision experiments. The first three tunings utilize the Laine-Schroeder equation of state for quark-gluon phase [28], and the fourth one uses the HotQCD Collaboration equation of state [29]. We perform the simulation of events from different centrality classes, as indicated in the column 2, in order to obtain close mean charged particle multiplicity values for all the considered collision types.

Experiment	Centrality (%)	$\langle dN_{\text{ch}}/d\eta \rangle$	α	ϵ_0 (GeV/fm ³)
Au+Au @ 200 AGeV	0–5	688	0.18	235
Pb+Pb @ 2.76 ATeV	19–28	693	0.24	679
Pb+Pb @ 5.02 ATeV	23–33	677	0.24	1067
Xe+Xe @ 5.44 ATeV	10–19	680	0.44	445

code [27] and is regulated by the main iHKM parameters, which are the initial maximal energy density ϵ_0 at the starting proper time τ_0 , and the parameter α , defining the proportion between the contributions from the “binary collisions” and the “wounded nucleons” models’ to the full GLISSANDO profile. These parameters are fixed for each collision type based on the experimental data to give the best fit to the measured dependence of charged particle multiplicity on collision centrality and the slope of pion spectrum in the most central events.

Earlier, in the articles [22–24] the iHKM parameters were adjusted for the description of Au+Au collisions at the RHIC energy 200A GeV and the Pb+Pb collisions at the LHC energies 2.76A and 5.02A TeV. Now we have also tuned iHKM to simulate the Xe+Xe collisions at the LHC energy 5.44A TeV (see the corresponding model parameter values in Tab. 1). An interesting fact is that for Xe+Xe collisions simulation in iHKM one should use a noticeably increased value of α parameter in comparison to the case of Pb+Pb collisions with close energy. Such an increase means a larger contribution to the initial energy-density profile, associated with the binary collisions in the Xe+Xe case. We think this modification is rather due to a smaller size of the Xe nucleus as compared to the Pb nucleus and can be hardly explained by the increased collision energy.

Having fixed the model parameters for each collision type, we can calculate the femtoscopy scales in each case and compare the obtained results. Performing such a comparison, in the current study we would like to find out, if the interferometry radii R_{out} , R_{side} , R_{long} and the corresponding interferometry volume $V_{\text{int}} = R_{\text{out}}R_{\text{out}}R_{\text{out}}$ depend almost linearly only on the mean charged particle multiplicity (this would confirm the “scaling hypothesis”, often assumed in the studies on correlation femtoscopy, e.g. [30,31]), or not, and apart from the multiplicity, one has also to take into account the geometric sizes of the colliding ions, as it was concluded in the previous studies by our research group [32–34], see Figure 1.

In Figure 2 one can see the dependencies of pion and kaon interferometry volume $V_{\text{int}} = R_{\text{out}}R_{\text{side}}R_{\text{long}}$ on the parameter S_T , characterizing the initial transverse area of the system, formed in a relativistic heavy-ion collision. The value of S_T is calculated for each collision type, presented in Table 1, as the area of near elliptical cross-section of the initial energy-density distribution by a transverse plane crossing it at a half-value of maximal initial energy density ϵ_0 :

$$S_T = \pi ab, \quad (1)$$

where a and b are the semi-axes of the ellipse.

The volume is divided by $\langle dN_{\text{ch}}/d\eta \rangle$ to eliminate the influence of small differences between charged particle multiplicity values for different experiments. The

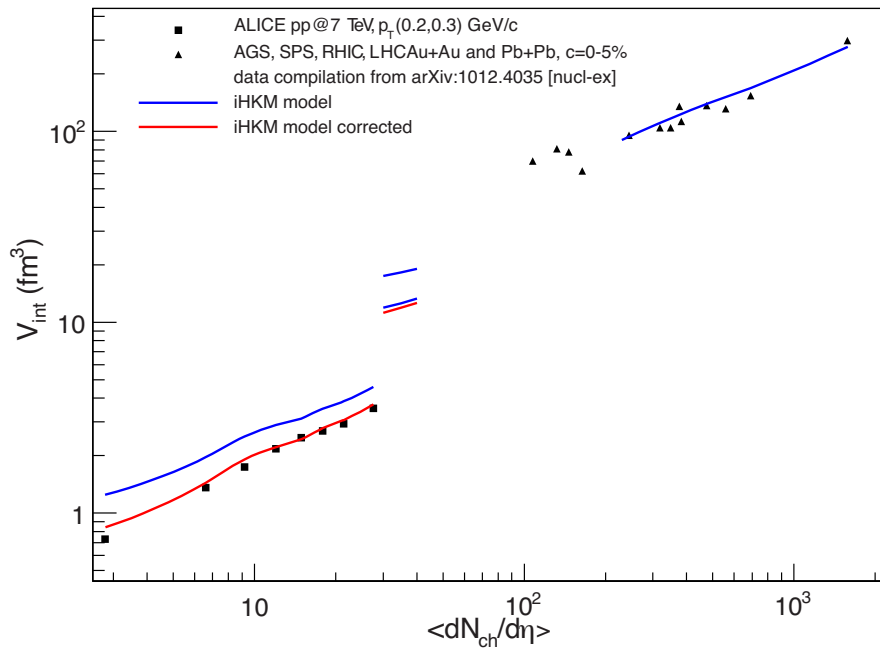


Fig. 1. The dependency of the interferometry volume $V_{\text{int}} = R_{\text{out}}R_{\text{side}}R_{\text{long}}$ on the mean charged particle multiplicity $\langle dN_{\text{ch}}/d\eta \rangle$. The iHKM results for $p + p$ LHC collisions at $\sqrt{s} = 7$ TeV are shown by the lines at the left. The lines at the right are related to different central $A + A$ collisions. The points corresponding to the experimental results for AGS, SPS, RHIC and LHC collisions [35], [36–43] are also shown for comparison. The red lines represent the model results with the quantum corrections applied according to [34], while the blue lines represent uncorrected results. The line segments in the middle are related to the model predictions for the $p + \text{Pb}$ LHC collisions at $\sqrt{s} = 5.02$ TeV obtained for the two different initial transverse sizes of the system — $R = 1.5$ fm (upper line) and $R = 0.9$ fm (lower lines).

shown femtoscopy radii and interferometry volumes correspond to the pair transverse momentum $0.2 < k_T < 0.3$ GeV/c. The corresponding correlation functions were constructed using particles with transverse momenta from the range $0.1 < p_T < 1.55$ GeV/c and pseudorapidity from the range $|\eta| < 0.8$.

The obtained values of pion interferometry volumes are about 2 times higher, than the kaon ones. For both particle sorts the volume grows with the initial transverse size of the system S_T . This size is minimal for the LHC Pb+Pb collisions at 5.02A TeV, after that goes the 2.76A TeV Pb+Pb collision size, then follows the size for 5.44A TeV Xe+Xe collision, and the maximal one is the S_T value for the top RHIC energy Au+Au collisions.

Another plot, demonstrated in Figure 3, presents the iHKM results on the individual kaon femtoscopy radii R_{out} , R_{side} , and R_{long} versus k_T and the rescaled initial transverse energy-density distributions $\epsilon(x)/\epsilon_0$ for the four considered collision types. From the plot one can conclude, that the difference between interferometry volumes for different systems is mainly due to the difference in R_{out} and a slight difference in R_{side} , while the R_{long} values in all the cases almost coincide.

The obtained results support the suggestion, set out in [32–34], that the difference in the interferometry volumes originates from the difference in transverse collective flows, developed in the considered systems. And the flow in turn depends on the initial geometrical size of the system and the corresponding energy/pressure gradient.

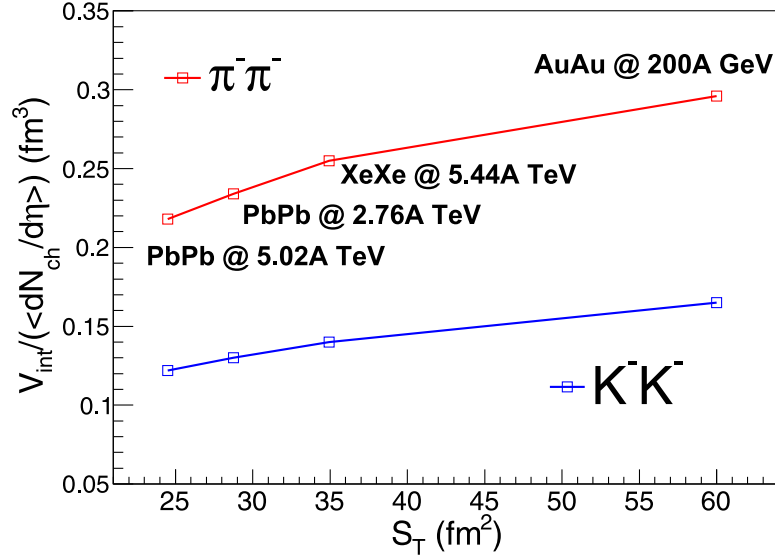


Fig. 2. The dependency of the ratio of pion and kaon interferometry volume $V_{\text{int}} = R_{\text{out}}R_{\text{side}}R_{\text{long}}$ to the mean charged particle multiplicity $\langle dN_{\text{ch}}/d\eta \rangle$ on the initial transverse overlapping area of colliding nuclei S_T , obtained in iHKM. The squares from left to right are related to the following experiments: Pb+Pb @ 5.02A TeV, Pb+Pb @ 2.76A TeV, Xe+Xe @ 5.44A TeV, and Au+Au @ 200A GeV.

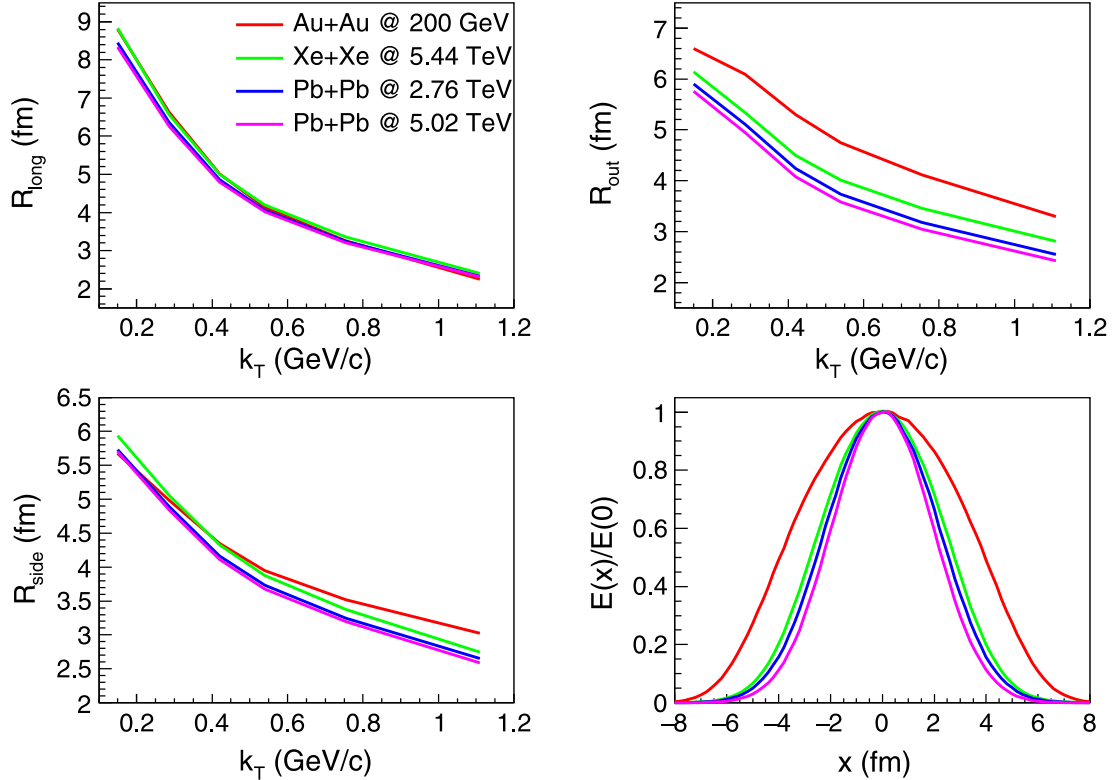


Fig. 3. The kaon interferometry radii for different collision types, listed in Table 1, and the corresponding (rescaled) initial transverse energy-density distributions $\epsilon(x)/\epsilon_0$.

3 Conclusions

The theoretical description of the interferometry scales in the LHC Pb+Pb collisions at 2.76A and 5.02A TeV, LHC Xe+Xe collisions at 5.44A TeV and in the RHIC

Au+Au collisions at 200.4 GeV, obtained within iHKM is presented. The events with close values of charged particle multiplicity were considered in order to study how interferometry volume and individual radii depend on the area of the overlapping region formed by the two colliding ions at the very beginning of the collision. Our analysis demonstrates that at the fixed charged particle multiplicity the interferometry volume and the transverse interferometry radii grow when the initial transverse overlapping size grows.

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Author contribution statement

Conceptualization: Yu.S. ; data curation: M.A. and V.S. ; funding acquisition: Yu.S. ; investigation: M.A. and V.S. ; methodology: Yu.S. ; project administration: Yu.S. ; supervision: Yu.S. ; visualization: M.A. and V.S. ; writing of original draft: V.S.

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1.5. Вплив дотермальної стадії на результуючі імпульсні спектри

THE PRETHERMAL STAGE OF HEAVY-ION COLLISION AND THE PARTICLE PRODUCTION*

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The influence of the intensity of matter evolution at the pre-thermal stage of heavy-ion collision on the observed particle spectra is investigated within the integrated hydrokinetic model (iHKM). The simulation results at different values of the thermalization time τ_{th} and the relaxation time τ_{rel} , characterizing the thermalization rate, at the fixed initial time τ_0 are analyzed for the case of central Pb+Pb collisions at the LHC energy $\sqrt{s_{NN}} = 2.76$ TeV.

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1. Introduction and model description

The early stage of the matter evolution in ultra-relativistic $A + A$ collisions is expected to be very important for the formation of bulk observables. In iHKM, the pre-thermal dynamics of the system's expansion is effectively described within energy-momentum transport approach in relaxation time approximation [1].

At the initial time τ_0 , the system's energy-momentum tensor has a form of $T_0^{\mu\nu}(x)$. As the system expands, the tensor evolves and at the thermalization time τ_{th} acquires the hydrodynamical Israel–Stewart form of $T_{\text{hydro}}^{\mu\nu}(x)$. Between τ_0 and τ_{th} the energy-momentum tensor has the form of

$$T^{\mu\nu}(x) = T_{0,\text{free-evolving}}^{\mu\nu}(x)\mathcal{P}(\tau) + T_{\text{hydro}}^{\mu\nu}(x)[1 - \mathcal{P}(\tau)] \quad (1)$$

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in relaxation time approximation for Boltzmann equation [2]. Here, $\mathcal{P}(\tau) = \left(\frac{\tau_{\text{th}} - \tau}{\tau_{\text{th}} - \tau_0}\right)^{\frac{\tau_{\text{th}} - \tau_0}{\tau_{\text{rel}}}}$ is the weight function, such that $\mathcal{P}(\tau_0) = 1$ and $\mathcal{P}(\tau_{\text{th}}) = 0$. The initial energy-density profile defining $T_0^{\mu\nu}(x)$ is obtained using the GLIS-SANDO code for Monte Carlo Glauber simulations [3].

Once the system is thermalized at $\tau = \tau_{\text{th}}$, its subsequent locally equilibrated evolution is described in terms of viscous hydrodynamics. Here, for the quark–gluon matter, one uses the equation of state based on the lattice QCD calculations; in this note we use the Laine–Schroeder [4] one.

Gradually, the matter loses the local chemical and thermal equilibrium and decouples into hadrons. At the “particlization” temperature T_p , the hydrodynamical description is switched to the description in terms of particles. At this stage, treated with UrQMD model [5], the resonance decays and multiple particle scatterings (elastic and inelastic) take place.

Finally, in the output of the model, one obtains the set of particle last collision points and momenta, which are utilized for constructing and analyzing various observables.

2. Results and conclusions

In our previous studies [1], we found that the iHKM results for bulk observables are very sensitive to the initial-state formation time τ_0 . In this paper, we analyze the influence of the thermalization rate parameters (relaxation time τ_{rel} and thermalization time τ_{th}) on the particle momentum spectra, considering the maximal initial energy density in the center of the fireball $\epsilon_0 \equiv \epsilon(\tau_0)$ as a free parameter at fixed $\tau_0 = 0.1 \text{ fm}/c$.

At first, we perform simulations with two different thermalization times $\tau_{\text{th}} = 1.0 \text{ fm}/c$ and $\tau_{\text{th}} = 1.5 \text{ fm}/c$ at the same relaxation time $\tau_{\text{rel}} = 0.25 \text{ fm}/c$ for both cases. The respective ϵ_0 values are $834 \text{ GeV}/\text{fm}^3$ and $681 \text{ GeV}/\text{fm}^3$. In Fig. 1, one can see a comparison of the pion, kaon and proton spectra for the central Pb+Pb collisions ($c = 0\text{--}5\%$) at the LHC energy $\sqrt{s_{NN}} = 2.76 \text{ TeV}$, calculated in iHKM, with the experimental data presented by the ALICE Collaboration [6]. The plot demonstrates that varying the maximal initial energy density parameter ϵ_0 , one can describe the measured spectra equally well at both thermalization times τ_{th} .

In a similar way, the possibility to compensate the modification of the spectra, associated with the change of relaxation time τ_{rel} , by the variation of the initial energy density ϵ_0 should also be verified. In Fig. 2, we compare with the experimental data the iHKM results on pion, kaon and proton spectra, obtained at fixed thermalization time $\tau_{\text{th}} = 1.5 \text{ fm}/c$ and the two different relaxation times, $\tau_{\text{rel}} = 0.25 \text{ fm}/c$ and $\tau_{\text{rel}} = 0.6 \text{ fm}/c$. Although the

latter is fairly larger than the former, it still remains smaller than τ_{th} . As one can see, in this case we also obtain similar results for both τ_{rel} values, tuning our free parameter ϵ_0 , which is put to be 630 GeV/fm^3 for $\tau_{\text{rel}} = 0.6 \text{ fm}/c$.

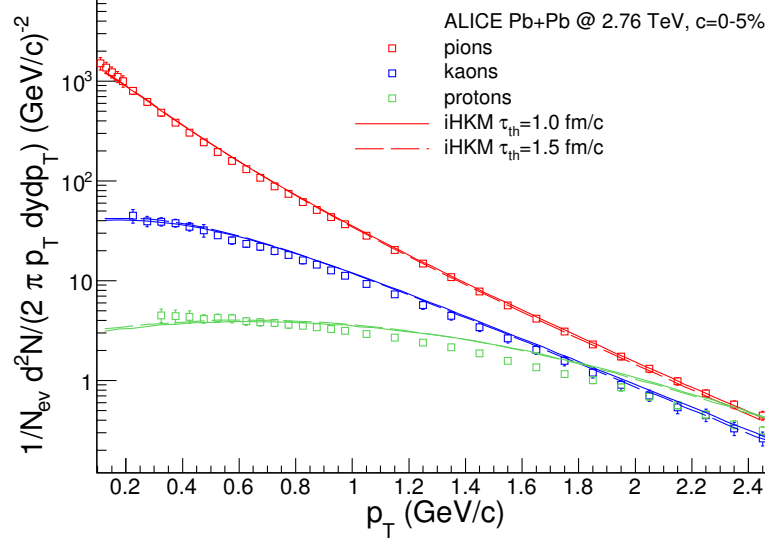


Fig. 1. The pion, kaon and proton spectra in the central ($c = 0-5\%$) Pb+Pb collisions at the LHC energy $\sqrt{s_{NN}} = 2.76 \text{ TeV}$ obtained in iHKM at the two different thermalization time values, $\tau_{\text{th}} = 1.0 \text{ fm}/c$ (solid lines) and $\tau_{\text{th}} = 1.5 \text{ fm}/c$ (dashed lines). The square markers represent the experimental data from the ALICE Collaboration [6].

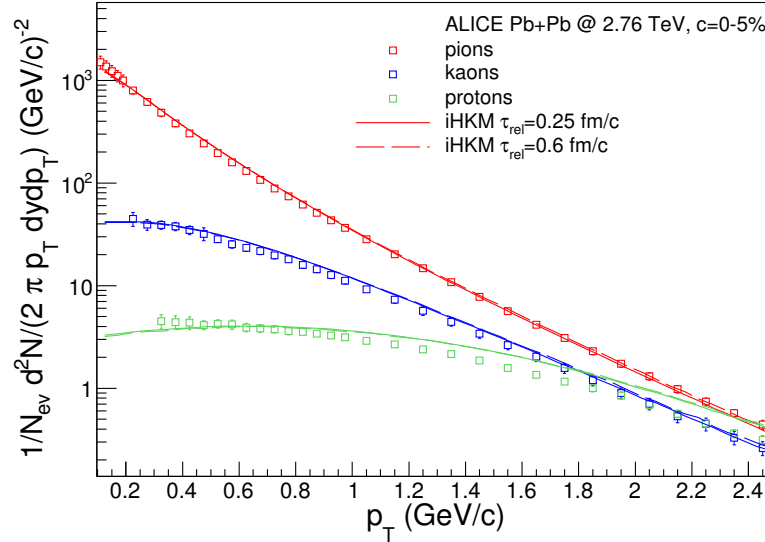


Fig. 2. The comparison of the pion, kaon and proton spectra in the central ($c = 0-5\%$) Pb+Pb collisions at the LHC energy $\sqrt{s_{NN}} = 2.76 \text{ TeV}$ obtained in iHKM at the two different relaxation times, $\tau_{\text{rel}} = 0.25 \text{ fm}/c$ (solid lines) and $\tau_{\text{rel}} = 0.60 \text{ fm}/c$ (dashed lines). The thermalization time in both cases is $\tau_{\text{th}} = 1.5 \text{ fm}/c$. The square markers represent the experimental data measured by the ALICE Collaboration [6].

The obtained results show that the characteristics of the thermalization rate cannot be unambiguously extracted from the experimental data if the initial energy density ϵ_0 is unknown — choosing appropriate ϵ_0 value, one can with equal success describe the observed data at various values of thermalization and relaxation times.

The research was carried out within the scope of the European Research Network “Heavy ions at ultrarelativistic energies” and corresponding Agreement with NAS of Ukraine. It is partially supported by the Tomsk State University Competitiveness Improvement Program (project 5-100), and NAS of Ukraine Targeted research program “Fundamental research on high-energy physics and nuclear physics (international cooperation)” Agreement F7-2018.

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1.6. Порівняння початкових умов і результатів моделювання для зіткнень ядер золота, свинцю та ксенону на RHIC та LHC



Review

Femtoscopy scales and particle production in the relativistic heavy ion collisions from Au+Au at 200 AGeV to Xe+Xe at 5.44 ATeV within the integrated hydrokinetic model

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Abstract The recent results on the main soft observables, including hadron and photon yields and particle number ratios, p_T spectra, flow harmonics, as well as the femtoscopy radii, obtained within the integrated hydrokinetic model (iHKM) for high-energy heavy-ion collisions are reviewed and re-examined. The cases of different nuclei colliding at different energies are considered: Au+Au collisions at the top RHIC energy $\sqrt{s_{NN}} = 200$ GeV, Pb+Pb collisions at the LHC energies $\sqrt{s_{NN}} = 2.76$ TeV and $\sqrt{s_{NN}} = 5.02$ TeV, and the LHC Xe+Xe collisions at $\sqrt{s_{NN}} = 5.44$ TeV. The effect of the initial conditions and the model parameters, including the utilized equation of state (EoS) for quark-gluon phase, on the simulation results, as well as the role of the final afterburner stage of the matter evolution are discussed. The possible solution of the so-called “photon puzzle” is considered. The attention is also paid to the dependency of the interferometry volume and individual interferometry radii on the initial transverse geometrical size of the system formed in the collision.

1 Introduction

Present-day the ultrarelativistic heavy-ion collision experiments, carried out at the BNL Relativistic Heavy Ion Collider (RHIC) and at the CERN Large Hadron Collider (LHC), provide the only laboratory method of obtaining a new unusual quark-gluon state of matter, characterized by extremely high temperature and energy densities. It is believed that this state is quite similar to the one the matter had in the very early Universe at times about 10^{-6} s after the Big Bang. Naturally, the comprehensive study of the properties of matter under

such extreme conditions and the dynamics of its evolution constitutes a fundamental physical problem.

In ultrarelativistic heavy-ion collision, in the energy range starting from top RHIC and higher, due to strong Lorentz contraction the two colliding nuclei can be considered as ultra-thin “pancakes” of quarks and gluons moving towards each other at a great speed.¹ After the nuclei pass through each other and carry away practically all the net baryon charge, the space region between them becomes occupied by a hot and dense system of partons with small (for RHIC) or practically zero (for LHC) baryon chemical potential. Such a system fastly expands, cools down, and eventually disintegrates into a system of about several thousand hadrons and resonances (together with some amount of leptons, photons, electrons etc.). At this stage hadrons intensively interact with each other, experiencing elastic and inelastic scatterings, and the resonance decays take place. Finally, when all the interactions cease, produced free particles travel to the detectors, allowing the experimentalists to collect sets of data, from which one extracts various information about the created system. The entire complicated process of the system’s evolution in each collision is extremely fast and takes only about 10^{-22} s, or 10 fm/c.

A thorough analysis of a great massive of measured data indicates that at the early stage of the collision the created strongly interacting matter gets thermalized and demonstrates the collective properties. And although the precise mechanism of the thermalization is still not clear (see [1–9])

¹ The corresponding Lorentz γ factor in the center of mass frame is of the order of 10^2 for top RHIC energy and of the order of 10^3 for the LHC. Thus, the thickness of the partonic “pancake” in each case is of the order of 10^{-2} fm and 10^{-3} fm respectively. At the same time in the momentum space the corresponding momentum distributions are respectively Lorentz elongated, that can be accounted for by using a wide (in longitudinal direction) initial momentum distribution.

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for the corresponding discussion), the hypothesis that it takes place, and at some moment the system comes to a state of local equilibrium, allows one to utilize the relativistic hydrodynamics approximation to describe the system's evolution during the time, while such equilibrium is preserved.² The application of hydrodynamical formalism implies utilization of certain equation of state (EoS) for the considered quark-gluon fluid, which cannot be strictly defined at the moment and thus is model dependent.

The representation of the system in terms of continuous medium is convenient, however, at the late times the matter loses the local equilibrium, the hydrodynamics becomes inapplicable, and the system transforms into a gas of particles (this transformation is usually considered to be connected with the restoration of QCD chiral symmetry and happening at the temperature close to 150 MeV). Thus, for a proper description of the collision's final "afterburner" stage another type of a model is required, e.g. a hadron cascade model [10–12]. So, now a theorist who wishes to develop an adequate approach for the relativistic heavy-ion collisions most likely will need to combine in the model different approaches for simulation of different stages of the matter evolution—such collision models are known as hybrid ones [13–18].

In this paper we review the recent results on soft physics observables obtained for the high-energy heavy-ion collisions within the integrated hydrokinetic model (iHKM) [19, 20]. The iHKM can be considered as hybrid model as well, since it includes viscous hydrodynamics for description of locally equilibrated phase and switching to the UrQMD hadron cascade [10, 11] at the particlization isothermal hypersurface T_p . However, the advantage of iHKM is that it includes treatment of pre-equilibrium dynamics of the system, which most other models lack.

Typically the collision simulation in a hybrid model starts right from the hydrodynamics at relatively large times, $\tau \sim 0.6\text{--}1$ fm/c, when the system can be expected to become nearly thermalized. However, the initial state of the matter is associated with a certain model energy (or entropy) density distribution, related to a very early evolution moment just after the collision of the two nuclei (e.g. the Monte Carlo Glauber GLISSANDO model [21] or the Monte Carlo Kharzeev–Levin–Nardi model [22]). Such states are typically characterized by large anisotropy in the local rest frame momentum spaces and are far from equilibrium [23]. It means that they cannot serve as proper initial conditions for hydrodynamics. Some equilibration process, transforming this primordial distribution to a nearly hydrodynamical form, must

precede the system expansion described in terms of a continuous fluid.

In contrast to other models, in iHKM the simulation starts at the early time $\tau_0 \sim 0.1$ fm/c with the pre-thermal dynamics stage, during which a far-from-equilibrium energy–momentum tensor of the system gradually evolves to a relativistic viscous hydrodynamics locally (partially) equilibrated tensor, which in turn serves as the initial condition for the hydrodynamical stage. Such an early beginning of the matter evolution is important for successful description of data, since the collective effects develop noticeably just because of finiteness and azimuthal asymmetry of the system already at the pre-equilibrium stage without any pressure gradients [19].

Another known issue related to hybrid models concerns possible violations of the energy–momentum conservation law, when one switches from continuous medium to particle gas expansion at the hypersurface, which contains non-spacelike elements, using Cooper–Frye prescription. The hydrokinetic approach, previously developed in Refs. [24–28] (see also Ref. [29]) and implemented in the hydrokinetic model (HKM)—the predecessor of iHKM—allows to avoid this problem and consider the realistic continuous emission of particles from the system all along the process of its hydrodynamical evolution through the use of escape function formalism instead of the distribution function one. However, in [28] it was also shown that if one uses smooth initial conditions, obtained by averaging of a large set of single-event density profiles, the contribution from non-spacelike parts of the hadronization hypersurface will be less than 2% and can be neglected without bringing noticeable distortion to the final simulation results (this can be explained by small spatial sizes of the corresponding hypersurface elements at small τ and strong collective flows at large τ). For this reason, and because of rather time-consuming calculations required for a full-value hydrokinetic approach realization, we used sudden switching to hadronic cascade at the particlization isotherm in the iHKM studies discussed here.

In the recent papers [20, 30–32] a simultaneous description and prediction of various bulk observables including the femtoscopy radii were obtained in the iHKM for the LHC Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV and $\sqrt{s_{NN}} = 5.02$ TeV, as well as for Au+Au collisions at the top RHIC energy $\sqrt{s_{NN}} = 200$ GeV, using a single set of model parameters for each collision type. In Ref. [30] the role of the post-hydrodynamic stage of the system's evolution in particle production was investigated. In [33] the problem of resonances observability in view of the interaction of their decay products with the hadronic medium created at the late stage of the collision was analyzed within iHKM. The model allowed also to describe photon production (momentum spectra and v_2 coefficients) for the cases of top RHIC energy and LHC energy 2.76A TeV [34, 35]. As a result, an all-around detailed

² It is interesting, that although a new state of matter, created in relativistic nucleus-nucleus collisions is often referred to as "quark-gluon plasma" (QGP), to stress its property to contain free color charges, it actually behaves rather like a nearly perfect liquid, than like "plasma", i.e. like ionized gas.

picture of the matter evolution and particle emission in course of a relativistic heavy-ion collision was obtained, and the space-time structure of the created system was revealed. Here we summarize the works [20,30–33] concerning hadronic observables, direct photon production [34,35] and present a newly obtained results for Xe+Xe collisions at the LHC energy $\sqrt{s_{NN}} = 5.44$ TeV.

2 The model description

The full complicated evolution process of the system, created in ultrarelativistic heavy-ion collision, can be divided into several stages. Each stage description within iHKM is realized using an appropriate formalism.

2.1 Initial pre-equilibrium state formation

This stage models the earliest far-from-equilibrated energy–momentum distribution of partonic system, formed just after the nuclei collision. It is attributed to the initial proper time $\tau_0 \sim 0.1$ fm/c and serves as a starting point for the subsequent pre-thermal relaxation energy–momentum transport dynamics, which gradually transforms the non-equilibrated energy–momentum tensor to the one describing a thermalized state of the system in nearly local equilibrium. The latter should be reached by the thermalization time, believed to be close to the inverse pion mass, $\tau_{th} \approx 1$ fm/c. We assume the initial parton distribution function to have a factorized form

$$f(t_{\tau_0}, \mathbf{r}_{\tau_0}, p) = \epsilon(b, \tau_0, \mathbf{r}_T) f_0(\eta, p). \quad (1)$$

(Here $(t_{\tau_0}, \mathbf{r}_{\tau_0})$ are the Cartesian coordinates at the hypersurface $\sigma_0 : \tau = \tau_0$, which corresponds to the initial far-from-equilibrium state of the system.)

The transverse energy–density profile $\epsilon(b, \tau_0, \mathbf{r}_T)$ is generated using the GLISSANDO code [21], that implements Monte Carlo Glauber approach with parameter α that defines contribution to energy density from binary collisions (*bin*), correspondingly $1 - \alpha$ is related to contribution from wounded (*w*) nucleons:

$$\epsilon(b, \tau_0, \mathbf{r}_T) = \epsilon_0(\tau_0) \times \frac{(1 - \alpha)N_w(b, \mathbf{r}_T)/2 + \alpha N_{bin}(b, \mathbf{r}_T)}{(1 - \alpha)N_w(b = 0, \mathbf{r}_T = 0)/2 + \alpha N_{bin}(b = 0, \mathbf{r}_T = 0)}. \quad (2)$$

The boost-invariant momentum function $f_0(\eta, p)$, where $\eta = \tanh^{-1}(z/t)$ is the space-time rapidity, is assumed to have the form [36]:

$$f_0(\eta, p) = g \exp \left(-\sqrt{\frac{(p \cdot U)^2 - (p \cdot V)^2}{\lambda_{\perp}^2} + \frac{(p \cdot V)^2}{\lambda_{\parallel}^2}} \right), \quad (3)$$

where $U^{\mu} = (\cosh \eta, 0, 0, \sinh \eta)$ and $V^{\mu} = (\sinh \eta, 0, 0, \cosh \eta)$ are the flow and the longitudinal direction four-vectors respectively.

We use the approximation of massless partons. Then the parameters $\lambda_{\parallel}$ and $\lambda_{\perp}$ in (3) can be interpreted as the two effective temperatures, parallel and perpendicular to the beam axis, since in the “rest frame” $\eta = 0$, $(p \cdot U)^2 - (p \cdot V)^2 = p_{\perp}^2$ and $(p \cdot V)^2 = p_{\parallel}^2$. In this sense one can consider the utilized momentum distribution as the one, corresponding to the Color Glass Condensate (CGC) effective gluon field theory [37].

In all the presented calculations we use $\lambda_{\perp} = 1.4$ GeV. The normalization constant $g = g(\lambda_{\perp}, \lambda_{\parallel})$ is defined from the requirement that the T^{00} component of the initial energy–momentum tensor, formally associated with the function $f_0(\eta, p)$ only and calculated according to Eq. (5) with the substitution $f_{free}(x, p) \rightarrow f_0(\eta, p)$, is unity.

One can further introduce a model parameter $\Lambda = \lambda_{\perp}/\lambda_{\parallel}$, defining the initial state momentum anisotropy. Its value is fixed at $\Lambda = 100$ for all the considered experimental set-ups. Such a large anisotropy of the initial state is specific for the models based on the CGC theory.

The energy density distribution $\epsilon(b, \tau_0, \mathbf{r}_T)$ in (2) includes two contributions: $N_{bin}(b, \mathbf{r}_T)$ from the binary collisions and $N_w(b, \mathbf{r}_T)$ from the wounded nucleons models, where b is the impact parameter denoting the collision centrality. The parameter α regulates the proportion between these two terms, and $\epsilon_0(\tau_0)$ defines the maximal energy density in the center of the system at the initial time τ_0 in the most central events. The values of α and $\epsilon_0(\tau_0)$ are fixed based on experimental mean charged particle density dependence on centrality and the pion p_T spectrum slope in central events (see Fig. 1) and serve as the two main iHKM parameters that adjust the model for the description of a particular collision type. These parameters have the same values for all the centrality classes, while the collision centrality is regulated by the GLISSANDO options, specifying cuts on the participant nucleons number.

Note also, that while GLISSANDO allows one to generate density profiles of single collision events, which can be then used for an event-by-event analysis, in our current studies we used a single smooth initial state profile, obtained by averaging of 50000 GLISSANDO events, for simulation of each particular collision type at given centrality within iHKM. To be able to perform v_n analysis we performed the averaging in such a way that each single-event profile is shifted to match its center-of-mass with the common center-of-mass of the fireball and then rotated to align its major principal axis of the n^{th} harmonic moment to that of the fireball.

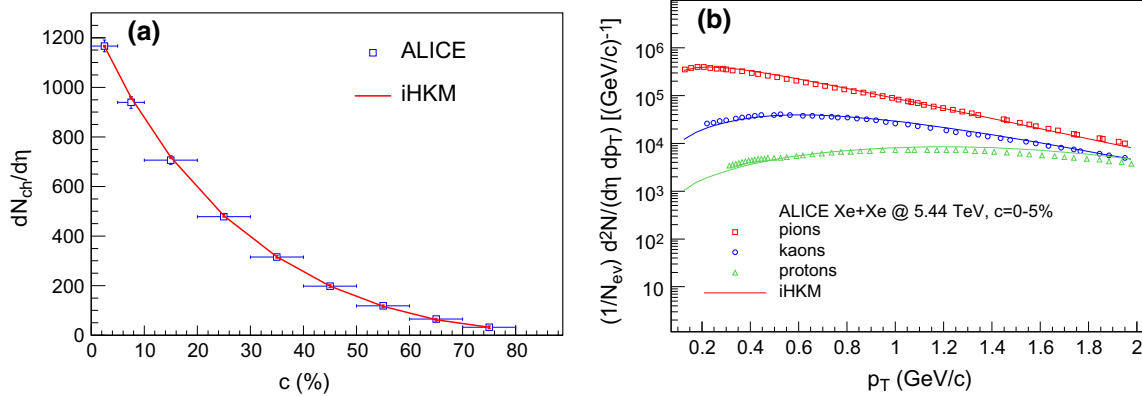


Fig. 1 The calibration of the iHKM parameters is performed based on the mean charged particle density $\langle dN_{ch}/d\eta \rangle$ dependency on centrality (a) and the p_T spectra slope in the most central events (b). The best combined fit within the model defines for a given collision type the

2.2 Pre-thermal matter evolution

During this stage the non-equilibrated initial energy-momentum tensor $T_0^{\mu\nu}(x)$ smoothly evolves, being little by little “mixed” with the Israel-Stewart tensor $T_{hydro}^{\mu\nu}(x)$, corresponding to viscous hydrodynamics regime. We use the relaxation time approximation associated with the equations of energy-momentum conservation for the tensor $T^{\mu\nu}(x)$ to evaluate the latter in course of its pre-equilibrium evolution. An important advantage of the applied formalism is that it does not require any additional assumptions, like the “anisotropic equilibrium” [40–42] or the Landau matching conditions, to continuously transfer the initial anisotropic and non-thermalized state at τ_0 to the nearly thermalized one at τ_{th} . Also, the utilized method is suitable for event-by-event simulations, since it allows to take into account large inhomogeneities of the system initial state, which will likely result in unusual transverse dynamics.

In the relaxation time approximation for Boltzmann equation the energy-momentum tensor of expanding matter $T^{\mu\nu}(x)$ at arbitrary moment of time between τ_0 and τ_{th} can be written as follows [43]

$$T^{\mu\nu}(x) = T_{free}^{\mu\nu}(x)\mathcal{P}(\tau) + T_{hydro}^{\mu\nu}(x)[1 - \mathcal{P}(\tau)]. \quad (4)$$

Here $T_{free}^{\mu\nu}(x)$ is an energy-momentum tensor, constructed based on the distribution function $f_{free}(t, \mathbf{r}, \mathbf{p}) = f(t_{\tau_0}, \mathbf{r} - \frac{\mathbf{p}}{p_0}(t - t_{\tau_0}), \mathbf{p})$, which corresponds to a free-streaming evolution of the initial non-equilibrium distribution (1), i.e.

$$T_{free}^{\mu\nu}(x) = \int d^3p \frac{p^\mu p^\nu}{p_0} f_{free}(x, p). \quad (5)$$

corresponding values of α and $\epsilon_0(\tau_0)$ parameters. In this figure the data and model curves for the LHC Xe+Xe collisions at $\sqrt{s_{NN}} = 5.44$ TeV are shown. The ALICE Collaboration experimental points are taken from [38,39]

The tensor $T_{hydro}^{\mu\nu}(x)$ is defined by the Israel-Stewart relativistic viscous hydrodynamics formula [44–46]:

$$T_{hydro}^{\mu\nu}(x) = [\epsilon_{hydro}(x) + p_{hydro}(x) + \Pi]u_{hydro}^\mu(x)u_{hydro}^\nu(x) - [p_{hydro}(x) + \Pi]g^{\mu\nu} + \pi^{\mu\nu}, \quad (6)$$

where ϵ_{hydro} is the local rest frame energy density, p_{hydro} is the local rest frame pressure, $\pi^{\mu\nu}$ is the shear stress tensor, $u_{hydro}^\mu(x)$ is the four-vector of energy flow, $g^{\mu\nu}$ is the metric tensor and Π is the bulk pressure. In our current studies we neglect the bulk pressure term, and so put $\Pi = 0$. The shear stress tensor, being independent dynamical variable, requires its own separate equation of motion, which we write, neglecting the vorticity terms, as follows:

$$\langle u^\gamma \partial_{;\gamma} \pi^{\mu\nu} \rangle = -\frac{\pi^{\mu\nu} - \pi_{NS}^{\mu\nu}}{\tau_\pi} - \frac{4}{3}\pi^{\mu\nu} \partial_{;\gamma} u^\gamma. \quad (7)$$

In this equation the semicolon denotes a covariant derivative, $\pi_{NS}^{\mu\nu}$ is the Navier-Stokes shear stress tensor, $\pi_{NS}^{\mu\nu} = \eta(\Delta^{\mu\lambda} \partial_{;\lambda} u^\nu + \Delta^{\nu\lambda} \partial_{;\lambda} u^\mu) - \frac{2}{3}\eta \Delta^{\mu\nu} \partial_{;\lambda} u^\lambda$, and brackets mean the following operation:

$$\langle A^{\mu\nu} \rangle = \left(\frac{1}{2} \Delta_\alpha^\mu \Delta_\beta^\nu + \frac{1}{2} \Delta_\alpha^\nu \Delta_\beta^\mu - \frac{1}{3} \Delta^{\mu\nu} \Delta_{\alpha\beta} \right) A^{\alpha\beta}, \quad (8)$$

where $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$.

The function $\mathcal{P}(\tau)$ in Eq. (4) is the weight function, which has to satisfy the following conditions:

$$0 \leq \mathcal{P}(\tau) \leq 1, \quad \mathcal{P}(\tau_0) = 1, \\ \mathcal{P}(\tau_{th}) = 0, \quad \text{and} \quad \partial_\mu \mathcal{P}(\tau_{th}) = 0. \quad (9)$$

Within the Boltzmann relaxation kinetics formalism, one can express the weight function in the following general

form [43]:

$$\mathcal{P}(\tau, \mathbf{r}, p) = \exp \left(- \int_{\tau_0}^{\tau} 1/\tau_{\text{rel}}(\tau', \mathbf{r}, p) d\tau' \right). \quad (10)$$

If one assumes that the relaxation time $\tau_{\text{rel}}(x, p)$ here depends only on τ for each fluid element, then $\mathcal{P}(\tau, \mathbf{r}, p)$ also will depend only on τ . This consideration corresponds to the Bjorken picture [47], assuming the thermalization of the matter to develop synchronously in proper time of different fluid elements. Then one will have:

$$\mathcal{P}(\tau) = \exp \left(- \int_{\tau_0}^{\tau} \frac{1}{\tau_{\text{rel}}(\tau')} d\tau' \right). \quad (11)$$

Taking $\tau_{\text{rel}}(\tau')$ in the form $\tau_{\text{rel}}(\tau') = \tau_{\text{rel}}(\tau_0) \frac{\tau_{\text{th}} - \tau'}{\tau_{\text{th}} - \tau_0}$, one can easily perform the integration in (11), and obtain the expression for $\mathcal{P}(\tau)$:

$$\mathcal{P}(\tau) = \left(\frac{\tau_{\text{th}} - \tau}{\tau_{\text{th}} - \tau_0} \right)^{\frac{\tau_{\text{th}} - \tau_0}{\tau_{\text{rel}}(\tau_0)}}. \quad (12)$$

Here $\tau_{\text{rel}}(\tau_0) \equiv \tau_{\text{rel}}$ is one of the iHKM parameters. In accordance with Eq. (9), it is required that $(\tau_{\text{th}} - \tau_0)/\tau_{\text{rel}} > 1$.

The evolution of the system's energy-momentum tensor will be governed by the relativistic hydrodynamics equations

$$\partial_{;\mu} \left\{ [1 - \mathcal{P}(\tau)] T_{\text{hydro}}^{\mu\nu}(x) \right\} = -T_{\text{free}}^{\mu\nu}(x) \partial_{;\mu} \mathcal{P}(\tau), \quad (13)$$

following from the energy-momentum conservation law for the full energy-momentum tensor, $\partial_{;\mu} T^{\mu\nu}(x) = 0$. One can see, that (13) can be re-written as the hydrodynamics equations for a new energy-momentum tensor $\tilde{T}_{\text{hydro}}^{\mu\nu}(x) = [1 - \mathcal{P}(\tau)] T_{\text{hydro}}^{\mu\nu}(x)$, such that $\tilde{T}_{\text{hydro}}^{\mu\nu}(x) = 0$ at $\tau = \tau_0$ for all x , with rescaled energy density $\tilde{\epsilon} = [1 - \mathcal{P}(\tau)]\epsilon$ and pressure $\tilde{p} = [1 - \mathcal{P}(\tau)]p$, and with an additional source term on the right-hand side:

$$\partial_{;\mu} \tilde{T}_{\text{hydro}}^{\mu\nu}(x) = -T_{\text{free}}^{\mu\nu}(x) \partial_{;\mu} \mathcal{P}(\tau). \quad (14)$$

The shear stress tensor also has to be re-defined as $\tilde{\pi}^{\mu\nu} = [1 - \mathcal{P}(\tau)]\pi^{\mu\nu}$, so that the corresponding equation of motion (7) will take the form:

$$\begin{aligned} [1 - \mathcal{P}(\tau)] \left\langle u^\gamma \partial_{;\gamma} \frac{\tilde{\pi}^{\mu\nu}}{(1 - \mathcal{P}(\tau))} \right\rangle \\ = - \frac{\tilde{\pi}^{\mu\nu} - [1 - \mathcal{P}(\tau)]\pi_{\text{NS}}^{\mu\nu}}{\tau_\pi} - \frac{4}{3} \tilde{\pi}^{\mu\nu} \partial_{;\gamma} u^\gamma. \end{aligned} \quad (15)$$

Since $T_{\text{free}}^{\mu\nu}(x)$ and $\mathcal{P}(\tau)$ are known, one can numerically find the solutions of the above equations with respect to $\tilde{T}_{\text{hydro}}^{\mu\nu}(x)$, and thus describe the pre-equilibrium dynamics of the system and gradual transition of the tensor $\tilde{T}_{\text{hydro}}^{\mu\nu}(x)$

to hydrodynamical regime by the time of thermalization, $\tilde{T}_{\text{hydro}}^{\mu\nu}(x) \rightarrow T_{\text{hydro}}^{\mu\nu}(x)$ when $\tau \rightarrow \tau_{\text{th}}$. However, to obtain such a description one also needs to close the system of hydrodynamical equations, which define the matter evolution, and specify the equation of state for considered quark-gluon fluid.

2.3 Hydrodynamics stage

At the hydrodynamical stage, which starts at $\tau = \tau_{\text{th}}$, the evolution of the system is described in terms of quark-gluon fluid expansion within the framework of Israel-Stewart relativistic viscous hydrodynamics. In Ref. [20], performing the calibration of iHKM parameters for the description of the LHC Pb+Pb collisions at 2.76A TeV, we found that the ratio of shear viscosity to entropy density parameter should be fixed to a value $\eta/s = 0.08 \approx 1/4\pi$, close to its minimum possible value according to Refs. [48, 49]. The viscous hydrodynamics equations (14) and (15) are numerically solved in iHKM using the vHLL code [50].

The hydrodynamical evolution of the system continues until it loses local equilibrium and transforms into hadron-resonance gas. It happens when the local temperature becomes lower than the particlization temperature T_p . The value of this temperature depends on the utilized equation of state for quark-gluon matter. In our studies we used the two QCD-inspired equations of state: Laine-Schroeder EoS [51] with the corresponding particlization temperature $T_p = 165$ MeV (it was chosen as giving the best description of experimental particle number ratios within the thermal model, see [52–54]) and HotQCD Collaboration EoS [55] with $T_p = 156$ MeV, consistent with the latest thermal model estimates as for the temperature of chemical freeze-out, $T_{\text{ch}} = 156 \pm 1.5$ MeV [56]. Also, both particlization temperatures are close to the Lattice QCD estimate for pseudo-critical temperature in the cross-over scenario, $T_{\text{ps}} = 154 \pm 9$ MeV.

In case of the RHIC energies, unlike in the LHC case, the utilized EoS needs to be corrected for a non-zero baryon and strange chemical potentials [57]:

$$\begin{aligned} \frac{p(T, \mu_B, \mu_S)}{T^4} &= \frac{p(T, 0, 0)}{T^4} + \frac{1}{2} \frac{\chi_B}{T^2} \left(\frac{\mu_B}{T} \right)^2 \\ &+ \frac{1}{2} \frac{\chi_S}{T^2} \left(\frac{\mu_S}{T} \right)^2, \end{aligned} \quad (16)$$

where $p(T, 0, 0)$ is the uncorrected equation of state at zero chemical potentials and

$$\frac{\chi_q}{T^2} = \frac{1}{VT^3} \frac{\partial^2 \ln Z}{\partial (\mu_q/T)^2}, \quad q = B, S. \quad (17)$$

In our studies, devoted to the Au+Au collisions at RHIC [32], we use the approximation of isentropic expansion of the Boltzmann massless gas to define the character of chemical

potential evolution. In this approximation one has $\mu_q/T = \text{const}_q$ along the world lines of each fluid element [58]. At the particlization isotherm $T_p = 165$ MeV we put $\mu_B = 21$ MeV to ensure the best description of the proton to antiproton yields ratio in the central events with $c = 0\text{--}5\%$. The strange chemical potential value $\mu_S = 5$ MeV at $T = T_p$ was fixed based on the condition of zero strangeness at the particlization hypersurface:

$$S|_{\sigma_p} = \sum_i [N(i) - \bar{N}(i)] \mu_{S,i} = 0, \quad (18)$$

where the summation is done over all the particle sorts, $N(i)$ and $\bar{N}(i)$ are the numbers of particles and antiparticles of the i th species at the hypersurface T_p , and $\mu_{S,i}$ is the particle strange chemical potential.

2.4 Particlization stage

At the particlization stage we switch from the continuous medium to particles cascade in the matter evolution description, using the common Cooper–Frye prescription [59]:

$$p^0 \frac{d^3 N_i(x)}{d^3 p} \Big|_{d\sigma(x)} = d\sigma_\mu(x) p^\mu \times f_i(p \cdot u(x), T(x), \mu_i(x)). \quad (19)$$

The subscript i here denotes the particle sort number and $d\sigma(x)$ is the element of the switching hypersurface σ_{sw} , related to the space-time point x .

As it was marked first by Van Hove [60] in 1983, the original Cooper–Frye formula for particle distribution at freeze-out (in our case—particlization) hypersurface can give negative contributions for momenta of some particles, if they radiate from non-space-like sites of the hypersurface. The treatment prescription for this problem, incorporating the two θ -functions into the original expression (19) was proposed soon in Refs. [61–63]. It was utilized in many works, particularly, in the package Fast Monte-Carlo Generator for freeze-out [64].

Later on, in one decade, simple modification of the Cooper–Frye formula with the only one θ -function, that just eliminated negative contribution, was also proposed and used in Refs. [65–67]. Despite slightly simpler form, such a modification violates particle number conservation in freeze-out area, in contrast to initial prescription [61–63]. (See detailed discussion of this issue in the review article [29]).

Considering the corrections to observables due to presence of non-space-like sectors, performing calculations first within HKM, and later within iHKM model, we found that these corrections are small, only a few percent. The reason for this is that proper times in such sectors are typically fairly small, and in addition, corresponding fluid elements move

very fast in transverse plane in case of ultrarelativistic nuclear collisions.

To build the particlization hypersurface we utilize the Cornelius routine [68]. The functions $f_i(x, p)$ in (19) are obtained after application of the Grad ansatz [69, 70] viscous corrections to the corresponding local equilibrium distribution functions $f_i^{\text{1.eq.}}(x, p)$. Assuming the same corrections for all hadron types, one can write (19) for starred variables related to the fluid local rest frame as follows:

$$\frac{d^3 \Delta N_i}{dp^* d(\cos \theta^*) d\phi^*} = \frac{\Delta \sigma_\mu^* p^{*\mu}}{p^{*0}} p^{*2} f_i^{\text{1.eq.}}(p^{*0}; T, \mu_i) \times \left[1 + (1 \mp f_i^{\text{1.eq.}}) \frac{p_\mu^* p_\nu^* \pi^{*\mu\nu}}{2T^2(\epsilon + p)} \right]. \quad (20)$$

Using this distribution, one can calculate the average particle number N_i for each species i . The sum of all these numbers will give the average total number of particles N_{tot} at the particlization hypersurface σ_{sw} . Then one can use Monte Carlo method to generate particles with their momenta and coordinates in accordance with (20). The total number of particles in a given event obeys the Poisson distribution with the mean value N_{tot} , and the species of each particle is selected randomly with probability N_i/N_{tot} .

2.5 Afterburner stage

At the final post-hydrodynamical stage of the system's evolution all the particles, generated at the switching hypersurface are passed to the UrQMD hadron cascade code [10, 11]. In iHKM we aim to account for all well-established hadron resonance states, composed of u, d , and s quarks, and since many of them are not processed by UrQMD, the heavy resonances not present in the UrQMD particle database, are decayed right at the particlization isotherm T_p to ensure the energy–momentum conservation.³

The UrQMD cascade code performs elastic and inelastic scatterings between hadrons and decays the particles known to be unstable. The program has multiple options, which allow one to change tuning of the model, e.g. switch baryon–antibaryon annihilation on/off or disallow some resonance decays to adjust the simulation to the experimental treatment of secondary particles.

As our previous studies demonstrate [30, 31, 33], the afterburner stage plays an important role in the formation of parti-

³ The utilized list of hadron states is consistent with the Laine–Schroeder EoS, however for the HotQCD EoS it leads to small violation of conservation laws. As it is shown in [71], to reach the full correspondence of HotQCD and HRG equations of state one needs to include in the latter one not only all the present states from Particle Data Group table (including not well-established), but also a large number of hypothetical states predicted by quark model. In our current analysis we do not account for these states.

cle yields, their ratios and spectra. This supports the hypothesis of continuous (not sharp) character of chemical and thermal freeze-out in heavy-ion collisions.

3 Results and discussion

3.1 Model calibration

To start a certain type of collision simulation in iHKM, one firstly needs to calibrate the model, setting the fitting values of its parameters, that would result in a proper description of the system's evolution process.

As it was shown in [20,31], the main time parameter, strongly affecting the formation of observables is the initial time τ_0 , while the thermalization time τ_{th} , defining the moment of switching from pre-thermal dynamics to hydrodynamics, when the system comes to a nearly thermalized state, and the relaxation time τ_{rel} , defining the rate of thermalization process, can be varied within quite wide limits and still give a good description of measured data, if the initial maximal energy–density parameter $\epsilon_0(\tau_0)$ is accordingly re-adjusted. Thus, for all the collision set-ups considered here the same setting of τ_{rel} and τ_{th} are used, namely $\tau_{rel} = 0.25$ fm/c and $\tau_{th} = 1$ fm/c. The same concerns the initial state momentum anisotropy parameter, Λ . As it was mentioned above, its value is fixed to $\Lambda = 100$ for all types of collisions under consideration.

The parameter α , defining the fraction of binary collision model contribution to the initial energy density distribution (2), has the smallest value in iHKM for Au+Au collisions at the top RHIC energy, $\alpha = 0.18$, a higher value $\alpha = 0.24$ for Pb+Pb collisions at both 2.76A TeV and 5.02A TeV energies and the maximum value $\alpha = 0.44$ for Xe+Xe collisions at 5.44A TeV. The increasing of α parameter could be connected with the increase of the inelastic nucleon-nucleon cross-section from $\sigma = 42$ mb in the RHIC Au+Au collisions case to $\sigma = 64$ mb for the LHC Pb+Pb collisions and $\sigma = 68.4 \pm 5.0$ mb for the LHC Xe+Xe collisions.

Apart from that, gold and lead nuclei have close radius values, about 6.37 fm and 6.62 fm respectively, while xenon nucleus has noticeably smaller radius about 5.36 fm. This also can lead to increasing of α value when going from Au+Au and Pb+Pb to Xe+Xe collisions, since the binary collisions model produces more narrow density distribution, which should be more appropriate in case of collision of smaller nuclei. Note also, that constructing the initial state for a Xe+Xe collision, one should account for a slightly prolate shape of xenon nuclei, characterized by the deformation parameter $\beta = 0.18 \pm 0.02$ [38]. Such an accounting can be realized using native GLISSANDO settings, so in our simulations we just put the values of corresponding GLISSANDO parameters BETA2A and BETA2B to 0.18.

The parameter τ_0 defines the initial time at which the pre-equilibrium stage of the matter evolution in iHKM starts. This time is associated with a far-from-equilibrium system state, formed very soon after the two nuclei overlapping in course of their collision. The estimates made in [72] suggest the values $\tau_0 \approx 0.1$ fm/c for the top RHIC energy collisions and $\tau_0 \approx 0.07$ fm/c for the full LHC energy case. In our works we primarily used $\tau_0 = 0.1$ fm/c for all the considered colliding systems. For the description of the top RHIC energy Au+Au collisions the Laine–Schroeder equation of state for quark-gluon phase and the particlization temperature $T_p = 165$ MeV were used, while for the case of the LHC Xe+Xe collisions at $\sqrt{s_{NN}} = 5.44$ TeV a more recent HotQCD Collaboration EoS with $T_p = 156$ MeV was chosen. In case of LHC Pb+Pb collisions the simulations were carried out using both mentioned equations of state with their corresponding particlization temperatures. Here the basic value $\tau_0 = 0.1$ fm/c was used with the Laine–Schroeder EoS, and in case of the HotQCD EoS very close starting time values $\tau_0 = 0.15$ fm/c for $\sqrt{s_{NN}} = 2.76$ TeV and $\tau_0 = 0.12$ fm/c for $\sqrt{s_{NN}} = 5.02$ TeV were applied in order to reach the best fitting of pion p_T spectrum at the model calibration. The corresponding values of the initial maximal energy density parameter $\epsilon_0(\tau_0)$ and the fraction of the binary collisions contribution to the initial energy–density profile α are listed in the Table 1.

3.2 Quark-gluon equations of state and freeze-out problem

The comparison of the simulation results obtained using different quark-gluon fluid equations of state can help to clarify the interplay between the factors, defining the behaviour of experimentally measured observables at hydrodynamical and post-hydrodynamical stages of the matter evolution. Note, that the applied equation of state here should be considered rather as the effective one, since the expansion rates of the system, formed in the ultrarelativistic heavy ion collision, are extremely high (even much higher, than in the Early Universe), so that the considered process is essentially dynamical (non-static) and thus can hardly be adequately described by the EoS, based on the lattice QCD calculations for the static system.

In Fig. 2 one can see the graphical comparison of the two EoS, utilized in our analysis. The two $p(\epsilon)$ plots look close, however a steeper growth of the energy density at high temperatures is observed for the Laine–Schroeder EoS. To analyze how the utilized EoS affects the magnitudes of the collective flows at the hadronization hypersurface, in Fig. 3 we compare the dependencies of the speed of sound square c_s^2 on the temperature T and the radial flow v_r at $r_T = 3$ fm on the proper time τ in iHKM for the two applied EoS's. From the first plot it follows, that at the temperatures, close to T_p , the c_s^2 magnitudes for the two equations of state are differ-

Table 1 The iHKM parameters α and $\epsilon_0(\tau_0)$, used to tune the model to the description of the listed heavy-ion collision events

Experiment	EoS	α	τ_0 (fm/c)	ϵ_0 (GeV/fm ³)
Au+Au @ 200 AGeV	L.-S.	0.18	0.1	235
Pb+Pb @ 2.76 ATeV	L.-S.	0.24	0.1	679
Pb+Pb @ 2.76 ATeV	HotQCD	0.24	0.15	495
Pb+Pb @ 5.02 ATeV	L.-S.	0.24	0.1	1067
Pb+Pb @ 5.02 ATeV	HotQCD	0.24	0.12	870
Xe+Xe @ 5.44 ATeV	HotQCD	0.44	0.1	445

ent, which should result in different accelerations in hydrodynamical regime. However, the second plot demonstrates, that the radial flow values at the times near $\tau \approx 10$ fm/c, corresponding to the final stage of the matter particlization, appear to be fairly close for both EoS's.

The analysis, carried out in [30, 31] shows that the experimental particle yields and their ratios, transverse momentum spectra of identified particles (π , K , p), v_n flow harmonics (for $n = 2, 3, 4$) and the femtoscopy radii of pions and kaons in Pb+Pb collisions at the LHC energies can be simultaneously described in iHKM with an equally good accuracy, using both Laine–Schroeder and HotQCD Collaboration equations of state, if switching to another EoS is accompanied by re-tuning of $\epsilon_0(\tau_0)$ parameters (see Figs. 4, 5, 6, 7, 8). The obtained results speak in favor of the hypothesis about the continuous character of chemical and thermal freeze-outs in the process of heavy ion collisions. Within the model it means, that the possible modifications in the system's expansion as a continuous medium, caused by the change of the EoS and leading to the changes in the final simulation results, can be compensated during the afterburner dynamics. This assumption is supported by our results for particle number ratios, calculated in the two regimes: the full iHKM simulation and the one with inelastic processes turned off at the afterburner stage (Fig. 4). The baryonic ratios, calculated without inelastic processes, have different values for the different utilized EoS's, however the same ratios, calculated in the full mode, coincide and agree with the experimental data.

Indeed, the post-hydrodynamic stage of the matter evolution at the LHC energy scales has a long duration, so that elastic and inelastic hadron-hadron interactions taking place at this stage should strongly affect the formation of bulk observables, measured in the experiment and calculated in the simulations. In Refs. [73, 74] the ALICE Collaboration pointed out, that it was the accounting for the annihilation processes at the final stage of collision, that allowed to achieve a better agreement of the model (anti)proton p_T spectra and yields with the data in the HKM model [28]. The influence of the afterburner inelastic interactions on the measured particle yields is investigated, e.g. in [75].

From the theoretical point of view, an abrupt transition from a very fast expansion of chemically equilibrated matter

with intensive inelastic reactions passing within the system to chemically frozen matter evolution without any inelastic processes, supposed by a sudden chemical freeze-out concept, looks unrealistic. The same concerns a sudden kinetic freeze-out. It would mean a sharp switching from a quite large hadron-hadron interaction cross-section to a vanishing one. But what would be the physical reason for this? Even a phase transition between quark-gluon matter and hadron gas in the considered expanding system can hardly be sudden in time.

In the paper [33], analyzing in iHKM the $K^*(892)$ observability in view of the interaction of its decay daughters with the hadronic medium at the afterburner stage, we found that in case of the LHC Pb+Pb collisions at 2.76 A TeV the effective duration of the kinetic freeze-out can reach about 5 fm/c after the particlization. Note also, that the term “continuous freeze-out” does not mean only different freeze-out times for different hadron species, as supposed, e.g. in [76]. The emission picture, obtained within iHKM in [33], suggests also the continuous character of particle radiation for each selected hadron species.

Another question related to the influence of the state of created matter at the early stages of collision on the measured observables concerns the angular momentum effects. One should note that in non-central $A + A$ collisions the interacting system has a residual angular momentum due to the shift between the centers of mass of overlapping parts of the two colliding nuclei [77–80]. In ultrarelativistic nuclear collisions the system has longitudinally very elongated shape and initially expands with large gradient along the corresponding axis. Therefore, more angular momentum is carried out by hadrons at large rapidities in c.m.s. and one cannot expect for rotation as whole of the very central, midrapidity, part of the fluid (the system cannot be considered a rigid body). Thus, the vorticity in the vicinity of $\eta = 0$ is significantly weakened. Moreover, with increasing $\sqrt{s}$, the moment of inertia grows more rapidly, than the total angular momentum of QGP, and can make the rotation and vorticity decrease even in the whole rapidity interval.

Therefore, since we deal with ultrarelativistic heavy ion collisions and observables at midrapidity only and do not consider femtoscopy in peripheral collisions, we neglect the

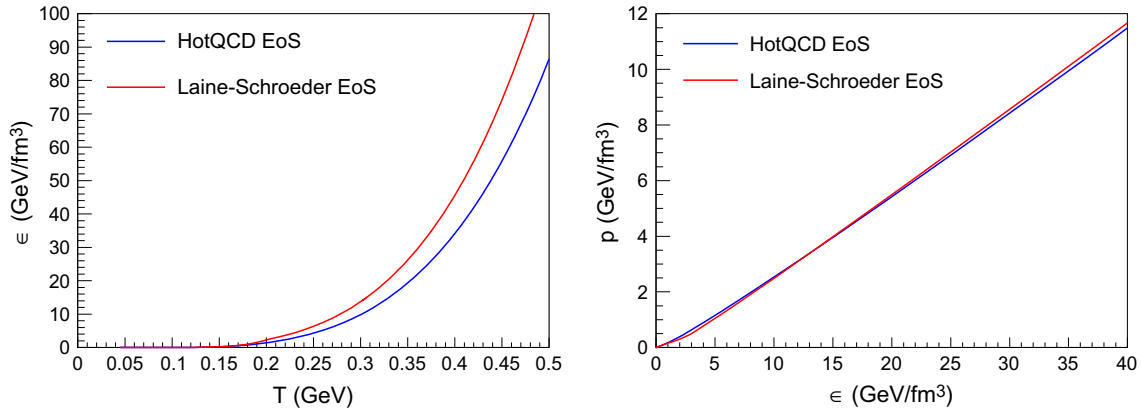


Fig. 2 The comparison of the two quark-gluon phase equations of state, applied in iHKM simulations. The red line is related to the Laine-Schroeder EoS [51] with the temperature of particlization $T_p = 165$ MeV, and the blue line corresponds to the HotQCD Collaboration EoS [55] with $T_p = 156$ MeV

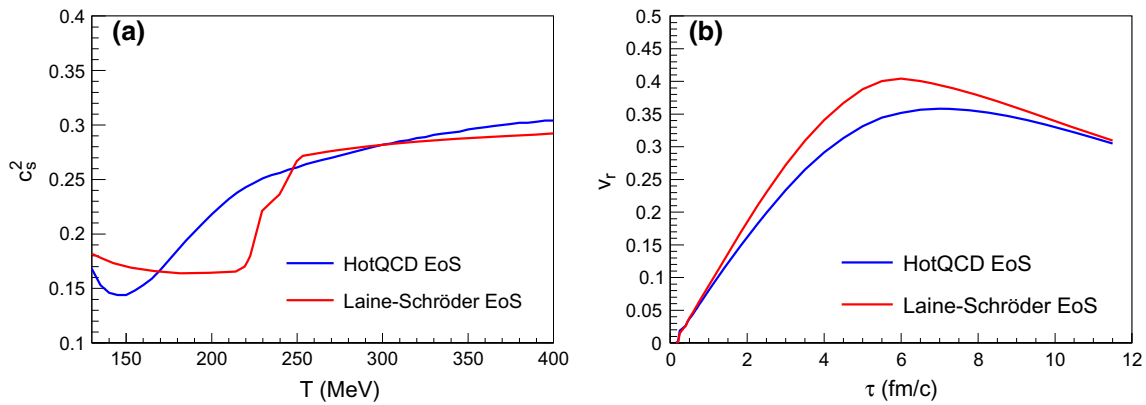


Fig. 3 The speed of sound square dependency on the temperature $c_s^2(T)$ (a) and the radial flow dependency on the proper time $v_r(\tau)$ at the transverse radial coordinate $r_T = 3$ fm (b), calculated within the iHKM

for the two quark-gluon phase equations of state: the Laine-Schroeder EoS (red) [51] and the HotQCD Collaboration EoS (blue) [55]

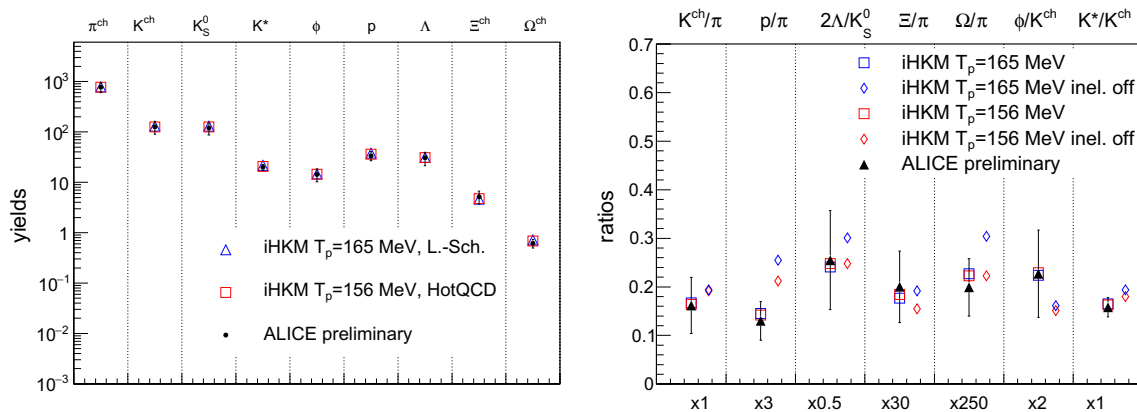


Fig. 4 The particle yields dN/dy (left) and particle number ratios (right) calculated in the iHKM using the two equations of state for quark-gluon matter with the corresponding particlization temperatures: the Laine-Schroeder EoS with $T_p = 165$ MeV and the HotQCD Collaboration EoS with $T_p = 156$ MeV. The results concern Pb+Pb collisions

at the LHC energy $\sqrt{s_{NN}} = 5.02$ TeV, $c = 0-10\%$ and are presented along with the ALICE Collaboration preliminary data [81, 82]. The particle number ratios were calculated in the two modes: the full iHKM simulation and the simulation without inelastic reactions at the afterburner stage

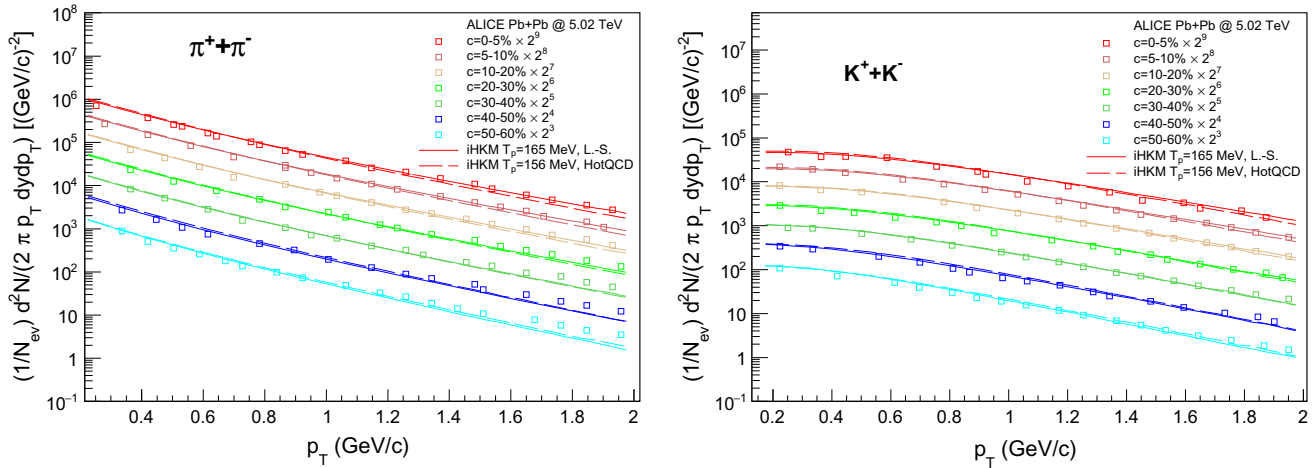


Fig. 5 The p_T spectra of pions (left) and kaons (right) for different centralities, calculated in the iHKM using the two equations of state for quark-gluon phase with the corresponding particlization temperatures: the Laine–Schroeder EoS with $T_p = 165$ MeV and the HotQCD Col-

laboration EoS with $T_p = 156$ MeV. The model results are compared to the ALICE Collaboration preliminary data [83]. The data for different centrality classes are scaled by the different powers of 2 for better readability

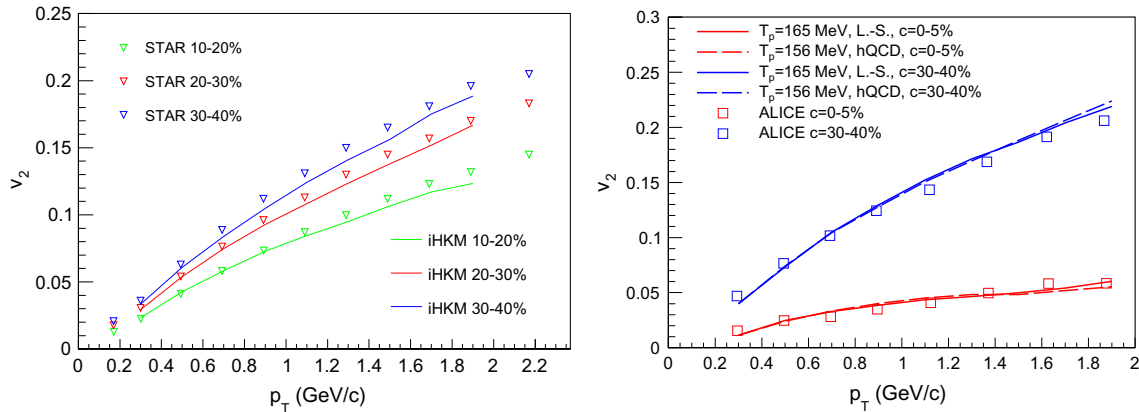


Fig. 6 The all charged particles $v_2(p_T)$ dependency, calculated in iHKM for the top RHIC energy Au+Au collisions (left) and for the LHC energy 5.02A TeV Pb+Pb collisions (right). The different centrality classes are concerned ($c = 10-20\%$, $c = 20-30\%$ and $c = 30-40\%$ in the RHIC case; $c = 0-5\%$ and $c = 30-40\%$ in the LHC case). In the LHC case the two equations of state for quark gluon phase with

their corresponding particlization temperatures were used for the simulations: the Laine–Schroeder EoS with $T_p = 165$ MeV and the HotQCD Collaboration EoS with $T_p = 156$ MeV. The experimental points correspond to the STAR and the ALICE Collaborations data, taken from [84] and [85] respectively

effects associated with angular momentum. Accordingly, the question about the influence of used equation of state on, e.g., Λ polarization, associated with vorticity, remains open in our model. Nevertheless, one can note some possibilities for the compensating effects again. For example, as it is seen in Table 1, the Laine–Schroeder EoS needs earlier starting time for hydrodynamics than the HotQCD one. It means, because of larger gradients, larger initial vorticity for Laine–Schroeder EoS. However, in this regime, the post-hadronization stage starts at larger energy densities, than in the case of HotQCD EoS, and so particle depolarization during interactions in expanding hadron-resonance gas is

expressed to a greater extent than at HotQCD EoS. Within the current studies we, however, do not check such a hypotheses of compensation.

3.3 Particle production

Once the model is calibrated for description of a particular collision type, based on the experimental charged particle multiplicity dependency on collision centrality and pion p_T spectrum slope in the most central events, a wide class of soft observables, including particle yields, particle number ratios, transverse momentum spectra, flow harmonics,

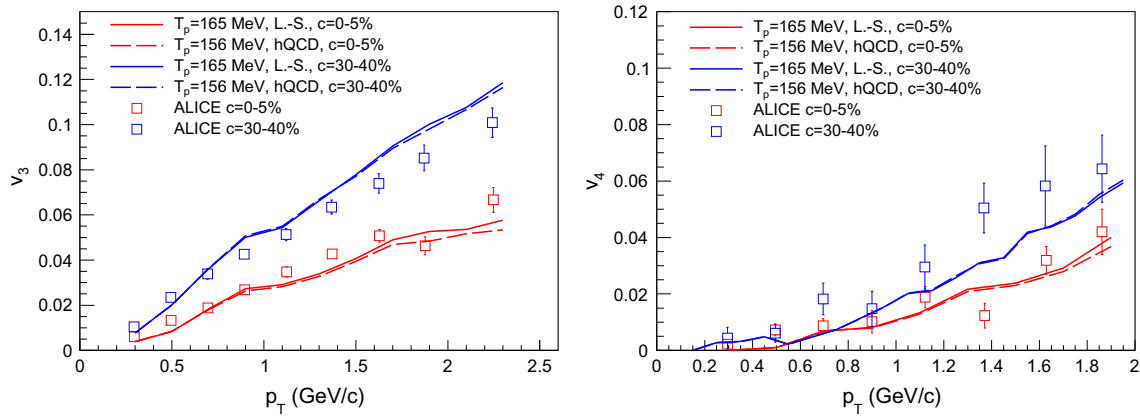
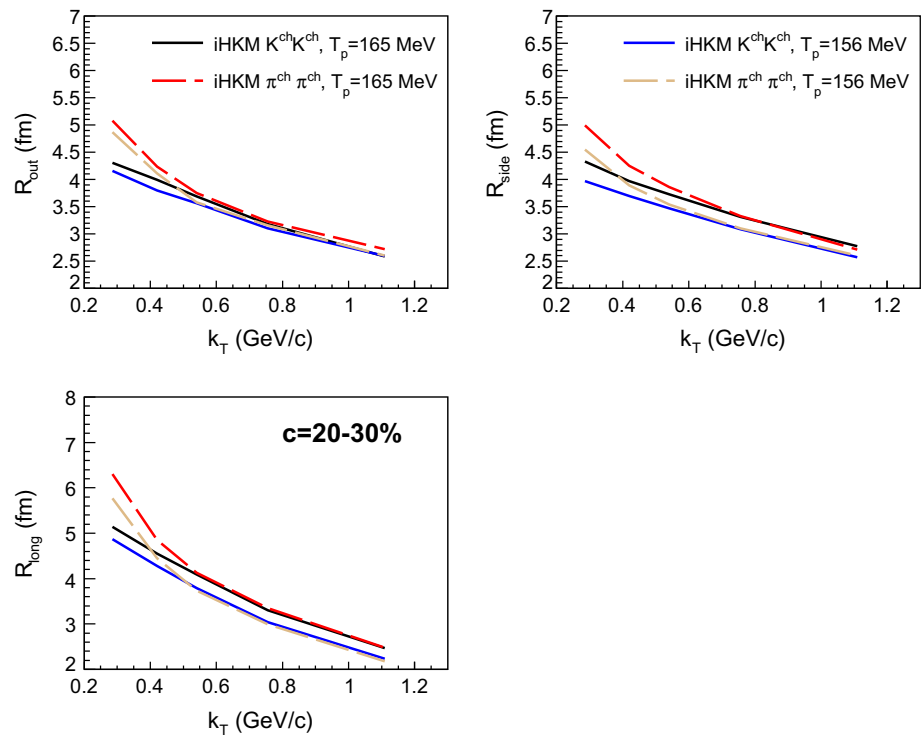


Fig. 7 The all charged particles v_3 and v_4 flow harmonics for different p_T , calculated in iHKM for Pb+Pb collisions at the LHC energy 5.02 A TeV. The Laine–Schroeder EoS with $T_p = 165$ MeV and the HotQCD Collaboration EoS with $T_p = 156$ MeV are used in the simu-

lations for quark–gluon matter. The results for the two centrality classes, $c = 0–5\%$ and $c = 30–40\%$, are presented. The ALICE Collaboration experimental points are taken from [85]

Fig. 8 The pion and kaon interferometry radii dependencies on k_T , as predicted within iHKM for Pb+Pb collisions at the LHC energy $\sqrt{s_{NN}} = 5.02$ TeV, $c = 20–30\%$. The Laine–Schroeder EoS with $T_p = 165$ MeV and the HotQCD Collaboration EoS with $T_p = 156$ MeV are used in the simulations for quark–gluon matter



femtoscopy scales, can be described simultaneously within iHKM, using the single found setting of model parameters for each experiment set-up (again see Figs. 4, 5, 6, 7, 8). The results, obtained in [20, 30–32], provide a detailed description of particle production in Au+Au collisions at the RHIC energy $\sqrt{s_{NN}} = 200$ GeV and in Pb+Pb collisions at the two LHC energies, $\sqrt{s_{NN}} = 2.76$ TeV and $\sqrt{s_{NN}} = 5.02$ TeV.

In Figs. 9, 10 one can also see the latest iHKM results for p_T spectra of all charged particles, pions, kaons and protons in Xe+Xe collisions at the LHC energy $\sqrt{s_{NN}} = 5.44$ TeV for eight centrality classes ($c = 0–5\%$, $c = 5–10\%$, $c = 10–20\%$, $c = 20–30\%$, $c = 30–40\%$, $c = 40–50\%$,

$c = 50–60\%$, $c = 60–70\%$). The simulation results are compared with the ALICE Collaboration experimental data [38, 39] and appear to reproduce them well in a wide p_T range, that includes soft momentum interval. The accuracy of the spectra description is comparable with previous iHKM results for the top RHIC energy Au+Au collisions and the LHC Pb+Pb collisions.

The iHKM results for various particle number ratios in the LHC Xe+Xe collisions at 5.44 A TeV are presented in Fig. 11 together with the results for other LHC collisions and the corresponding experimental points [81, 82, 86]. The shown model ratios are obtained in the full simulation regime, with

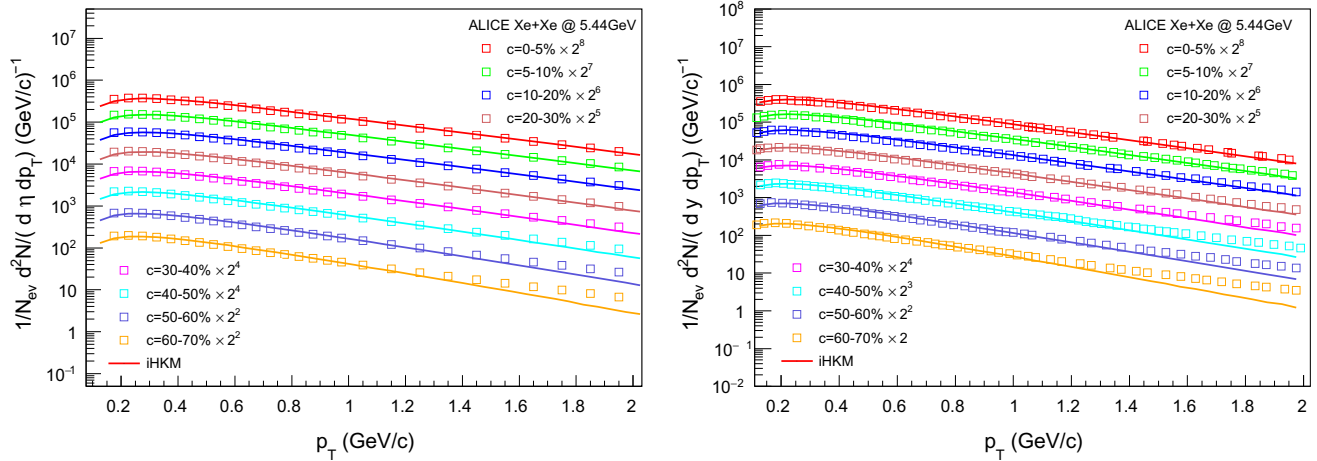


Fig. 9 The p_T spectra of all charged particles ($|\eta| < 0.8$, left) and pions ($|y| < 0.5$, right) in the LHC Xe+Xe collisions at 5.44 TeV, calculated in iHKM, in comparison with the ALICE Collaboration experimental points [38,39]

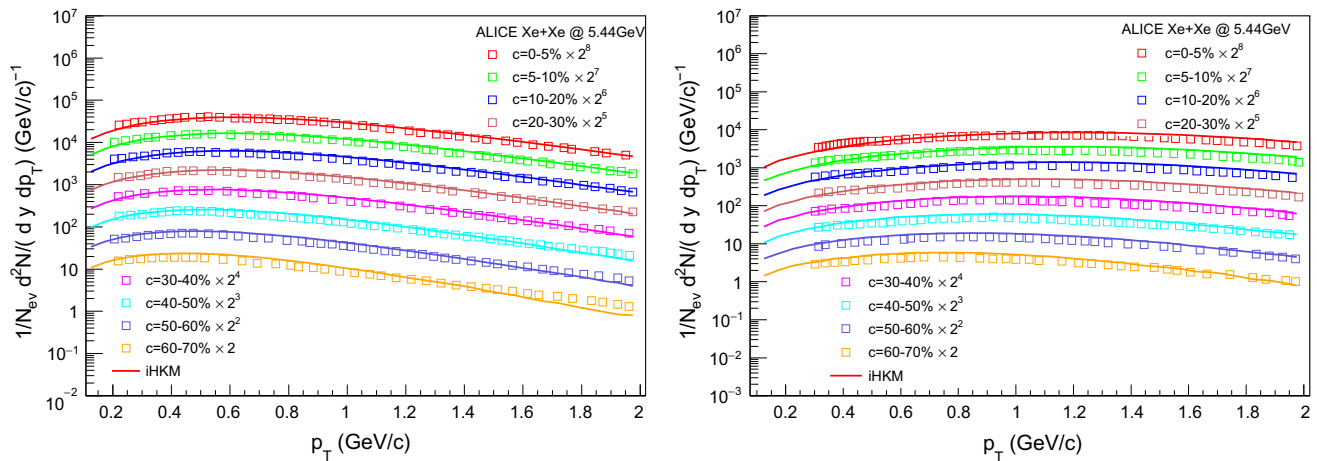


Fig. 10 The same as in Fig. 9 for kaons (left) and protons (right), $|y| < 0.5$ (The ALICE Collaboration experimental data are from the Ref. [39])

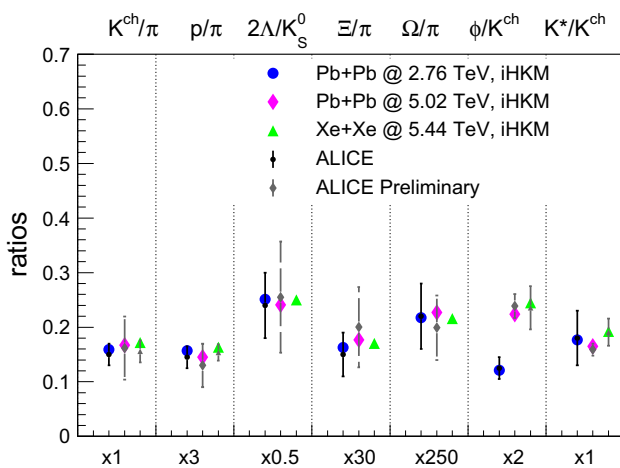


Fig. 11 The particle number ratios, calculated in iHKM for the LHC Pb+Pb collisions at 2.76 TeV and 5.02 TeV, as well as for the LHC Xe+Xe collisions at 5.44 TeV. The experimental data are also presented in cases when they are available [81,82,86]

inelastic processes at the post-hydrodynamic stage turned on. They agree with the measured data within the errors. The ratios of the same hadron yields in different collisions are close to each other, except for the ϕ/K^{ch} ratio in Pb+Pb collisions at the LHC energy $\sqrt{s_{NN}} = 2.76$ TeV. The latter fact can be possibly connected with a shorter in time and not so intensive afterburner dynamics in the 2.76 TeV case as compared to higher energy collisions at $\sqrt{s_{NN}} = 5.02$ TeV and $\sqrt{s_{NN}} = 5.44$ TeV. The afterburner dynamics can be important for the ϕ/K^{ch} ratio formation, since additional ϕ resonances appear at the late stage of the collision due to K^+K^- recombinations and correlations [33], increasing both the ϕ yield and the ϕ/K ratio. This effect should be smaller for collisions at smaller energy, so that the ϕ/K ratio in 2.76 TeV collisions will be smaller than those in 5.02 TeV and 5.44 TeV cases.

In order to describe the particle production in Au+Au collisions at the top RHIC energy [32] we had to introduce

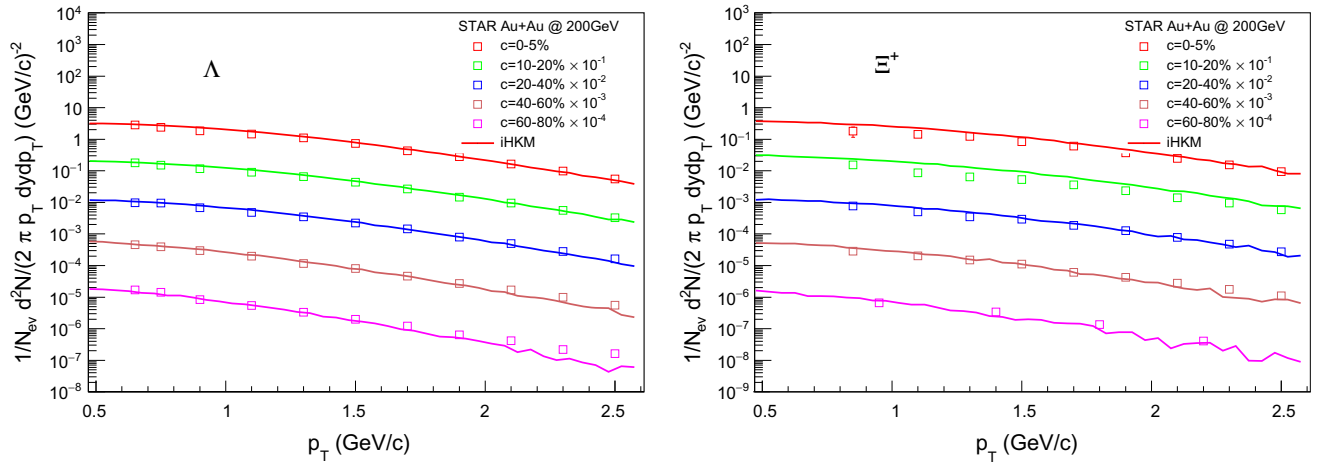


Fig. 12 The iHKM results for Λ and Ξ^+ hyperons p_T spectra in comparison with the STAR Collaboration experimental data [87]. The lambdas with $|y| < 1$ and cascade particles with $|y| < 0.75$ were selected for the analysis

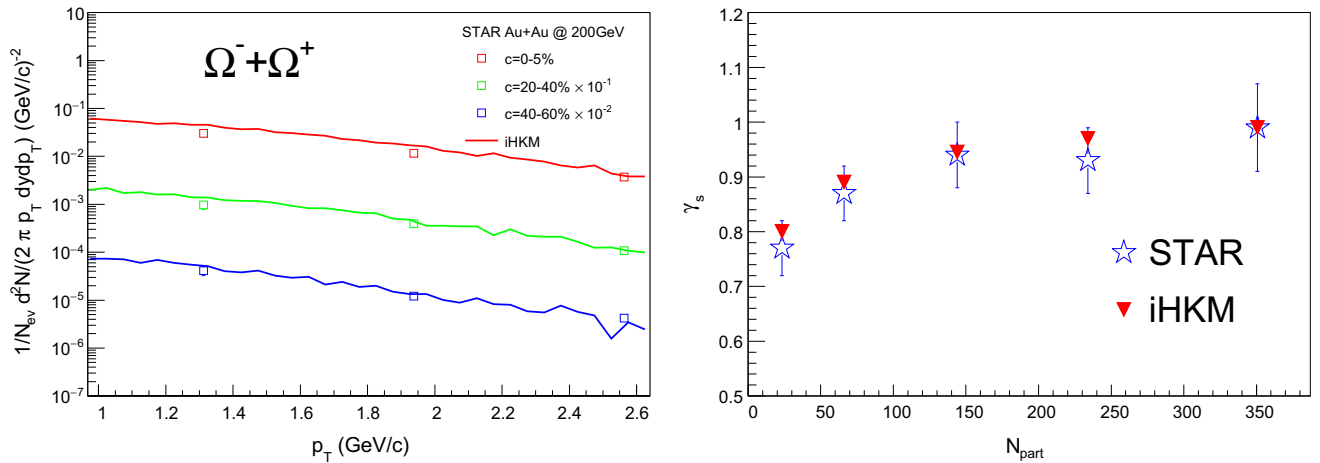


Fig. 13 The iHKM results for Ω hyperon p_T spectra (for $|y| < 0.75$) in comparison with the STAR Collaboration experimental data [87] and the γ_s factors, applied in iHKM simulations, together with those extracted by STAR in [87]

a downscaling strangeness suppression factor γ_s for non-central events. It was assumed to depend on the particlization time τ_p for the center of the system, calculated in iHKM and approximately characterizing the life-time of quark-gluon matter for each collision centrality.

$$\gamma_s(\tau_p) = A \exp(-b/\tau_p), \quad (21)$$

where $A = 1.1$ and $b = 0.8 \text{ fm}/c$.

The utilization of such a factor is motivated by the consideration that in non-central events the strange quarks do not have enough time to reach the local chemical equilibrium. This leads to a noticeable lowering of kaon and proton spectra (about a half of observed protons are produced as a result of strange resonances decays). The hyperon spectra are also affected. To correct the situation, each calculated hadron yield was multiplied by $\gamma_s(\tau_p)^{S_i}$, where S_i is the i th hadron sort strangeness. As a result, a good description for the mea-

sured kaon, proton and hyperon p_T spectra, as well as for the corresponding particle number ratios was reached.

In Figs. 12, 13 the results for Λ , Ξ and Ω hyperon spectra are demonstrated, as well as the comparison of γ_s values, used in the model, and those extracted within the experimental analysis from the Boltzmann fits to the measured spectra [87]. The model curves for hyperon spectra agree with the data and the utilized γ_s factors coincide with the experimental ones within the error bars.

3.4 Femtoscopy scales

The correlation femtoscopy is a powerful experimental technique, that allows to reveal the space-time structure of the system, formed in relativistic heavy-ion collision, presenting it in terms of the interferometry (femtoscopy) radii. The latter are extracted from the Gaussian fits $[1 + \lambda \exp(-q_i^2 R_i^2)]$

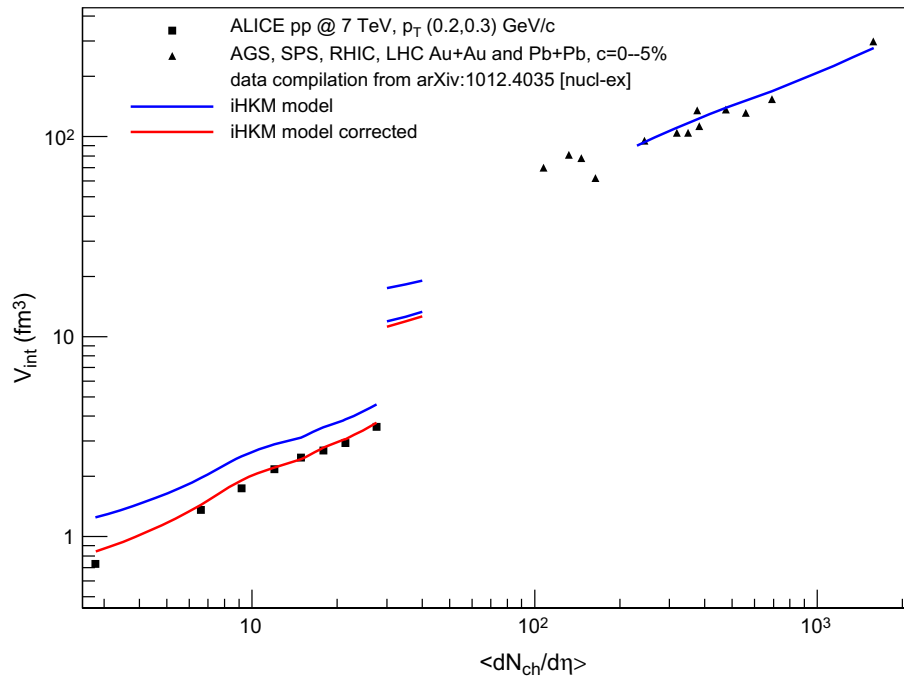


Fig. 14 The interferometry volume $V_{\text{int}} = R_{\text{out}} R_{\text{side}} R_{\text{long}}$ dependency on the mean charged particle density $\langle dN_{\text{ch}}/d\eta \rangle$. The blue line at the right corresponds to iHKM results for the central heavy-ion collisions at AGS, SPS, RHIC and LHC. The blue and red lines at the left describe the model results for the LHC $p + p$ collisions at the energy $\sqrt{s} = 7$ TeV. The corresponding experimental data are taken from the papers [98–106]. The line fragments in the middle correspond to the

model predictions for the case of $p + \text{Pb}$ collisions at the LHC energy $\sqrt{s} = 5.02$ TeV, calculated using the two different initial transverse system's sizes: $R = 1.5$ fm (upper line) and $R = 0.9$ fm (two lower lines). The red lines show the results after application of the quantum corrections following [97], and the blue lines demonstrate the results without corrections

to the experimentally measured two-particle momentum correlation functions $C(k_T, q)$ of identical particles in each k_T bin. Each $R_i(k_T)$ is commonly interpreted as the homogeneity length of the system in i th direction, related to the region, where the particles with momenta close to k_T are mostly emitted from [88–90].

The iHKM provides a satisfactory description of the femtoscopy radii in the considered high-energy collision experiments, for which the experimental data are available for comparison (except, maybe, for a somewhat overestimated pion *out* radii in the top RHIC energy Au+Au collisions at $c = 0$ –10% [32], as compared to the PHENIX Collaboration results), and allows to obtain predictions for not yet measured data (see Fig. 8).

The observed in iHKM behaviour of the radii with k_T is quite similar at all the collision energies. At the fixed centrality the radii values grow by 15–20%, when going from the RHIC to the LHC, and the radii, related to the different LHC energies are very close, with differences not exceeding 3–5%. Such similarity of the femtoscopy scales for the LHC collisions, that takes place despite the differences in the system geometric sizes at the freeze-out stage, can be explained by the fact, that collective flow velocity gradients are higher at

larger energies, and so this compensates the different geometry influence, resulting in close lengths of homogeneity in the considered cases [88,90].

An approximate k_T scaling, that can be observed in Fig. 8 between kaon and pion femtoscopy radii at $k_T > 0.4$ GeV/ c , was firstly noticed for the LHC Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV in iHKM simulations [91] and then confirmed in the experimental analysis [92].

In the current paper we also present the comparison of the interferometry scales, obtained in iHKM simulations for the high-energy collision events with close charged particle multiplicities. Such a comparison has an aim to check the so-called “scaling hypothesis” (see, e.g. [93,94]), supposing that the interferometry radii and the corresponding interferometry volume $V_{\text{int}} = R_{\text{out}} R_{\text{side}} R_{\text{long}}$ grow almost linearly with the mean charged particle density $\langle dN_{\text{ch}}/d\eta \rangle$. Despite the fact the scaling hypothesis is often assumed in femtoscopy studies, we believe, that the femtoscopy scales should depend also on geometrical sizes of the colliding ions (see Fig. 14), as it was previously noticed in [95–97].

To investigate this issue, for all the considered collision types we selected the events of different centralities, such that the mean charged particle multiplicities in each case were

Table 2 The centrality and the mean charged particle multiplicity $\langle dN_{\text{ch}}/d\eta \rangle$ at $|\eta| < 0.5$ of the events, chosen for the analysis of the femtoscopy scales dependency on the geometrical sizes of the colliding nuclei

Experiment	Centrality (%)	$\langle dN_{\text{ch}}/d\eta \rangle$
Au+Au @ 200 GeV	0–5	688
Pb+Pb @ 2.76 TeV	19–28	693
Pb+Pb @ 5.02 TeV	23–33	677
Xe+Xe @ 5.44 TeV	10–19	680

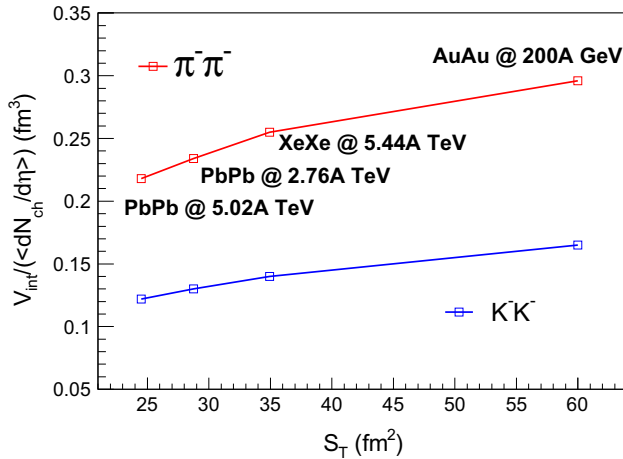


Fig. 15 The pion and kaon interferometry volume $V_{\text{int}} = R_{\text{out}} R_{\text{side}} R_{\text{long}}$, divided by the mean charged particle density $\langle dN_{\text{ch}}/d\eta \rangle$, calculated in iHKM for different relativistic heavy-ion collisions, characterized by the areas S_T of the colliding nuclei initial transverse overlapping. The corresponding collision centralities can be found in Table 2. The interferometry radii correspond to the pair transverse momentum $0.2 < k_T < 0.3$ GeV/c

nearly equal (see Table 2). Then we built the graphs of pion and kaon interferometry volume dependencies on S_T parameter, which characterizes the sizes of the initial transverse overlapping of the two colliding nuclei (see Fig. 15). This parameter is calculated as the area of nearly elliptical cross-section of the initial energy-density profile at the half-value of the maximal initial energy density $\epsilon_0(\tau_0)$,

$$S_T = \pi ab, \quad (22)$$

where a and b are the semi-axes of the ellipse. The V_{int} values were also divided by the corresponding mean charged particle multiplicities to avoid the effect of still existing small differences in $\langle dN_{\text{ch}}/d\eta \rangle$ between the experimental set-ups.

From Fig. 15 it follows that the plotted reduced volume grows with S_T , having the smallest value for the LHC Pb+Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV, a somewhat higher value in the $\sqrt{s_{NN}} = 2.76$ TeV case, after which follows the volume in Xe+Xe collisions at $\sqrt{s_{NN}} = 5.44$ TeV, and the highest one is the value for Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV.

The next plot in Fig. 16 shows the individual kaon interferometry radii dependencies on the pair k_T for the analyzed collision experiments together with the corresponding re-scaled initial energy density profiles $\epsilon(x)/\epsilon_0$. One can see, that the differences between the V_{int} values, observed in Fig. 15, are mainly due to the differences in the corresponding transverse radii. The contribution from R_{out} radius is the principal one, the R_{side} contribution is smaller, and the R_{long} radii in all the considered cases practically coincide. The obtained results confirm the assumption, made in [95–97], that the origin of the observed differences between the interferometry volumes is in different transverse flow velocities, which develop in the analyzed systems because of the different initial geometric sizes and the energy/pressure gradients.

Based on the combined fitting of the longitudinal femtoscopy radii dependency on k_T and the transverse momentum spectrum, one can estimate the time of maximal emission for particles of a given species [107], i.e. the effective time, that can be associated with the maximal intensity of the emission process, when the corresponding particles most actively leave the system. The presented method was applied by the ALICE Collaboration for the extraction of pion and kaon times of maximal emission from the experimental data [92].

The first step is to perform a combined fitting of pion and kaon p_T spectra with the formula

$$p_0 \frac{d^3 N}{d^3 p} \propto \exp[-(m_T/T + \alpha)(1 - \bar{v}_T^2)^{1/2}], \quad (23)$$

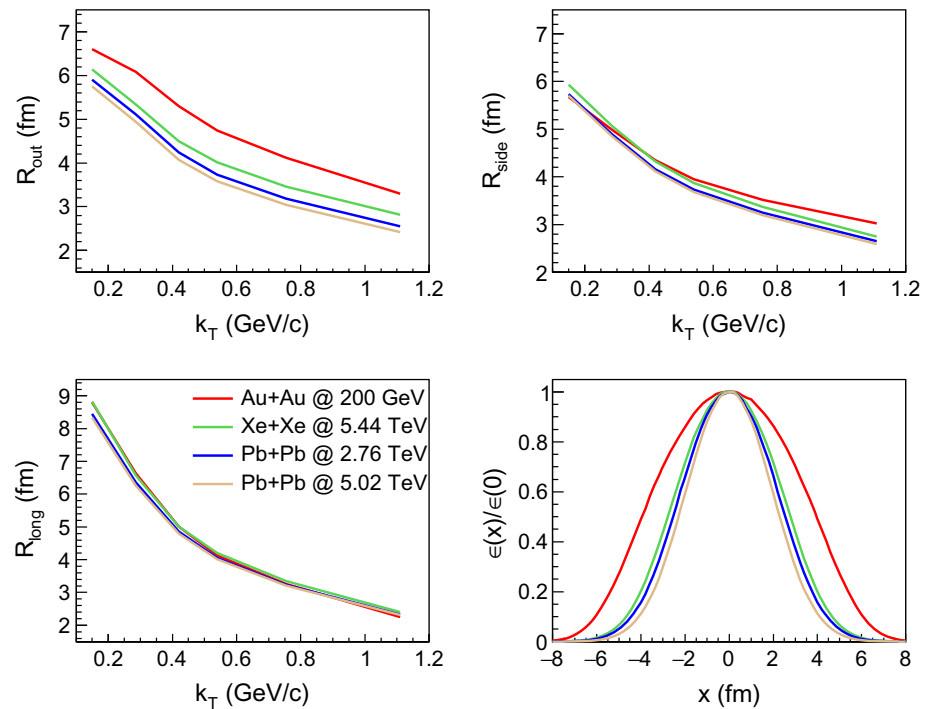
where T is the effective temperature, α is the parameter, describing the strength of collective flow in such a way, that the infinite α means zero flow, and small α means strong flow, $\bar{v}_T$ is the transverse collective velocity at the saddle point, $\bar{v}_T = k_T/(m_T + \alpha T)$ (see Ref. [107] for details). From the fit one extracts the common effective temperature of pions and kaons and the two values α for each particle sort.

At the second step one should carry out the fitting of $R_{\text{long}}(m_T)$ dependencies for pions and kaons using the formula

$$R_{\text{long}}^2(m_T) = \tau^2 \lambda^2 \left(1 + \frac{3}{2} \lambda^2\right), \quad (24)$$

where λ is expressed through the longitudinal homogeneity length of the system λ_l , $\lambda^2 = (\lambda_l/\tau)^2 = T/m_T \cdot (1 - \bar{v}_T^2)^{1/2}$, and τ is the maximal emission time, the one is interested in. When performing this fit, the T and α_π parameters should be constrained within the limits, defined earlier during the spectra fitting. However, the α_K parameter for kaons should be left unconstrained. As a result, one extracts from the fit the desired times of maximal emission τ_π and τ_K . In the paper [32] we obtained the values $\tau_\pi = 7.12 \pm 0.01$ fm/c and $\tau_K = 9.71 \pm 0.02$ fm/c. These results support the hypoth-

Fig. 16 The kaon interferometry radii, obtained in iHKM for different collision events with close charged particle multiplicity (see Table 2 for the details). The right lower figure demonstrates the corresponding re-scaled initial transverse energy-density distributions $\epsilon(x)/\epsilon_0$



esis about the continuous character of particle emission and freeze-out in high-energy heavy-ion collisions.

3.5 Direct photon production

In spite of the hadrons, that are formed at the last stage of superdense system evolution, the direct photon production is accumulated from the different sources along with the process of relativistic heavy ion collision developing. Those include the primary hard photons from the parton collisions at the very early stage of the process, the photons generated at the pre-thermal phase of dense matter evolution, then thermal photons at partially equilibrated hydrodynamic quark-gluon stage, and, finally, from the hadron gas phase. All the phases of evolution are presented in iHKM, therefore, the corresponding calculations with initial conditions providing a good description all the bulk hadron observables (see previous discussions) were done in the model for top RHIC [35] and LHC [34] energies.

Along the way a hadronic medium evolution is treated in two distinct, in a sense opposite, approaches: chemically equilibrium and chemically non-equilibrium, namely, chemically frozen expansion. We find the description of direct photon spectra, elliptic and triangular flow are significantly improved, for both RHIC the LHC energies, if an additional portion of photon radiation associated with the confinement processes, the “hadronization photons”, is included into consideration. In iHKM this contribution is introduced with only one free parameter, characterizing intensity of the photon radiation at the confinement process. This parameter is fixed from fitting the direct photon spectra in most central events,

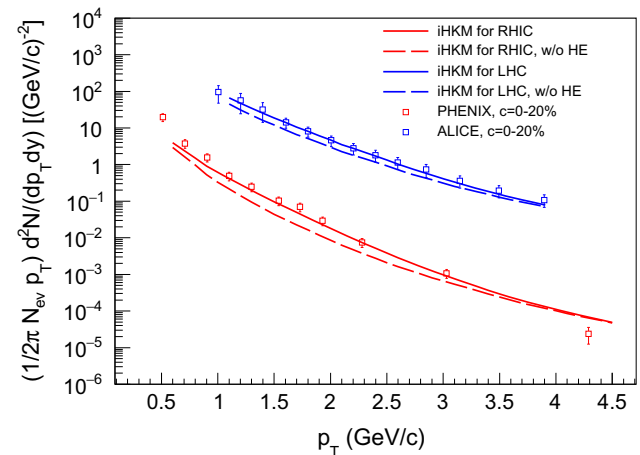


Fig. 17 The iHKM results for photon spectra in the top RHIC energy and the LHC 2.76A TeV nucleus-nucleus collisions ($c = 0-20\%$) compared to the experimental results [108, 110]. The model curves corresponding to the calculations with and without hadronization emission (HE) are shown

and used then at all centralities for spectra, as well as for description of elliptic and triangular photon flow. Figure 17 demonstrates the iHKM results for RHIC and LHC photon spectra in central events ($c = 0-20\%$) with and without this hadronization emission (HE). The iHKM results for RHIC (centrality 20-40%) photon elliptic and triangular flows are demonstrated in Fig. 18.

The problem of serious underestimation of direct photon production, as well as their flows in theoretical models as compared to the experimental data is called “direct photon puzzle”. From our results it can be concluded, that photon

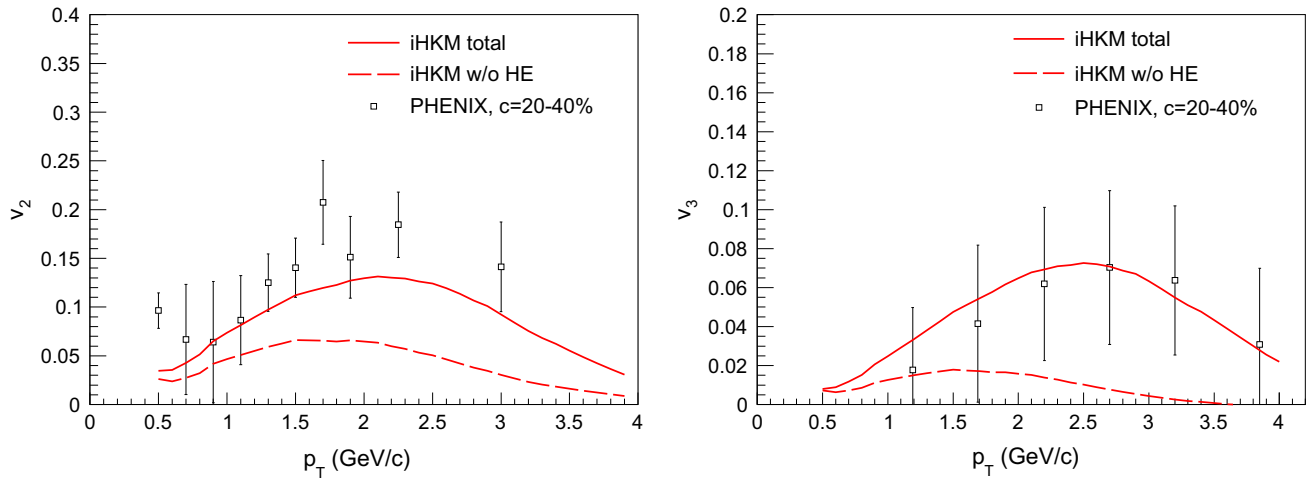


Fig. 18 The iHKM results for the photon v_2 and v_3 flow harmonics in the top RHIC energy Au+Au collisions ($c = 20\text{--}40\%$) compared to the experimental results [109]. The model curves corresponding to the calculations with and without hadronization emission component are shown

radiation in the pseudo-critical region is, probably, a key to solve the photon puzzle.

4 Conclusions

The integrated hydrokinetic model, which simulates the full process of the matter evolution in course of the relativistic heavy-ion collision, including early pre-equilibrium relaxation dynamics, that transforms the initially non-equilibrated state of the system to a nearly thermalized one, allows one to obtain a comprehensive description of measured bulk observables and to reveal underlying dynamics of the collision process and its femtoscopic space-time structure. The early start of the system's evolution in iHKM (at proper time about $\tau_0 = 0.1$ fm/c) allows to account for pre-thermal development of the collective effects and thus improve the data description.

The model provides a simultaneous description of particle yields and their ratios, hadron and photon p_T spectra and flow harmonics. Also a good description and successful prediction of pion and kaon femtoscopia radii for all the modern ultrarelativistic heavy-ion collision experiments (Au+Au collisions at the RHIC energy 200A GeV, Pb+Pb collisions at the LHC energies 2.76A TeV and 5.02A TeV, Xe+Xe collisions at the LHC energy 5.44A TeV) using a single parameter set in each case. To re-calibrate iHKM, switching from the simulation of one of the mentioned collision types to another, one basically needs to change only two parameters, α and $\epsilon_0(\tau_0)$, and to go from the LHC energy 2.76A TeV to 5.02A TeV—only one parameter, $\epsilon_0(\tau_0)$.

The simulations carried out using the two different equations of state for quark-gluon phase with the corresponding particlization temperatures demonstrate, that an equally

good description of data can be achieved in both cases, provided the initial maximal energy density parameter $\epsilon_0(\tau_0)$ is retuned. This result together with the results of probing the hadronic medium with $K^*(892)$ resonance, showing that intensive interactions between hadrons take place at least during 5 fm/c after particlization, speak in favor of continuous character of chemical and kinetic freeze-outs and indicate a great role of the afterburner stage of heavy ion collision in the formation of measured observables.

The iHKM results for the interferometry volumes and radii, obtained for different colliding nuclei in events with close charged particle multiplicities, allow to conclude, that the hypothesis about scaling of the femtoscopia radii with multiplicity is not confirmed. The transverse interferometry radii, and hence, the interferometry volume, depend also on geometric sizes of the colliding nuclei and grow with the transverse area of their overlapping right after collision.

The investigation of the “direct photon puzzle” within iHKM demonstrates that account for the photon radiation during the confinement process in the pseudo-critical region can help to solve the problem and provide a good description of the direct photon spectra and flow harmonics.

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Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: All the necessary data are presented in the tables and figures within the article. The model results demonstrated in the graphs can also be obtained from the authors in tabular form.]

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РОЗДІЛ 2

ЧАСИ ЕМІСІЇ ЧАСТИНОК У РЕЛЯТИВІСТСЬКИХ ЗІТКНЕННЯХ ЯДЕР

2.1. Народження резонансів $K^*(892)$ і $\phi(1020)$ та їх взаємодія з адронним середовищем на LHC та RHIC



$K^*(892)$ and $\phi(1020)$ production and their decay into the hadronic medium at the Large Hadron Collider

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Abstract

The production of the $K^*(892)$ strange resonance in Pb + Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV LHC energy is analyzed within the integrated hydrokinetic model (iHKM) at different equations of state of superdense matter. The similar analysis is done also for the RHIC top energy $\sqrt{s_{NN}} = 200$ GeV for comparison purposes. A modification of experimental $K^*(892)$ -identification is studied for different centralities in view of possible re-scattering of the decay products at the afterburner stage of the fireball evolution. We see quite intensive rescattering of the decay products as well as recombination processes for $K^*(892)$. In addition, the production of the much longer-long-lived $\phi(1020)$ resonance with hidden strange quark content is investigated.

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Keywords: Resonance; Rescattering; Identification; Lead–lead collisions; LHC

1. Introduction

The analysis of strange resonance yields in relativistic heavy ion collisions can provide valuable information about the properties of hot and dense matter, formed in such processes. The results of theoretical lattice QCD calculations as well as a multitude of experimental observa-

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tions on hadron production, hydrodynamic flow and jet quenching in ultra-relativistic nuclear collisions demonstrate the presence of quark–gluon matter in the hot fireball as well as the existence of a cross-over phase transition from the strongly coupled quark–gluon plasma (QGP) to hadronic matter. For recent reviews see [1,2] and references there.

Here we don't want to deal with a description of the QGP phase but rather concentrate on getting a better handle on the hadronic phase after chemical freeze-out. The hadrons containing strange quarks can play an important role in understanding inter alia the final hadronic stage of the matter evolution in $A + A$ collisions. At LHC energy, the chemical freeze-out temperature T_{chem} is very well determined to be 156.5 ± 1.5 MeV [3].

This temperature is very close to the best value for the pseudo-critical temperature T_{ps} of 154 ± 9 MeV [4,5]. Consequently, we conclude that the non-equilibrium, dilute hadronic stage commences for temperatures below T_{ps} , i.e. the system reaches there the final afterburner stage. At this non-equilibrium stage particles still collide, at least, elastically, the resonances decay, and hadrons *gradually* escape from the system and travel freely to the detectors. Strange hadron resonance states, such as the $K(892)^*$, that lives about 4 fm/c, can be used as a probe of the intensity of collision processes at this afterburner stage if one studies their yield by registering the hadronic decay channel $K(892)^* \rightarrow K\pi$. Taking into account the K^* lifetime, such information about the medium will deal mostly with the first 4–5 fm/c of the hadronic phase. The interactions of daughter hadrons may prevent identification of all produced K^* 's in the experiment. Consequently one expects for such resonances deviations from the overall chemical freeze-out picture determined using hadrons which are stable against strong decays. Such deviations are indeed observed [3].

Dealing with the modification of resonance yields due to modifications in the non-equilibrium hadronic phase after chemical freeze-out is a delicate matter. Processes which can lead to K^* losses involve:

1) elastic scattering of the decay pions and kaons with the constituents of the medium, in the present case mostly pions and kaons as well as some nucleons. A prominent case is re-formation of the K^* . Since the K^* is a pseudo-vector, the decay angular distribution in the process should be taken into account. Furthermore, forward angle Coulomb scattering should not be neglected, both from other charged mesons and baryons in the fireball. At LHC energies, the net charge of the fireball vanishes, contrary to the situation at lower beam energies. However, there may be large charges in the causally connected parts of the fireball due to fluctuations and their possible influence should be studied. The re-formed K^* may also survive in the medium and decay outside.

2) inelastic processes such as $K^* + \pi \rightarrow K + \pi + \pi$, i.e. processes involving higher K^* resonances can also lead to K^* losses.

3) the losses or gains should be confined to the life time of the hadronic phase. At LHC energy the ALICE collaboration has investigated this lifetime through measurements of Hanbury-Brown/Twiss interferometry (see [6]). From these measurements one concludes that the life time of the fireball is rather short, about 10 fm/c.¹

The studies below are still rather schematic in that the UrQMD model is used to describe the final hadronic cascade where all Coulomb effects are neglected and the main collision process which is taken into account is $\pi + K \leftrightarrow K^*$. Also the life time of the hadronic phase in UrQMD

¹ According to the results of [7–9] such a time corresponds to the temporal location of the maximal emission hypersurface for relatively soft pions. In evolutionary models hadron emission is continuous and typically has fairly long tail outside the maximum.

lasts longer than the 10 fm/c discussed above. Nevertheless, we believe that an integrated look at the process within the framework of the hydrokinetic model iHKM can lead to important insights.

In this note we, therefore, study the effects of hadronic rescatterings on K^* resonance observability within the integrated hydrokinetic model (iHKM) for the case of Pb + Pb collisions at the Large Hadron Collider (LHC) energy $\sqrt{s_{NN}} = 2.76$ TeV and compare the results to the detailed data recently reported by the ALICE collaboration [10]. Our results are also compared with the STAR data from RHIC [11].

2. Integrated hydrokinetic model

The ‘Integrated Hydrokinetic Model’ (iHKM) [12] of relativistic nuclear collisions is an extended version of the well-known HKM model [15], which includes, in addition to the latter, an energy–momentum transport model of the pre-thermal stage of the matter evolution [13,14] and viscous (not ideal as in HKM) hydrodynamics at the thermal stage. As well as its predecessor, HKM, the iHKM model simultaneously describes well a wide class of bulk observables in heavy ion collisions, such as multiplicities, hadron transverse momentum spectra and their anisotropy — flow harmonics, interferometry radii, source functions, etc. at different centralities [12,16,17]. Now the iHKM includes the five stages of the matter evolution and observable formation in $A + A$ collisions:

1) Formation of the initial state.

The estimates of the Glasma formation time (see [12] and references therein) as well as comparison of the iHKM results with the experimental data, point out on very early formation time for initial energy density profile at the LHC, namely, $\tau_0 \approx 0.1$ fm/c. To describe the energy density distribution at this initial state, the combined method is used [12]. According to the latter, the generally non-equilibrium boost-invariant (in the central region of rapidity) parton/gluon distribution function on the *initial* hypersurface σ_0 : $\tau = \tau_0$ is presented in the factorized form

$$f(t_{\sigma_0}, \mathbf{r}_{\sigma_0}, \mathbf{p}) = \epsilon(b; \tau_0, \mathbf{r}_T) f_0(p). \quad (1)$$

Here, according to the “mixed” Glauber Monte Carlo approach, implemented in the GLISSANDO code [18] the initial energy density profile $\epsilon(b; \tau_0, \mathbf{r}_T)$ is calculated as a mixture of contributions from two models: the wounded nucleon model and the binary collisions model. The proportion between the two types of the corresponding contributions to the initial energy density is described by means of the phenomenological parameter, $0 \leq \alpha \leq 1$. The distributions of numbers of wounded nucleons and binary collisions at τ_0 are calculated in GLISSANDO code. The initial energy density profile defined then by the weighed (with the coefficients α and $1 - \alpha$) sum of these distributions and the initial energy density ϵ_0 in the center of the system, $\mathbf{x}_T \approx 0$, at the central collisions. They are the main free parameters of the model. A possible momentum anisotropy of partons/gluons in the initial state, that is typical for approaches based on the Color Glass Condensate effective field theory, is taken into account by the function $f_0(p)$ in (1) in the way described in detail in Refs. [12,14]:

$$f_0(p) = g \exp \left(- \sqrt{\frac{(p \cdot U)^2 - (p \cdot V)^2}{\lambda_{\perp}^2} + \frac{(p \cdot V)^2}{\lambda_{\parallel}^2}} \right), \quad (2)$$

where $U^\mu = (\cosh \eta, 0, 0, \sinh \eta)$, $V^\mu = (\sinh \eta, 0, 0, \cosh \eta)$. In the rest frame of the fluid element, $\eta = 0$, $(p \cdot U)^2 - (p \cdot V)^2 = p_{\perp}^2$ and $(p \cdot V)^2 = p_{\parallel}^2$, so the parameters $\lambda_{\parallel}^2$ and $\lambda_{\perp}^2$ can be associated with the two temperatures along the beam axis and that orthogonal to it correspondingly. The parameter $\Lambda = \lambda_{\perp}/\lambda_{\parallel}$ thus defines the momentum anisotropy of the initial state.

The values of the initial transverse energy density ϵ_0 at τ_0 and $\mathbf{x}_T = 0$, and the parameter α are fixed based on the experimental mean charged particle multiplicity values. It turns out that these two parameters do not depend on the collision centrality at a given collision energy. The variations of other parameters, namely, the viscosity, relaxation time, and thermalization time, lead to the re-scaling of the ϵ_0 value (only!), necessary in order to re-adjust the model output to the experimental multiplicity [12] in central collisions (see details in Ref. [12]). A change of the thermodynamic equation of state is correlated with corresponding modifications of the initial time and energy density.

2) Pre-thermal stage.

The non-thermal energy–momentum tensor obtained from the distribution (1) is the starting point for the subsequent pre-thermal dynamics in the relaxation time approximation [12–14] that results in thermalization of the matter. Within this approximation, the system’s energy–momentum tensor gradually evolves from the free streaming form (at the initial proper time τ_0) to the hydrodynamical (local equilibrium) one (at some “thermalization time” $\tau_{th} \sim 1$ fm/c. At the intermediate times the non-equilibrium energy–momentum tensor has a form

$$T^{\mu\nu}(x) = T_{\text{free}}^{\mu\nu}(x)\mathcal{P}(\tau) + T_{\text{hyd}}^{\mu\nu}(x)(1 - \mathcal{P}(\tau)), \quad (3)$$

where $0 \leq \mathcal{P}(\tau) \leq 1$ is the interpolating function, taken in the form $\mathcal{P}(\tau) = \left(\frac{\tau_{th} - \tau}{\tau_{th} - \tau_0} \right)^{\frac{\tau_{th} - \tau_0}{\tau_{rel}}}$ with τ_{rel} being the relaxation time parameter. The pre-thermal stage in the iHKM lasts optimally from the time of initial stage formation, $\tau_0 \approx 0.1$ fm/c, till the thermalization time τ_{th} , when the initially non-equilibrated system reaches an approximate local thermal equilibrium. In previous and this iHKM analysis we use the value $\tau_{th} = 1$ fm/c. The value of τ_0 in the model is fixed based on the experimental pion p_T spectrum slope. The temperature in the very central part of the system at the thermalization time τ_{th} is about 400 MeV at the LHC energy $\sqrt{s_{NN}} = 2.76$ TeV, while the value average over transverse plane at this proper time τ_{th} is, of course, less.

3) The hydrodynamic stage.

At the thermalization time τ_{th} the energy–momentum tensor of the system takes the Israel–Stewart form for relativistic viscous hydrodynamics. The evolution of the system after time $\tau_{th} = 1$ fm/c is described by the hydrodynamics with the minimal possible ratio of the shear viscosity coefficient to the entropy density $\frac{\eta}{s} = \frac{1}{4\pi}$. In this paper we use the two equations of state (EoS). One is that was used in HKM model. It is the Laine–Schroeder equation of state [19] that guarantees that the subsequent transition from the continuous medium to the hadron gas (with 360 hadrons being taken into account) will pass without discontinuities in pressure and energy density. The other one is taken from the recent Lattice QCD data [4,5,20]. The hydrodynamic approach is utilized to describe the expansion of superdense quark–gluon and hadron matter, close to local chemical and thermal equilibrium, until the temperature at which both these types of equilibrium are violated, and therefore another approximation should be used to describe the further system evolution.

4) The particlization stage.

We assume that the chemically and thermally near-local-equilibrium evolution takes place until certain temperature T_p . Then one performs a switching from hydrodynamic representation to the system description in terms of particles. In the iHKM either gradual or sudden particlization can be realized. In the current analysis we compare the two variants of sharp particlization in iHKM with the two temperatures: $T_p = 163$ MeV that corresponds to the Laine–Schroeder equation of state for quark–gluon matter [19] and $T_p = 156$ MeV responding to HotQCD Collaboration equation of state [20] (for brevity we will refer to it in what follows as “HotQCD

EoS”). Both switching temperatures are still in agreement with the Lattice QCD data for pseudo-critical temperature in the cross-over scenario, $T_{ps} = 154 \pm 9$ MeV [4,5]. The higher temperature, $T_p = 163$ MeV, does not practically change our previous results [12,15,16] describing well the multitude of bulk observables at RHIC and LHC energies with the particlization temperature $T_p = 165$ MeV. The latter corresponds to the energy density $\epsilon = 0.5$ GeV/fm³ for the Laine–Schroeder EoS, and serve as the reference point for comparison. The lower temperature, $T_p = 156$ MeV, is considered as corresponding to the most recent estimates of chemical freeze-out temperature in thermal model, $T_{ch} = 156 \pm 1.5$ MeV [3]. The particlization hypersurface is built in iHKM with the help of the Cornelius routine [21]. For taking into account the viscous corrections to the hadron distribution function Grad’s 14-moment ansatz is used.

5) Hadronic cascade stage.

After particlization the system finally proceeds to the hadronic cascade within UrQMD model [22]. In such type of the evolutionary model, the chemical freeze-out as well as the thermal one, are not sudden in time (and not sharp in temperature), in opposite to the thermal models, where the recent analysis points out to the temperature of sharp chemical freeze-out at the LHC energies $T_{ch} = 156 \pm 1.5$ MeV [3]. From the point of view based on evolutionary hadron cascade models, the latter temperature should be considered rather as the effective temperature for continuous chemical freeze-out. The situation might be similar to the thermal continuous freeze-out. Namely, with regard to the latter, it is demonstrated [7] that in good approximation there is the duality between descriptions of the particle spectra within realistic continuous freeze-out and within the Cooper–Frye prescription for sharp freeze-out, if the latter one is attributed to the hypersurface of maximal continuous emission of the hadrons from expanding fireball. The detailed analysis will be done in separate work; in this article we are basing on such a hypothesis of the duality between sharp and continuous chemical freeze-out, so here we consider the temperatures $T_p = 163$ MeV and $T_p = 156$ MeV just as the particlization temperatures. At the further matter evolution the non-elastic processes (including annihilation but not the resonance decays) die out gradually so that the rate of decreasing of the inelastic collision number is maximal at some effective “chemical freeze-out” temperature $T_{ch} \leq T_p$.

In current calculations for the case of particlization temperature $T_p = 163$ MeV we use the set of the iHKM parameters, which optimize the description of the multiple bulk observables [8,12,23,24], including circumscribing and prediction of pion and kaon interferometry radii momentum behavior, at the LHC: $\tau_0 = 0.1$ fm/c, $\tau_{th} = 1$ fm/c, the relaxation time at the pre-thermal stage $\tau_{rel} = 0.25$ fm/c, $\epsilon_0 = 680$ GeV/fm³, $\alpha = 0.24$, the momentum anisotropy of the initial state at τ_0 — the ratio of transverse to longitudinal “temperatures” is $\Lambda = 100$, dissipative parameter $\eta/s = 0.08$. As for the case of particlization temperature $T_p = 156$ MeV, it differs from the previous one by the values of $\epsilon_0 = 495$ GeV/fm³ and $\tau_0 = 0.15$ fm/c. The latter parameters guarantee the correct dependence of the multiplicity on the centrality and correct pion transverse momentum spectra for the corresponding “HotQCD EoS”.

3. Results and discussion

The initial conditions (IC) for iHKM calculations are chosen to correspond to the simulation of Pb + Pb collisions at the LHC energy $\sqrt{s_{NN}} = 2.76$ TeV for the eight centrality classes: $c = 0$ –5%, $c = 5$ –10%, $c = 10$ –20%, $c = 20$ –30%, $c = 30$ –40%, $c = 40$ –50%, $c = 50$ –60%, and $c = 60$ –70%.

In Fig. 1 we present the comparison of $K(892)^*$ transverse momentum spectra calculated in iHKM with the experimentally measured ALICE points [10]. The iHKM results are very close

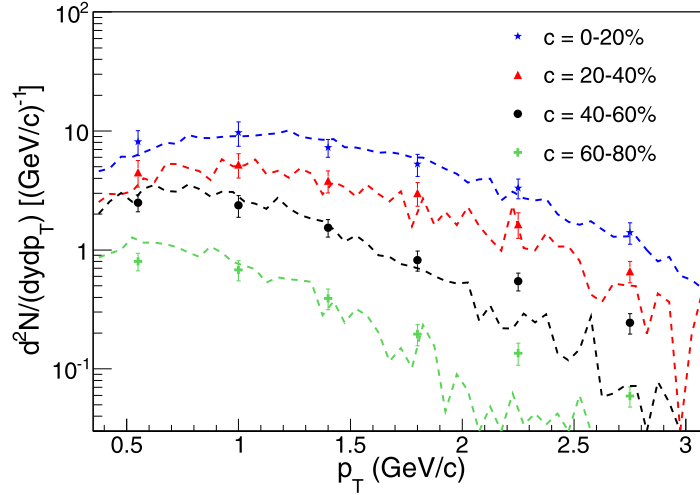


Fig. 1. The $K(892)^*$ resonance p_T spectra for Pb + Pb collision events with different centralities at the LHC energy $\sqrt{s_{NN}} = 2.76$ TeV obtained in iHKM simulations (lines) in comparison with the experimental data [10] (markers).

for both used EoS. One can see that iHKM reproduces well the experimental data for central collisions ($c = 0-20\%$ and $c = 20-40\%$ classes) in full p_T range and for the case of non-central collisions a good description is achieved for not very high $p_T < 1.8$ GeV/c.

The $K(892)^{*0}$ resonances are identified in the experiment, as well as in iHKM, by means of the products of their decays into $K^+\pi^-$ pairs. The result of such an identification is affected by several factors. First, since strange $K(892)^*$ resonance has intermediate lifetime, about 4 fm/c, the intensive rescattering that takes place for particles born in course of hadronization, leads to the two opposed effects: a reduction of the number of $K\pi$ pairs (direct and coming from resonance decays) identified as K^* because of rescattering of mesons forming such a pair and, on the other hand, an enhancement of identified K^* because of possible re-combination processes at the “afterburner” stage. Apart from that, in experimental analysis, different types of correlations, including event-by-event elliptic flow and residual ones, which exist between kaons and pions, can be misinterpreted as their bound resonance state. The common misidentification problem, when particles of one species are identified as those of another one, also contributes to this effect. And the last thing to note, the invariant mass criterion, used to select $K\pi$ pairs corresponding to the K^* decay, does not work perfectly and can lead to rejecting some pairs that actually come from the decay of interest or, on the contrary, to accepting some irrelevant pairs.

In order to investigate the influence of different above mentioned competing effects (except for experimental misidentification problem and event-by-event effects) on the K^* observability, we use iHKM simulations. Since the full iHKM calculation includes hadron cascade stage of system’s evolution modeled within UrQMD, one can compare the number of $K^+\pi^-$ pairs, which can be identified as coming from the K^{*0} decays after the UrQMD stage² and the number of actual primary K^{*0} ’s, produced in the course of medium particlization, plus those coming from subsequent resonance decays. For our analysis we select the $K^+\pi^-$ pairs with rapidity $|y| < 0.5$ and $0.3 < k_T < 5$ GeV/c in accordance with [10]. The criterion we use to tell that a pair comes from the desired decay is the following: we require all spatial coordinates of the particle last

² Usually in the experiment one also analyzes $\overline{K^{*0}}$ and $K^{*\pm}$ yields, however in our simulations we found that all the results on efficiency of identification for these resonances are very close to the results for K^{*0} , so in what follows we will talk only about K^{*0} , having in mind, that the same concerns also $\overline{K^{*0}}$ and $K^{*\pm}$.

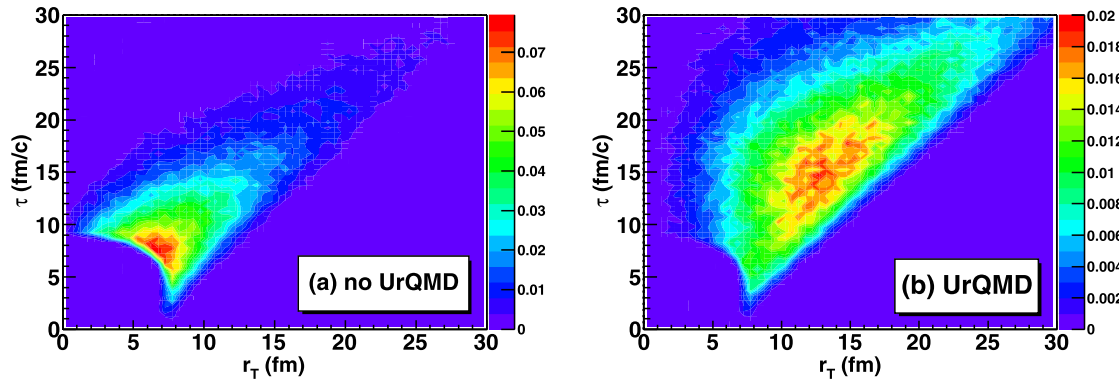


Fig. 2. The comparison of the emission functions $g(\tau, r_T)$, averaged over complementary space and momentum components, of $K^+\pi^-$ pairs, associated with $K(892)^{*0}$ decay products, for two cases: (a) free-streaming of the particles and resonances, and (b) UrQMD hadron cascade. The plots are obtained using iHKM simulations of Pb + Pb collisions at the LHC $\sqrt{s_{NN}} = 2.76$ GeV, $0.3 < k_T < 5$ GeV/c, $|y| < 0.5$, $c = 5\text{--}10\%$.

collision points to differ by less than 0.01 fm, $|x_i^K - x_i^\pi| < 0.01$ fm, and the pair invariant mass should fall in the range $0.77 < M_{\pi K} < 1.02$ GeV/c² around the $K(892)^{*0}$ invariant mass value, $M_{K^*} = 895.94$ MeV, that corresponds to the range utilized in the experimental analysis [10].

Now, let us present the detailed analysis of the post-hydrodynamic stage of a central collision ($c = 5\text{--}10\%$), based on the comparison of two scenarios after particlization: (1) free-streaming of the particles and resonances, and (2) UrQMD cascade. The emission pictures of not re-scattered decay products of $K(892)^{*0}$ in both scenarios are presented in Fig. 2 for L.-S. EoS. We see that in the UrQMD scenario the emission picture is blurred out and strongly suppressed at small times after the particlization (see color scales). The numerical comparison of K^* emission intensity at different proper times τ in two scenarios indicates that for the interval $\tau < 15$ fm/c in the UrQMD scenario the number of observed $K(892)^{*0}$'s is only 30% of this number in free-streaming case. Thus, at least 70% of direct K^* 's cannot be detected due to rescattering of their decay products in the dense hadronic medium, formed in the central collision. In fact, the actual loss of direct K^* 's can be even greater because of $K^+\pi^-$ recombination taking place and producing additional $K(892)^{*0}$ resonances — in UrQMD it is approximated by the coalescence mechanism. This effect also could explain the picture we see at the large times, $15 < \tau < 30$ fm/c, where the number of observed K^* 's in the UrQMD case becomes about 1.5 times larger than that in the free-streaming case. If then one compares the total numbers of identified $K(892)^{*0}$'s in the two cases, it appears that the K^{*0} reduction in UrQMD case is only about 20% as compared to the calculation without rescatterings. This indicates that recombination plays important role in K^* production and generates a great number of new K^* particles, at least 50% of direct ones, that partially compensate the substantial loss of the direct resonances taking place at small times after hadronization. Also this type of production can be the reason for comparably large time of maximal emission, obtained for kaons in HKM study [8] and ALICE experimental analysis [9].

Now let us demonstrate the dependence of the fraction of identified K^* resonances on the Pb + Pb collision centrality at the LHC energy $\sqrt{s_{NN}} = 2.76$ TeV, obtained in our analysis. Such dependencies are presented in Fig. 3 for both EoS's and corresponding particlization temperature values. As one can see, at $T_p = 163$ MeV this fraction increases from 0.8 for 5% most central collisions to unity for the near peripheral collisions with $c = 60\text{--}70\%$. In the case of $T_p = 156$ MeV the fraction value also increases when going from central to peripheral collisions, but for the central collisions it is greater, namely 0.88. Such a behavior is expected, since at the periphery

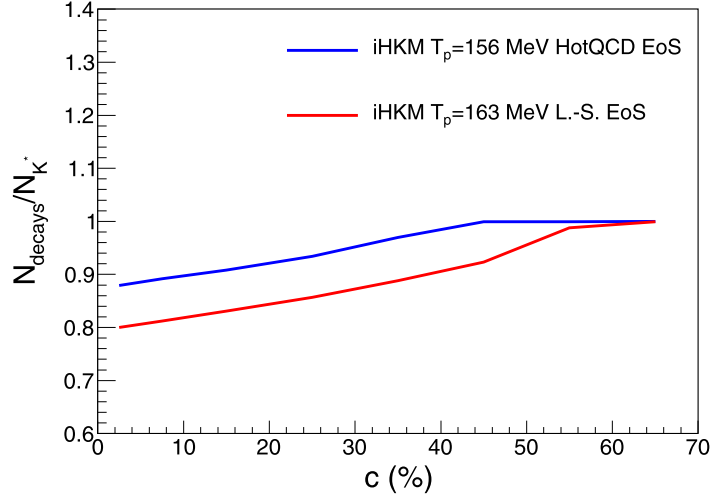


Fig. 3. The fraction of $K^+\pi^-$ pairs coming from $K(892)^*$ decay, which can be identified as daughters of K^* in iHKM simulations after the particle rescattering stage modeled within UrQMD hadron cascade. The simulations correspond to LHC Pb + Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV with different centralities. The iHKM results are presented for two cases: the Laine–Schroeder equation of state with particlization temperature $T_p = 163$ MeV (red line) and the HotQCD equation of state with $T_p = 156$ MeV (blue line). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

collisions most of colliding nuclei’s nucleons pass without intensive interaction and thus do not form large-volume hot and dense long-living system, so K^* decay products are not affected by re-scattering and can be easily identified. Also at higher particlization temperature one can expect longer duration of hadron re-scattering stage, leading to more strong reduction of observable K^* yield, as compared to that at the particlization stage, due to its decay products’ scattering. Note, as we will see further, that the final observed yields of K^{*0} are very similar at both EoS.

As for the relatively longer living resonances, such as $\phi(1020)$ with lifetime about 50 fm/c which decays into K^+K^- and $K_L^0 K_S^0$ pairs, one cannot expect that the rescatterings of daughter particles will be noticeable to reduce observed resonance number. Since KK interaction cross-section is not very large, the recombination effect is also not big. The results of iHKM simulations are in agreement with these considerations. The $\phi(1020)$ resonances were restored using $\phi \Rightarrow K^+K^-$ decay products selected based on the criterion $|x_i^{K^+} - x_i^{K^-}| < 0.01$ fm and $1.00 < M_{KK} < 1.07$ GeV/c², similar to the K^* case (the invariant mass range is taken from [10]). The fraction of observable $\phi(1020)$ in our simulations for all the centralities is about 20% for non-central and 30% for central collisions larger than unity. We connect such an enhancement with the manifestation of KK correlations, implemented in UrQMD through coalescence feature — it transforms close KK pairs into $\phi(1020)$ resonances.

The $\phi(1020)$ p_T -spectra from iHKM simulations compared to the ALICE data are presented in Fig. 4. The ϕ mesons with rapidity $|y| < 0.5$ and $0.5 < p_T < 5$ GeV/c were chosen for the analysis. We observe the same situation as for K^* spectra in Fig. 1 — the model describes well the experimental spectra in the case of central events (up to $c = 20$ –30%) in full p_T range, $0.5 < p_T < 5.0$ GeV/c, while for more peripheral events model gives good description for the reduced range $0.5 < p_T < 1.8$ GeV/c.

In Tables 2, 1 one can find the momentum integrated K^{*0} and $\phi(1020)$ yields for different LHC collision centrality classes. The results of iHKM simulations agree within the errors with the experiment.

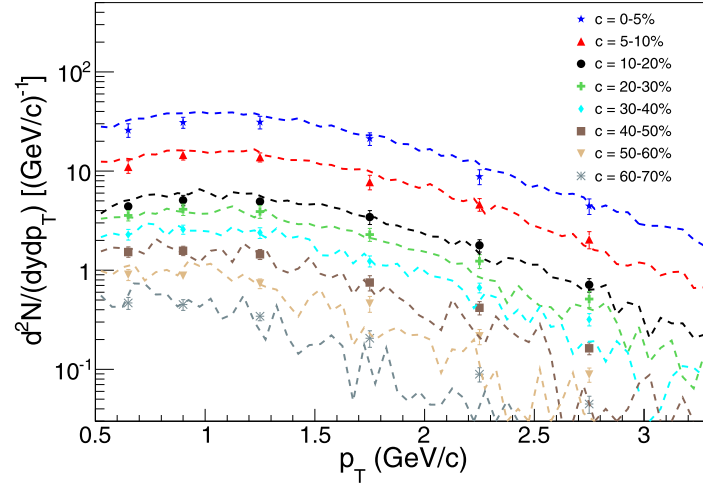


Fig. 4. The $\phi(1020)$ resonance p_T spectra for Pb + Pb collision events with different centralities at the LHC energy $\sqrt{s_{NN}} = 2.76$ TeV obtained in iHKM simulations (lines) in comparison with the experimental data [10] (markers).

Table 1

The K^* and $\phi(1020)$ p_T -integrated yields, K^*/K^+ and ϕ/K^+ ratios calculated in iHKM for the case of LHC Pb + Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV for the events from different centrality classes compared with the ALICE experimental data [10].

K^*				
Centrality	dN/dy iHKM	K^*/K iHKM	dN/dy ALICE	K^*/K ALICE
0–20%	14.8	0.18	$16.6 \pm 0.6 \pm 2.5 \pm 0.1$	$0.20 \pm 0.01 \pm 0.03$
20–40%	7.3	0.19	$9.0 \pm 0.8 \pm 1.1 \pm 0.1$	$0.24 \pm 0.02 \pm 0.03$
40–60%	3.43	0.20	$3.9 \pm 0.3 \pm 0.4 \pm 0.1$	$0.28 \pm 0.02 \pm 0.03$
60–80%	1.01	0.20	$1.13 \pm 0.09 \pm 0.11 \pm 0.07$	$0.31 \pm 0.02 \pm 0.03$

ϕ				
Centrality	dN/dy iHKM	ϕ/K iHKM	dN/dy ALICE	ϕ/K ALICE
0–5%	14.4	0.157	$13.8 \pm 0.5 \pm 1.7 \pm 0.1$	$0.127 \pm 0.004 \pm 0.014$
5–10%	11.8	0.140	$11.7 \pm 0.4 \pm 1.4 \pm 0.1$	$0.130 \pm 0.004 \pm 0.014$
10–20%	8.6	0.135	$9.0 \pm 0.2 \pm 1.0 \pm 0.1$	$0.134 \pm 0.003 \pm 0.013$
20–30%	5.6	0.131	$7.0 \pm 0.1 \pm 0.8 \pm 0.1$	$0.152 \pm 0.003 \pm 0.015$
30–40%	3.56	0.128	$4.28 \pm 0.09 \pm 0.48 \pm 0.09$	$0.144 \pm 0.003 \pm 0.014$
40–50%	2.06	0.125	$2.67 \pm 0.05 \pm 0.30 \pm 0.06$	$0.148 \pm 0.003 \pm 0.014$
50–60%	1.11	0.126	$1.49 \pm 0.03 \pm 0.16 \pm 0.05$	$0.145 \pm 0.003 \pm 0.014$
60–70%	0.52	0.118	$0.72 \pm 0.02 \pm 0.08 \pm 0.04$	$0.140 \pm 0.004 \pm 0.013$

Table 2

The comparison of K^*/K^+ ratio calculated in iHKM for the case of RHIC Au + Au collisions at $\sqrt{s_{NN}} = 200$ GeV and the experimental data [11] for different centrality classes.

c	K^*/K STAR	K^*/K iHKM
0–10%	$0.23 \pm 0.01 \pm 0.05$	0.21
10–30%	$0.24 \pm 0.02 \pm 0.05$	0.21
30–50%	$0.26 \pm 0.02 \pm 0.06$	0.22
50–80%	$0.26 \pm 0.02 \pm 0.05$	0.23

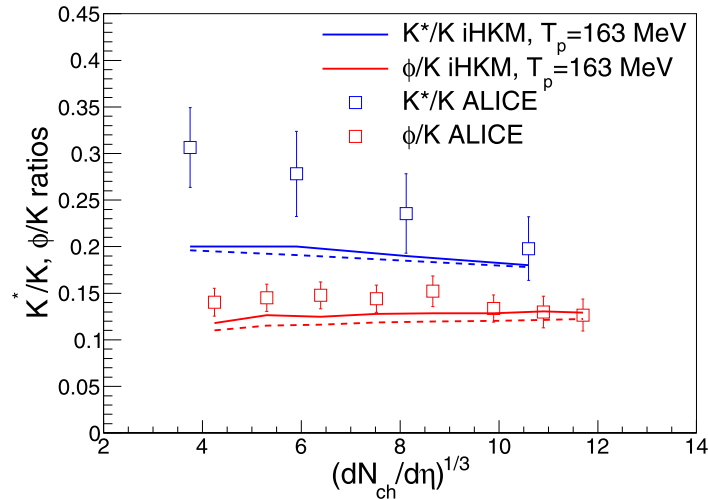


Fig. 5. The comparison of K^*/K^+ and ϕ/K^+ ratios dependency on particle multiplicity $(dN_{ch}/d\eta)^{1/3}$ calculated in iHKM for the case of LHC Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV and the corresponding ALICE experimental data [10]. The solid lines correspond to the iHKM calculations, performed using hadronization temperature $T_p = 163$ MeV and Laine–Schroeder equation of state [19], while the dashed lines are related to the HotQCD equation of state [20] and $T_p = 156$ MeV.

Additionally, we present the K^*/K^+ and ϕ/K^+ particle number ratios, calculated in the iHKM model for the LHC Pb + Pb collisions at the energy $\sqrt{s_{NN}} = 2.76$ TeV and for the top RHIC energy Au + Au collisions. The results related to $T_p = 163$ MeV are presented in Tables 2, 1. The corresponding values in the case of $T_p = 156$ MeV for LHC are close to those in Table 2 and are presented only graphically in Fig. 5.

The centrality behavior of K^*/K and ϕ/K ratios at the LHC, demonstrated in Fig. 5 as the ratios' dependence on $(dN_{ch}/d\eta)^{1/3}$, is such that the “afterburner” K^*/K ratio slightly decreases at increasing collision centrality (and multiplicity), while the ϕ/K ratio, on the contrary, slightly increases with $(dN_{ch}/d\eta)^{1/3}$. The experimental ϕ/K points show not monotonic behavior — at first the experimental ϕ/K ratio slightly increases, and then slightly decreases with the multiplicity. However, in general one can say that the iHKM satisfactory describes the ϕ/K ratio and, for central events, also the K^*/K ratio, but somehow underestimates the latter in case of the peripheral events.

The results on K^*/K ratio for RHIC are shown in Fig. 6 and are also compared with the corresponding experimental data [11]. Here the ratio behavior is shown as dependence on collision centrality c . Again, the K^*/K ratio weakly decreases as the considered collisions become more central. As one can see, our modeling results are in agreement with the RHIC experiment within the errors. The presented experimental and model ratio values are related to the end of the collision's final stage. Within iHKM one can also compare them with the values on hadronization hypersurface, which for all the centralities appear to be approximately twice larger than the “afterburner” ones.

The lower K^*/K values for the central events and the absence of clear centrality dependence for ϕ/K ratio, observed in the experiment and partially reproduced in the model calculations, are in agreement with above iHKM results concerning the K^* and ϕ resonance observability — in the K^* case there is a reduction of its observed yield, which is the larger, the more central collision is considered, and in the ϕ case one has the enhancement of its observed number, which does not depend on centrality. The thermal model results in the case of central LHC collisions [3] overestimate the K^*/K ratio by the factor of about 1.5. This fact supports the conception of

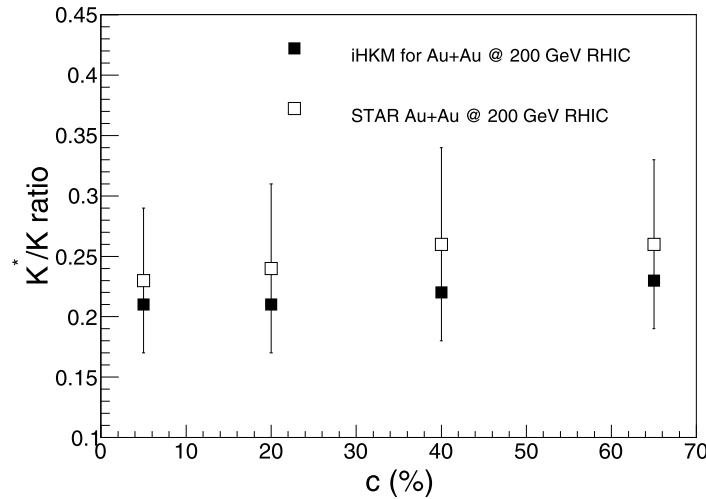


Fig. 6. The comparison of K^*/K^+ ratio calculated in iHKM for the case of RHIC Au + Au collisions at $\sqrt{s_{NN}} = 200$ GeV and the experimental data [11] for different centrality classes.

continuous freeze-out, realized inter alia through the particle interactions at the afterburner stage of collision.

4. Conclusions

The production of strange K^{*0} and long-lived $\phi(1020)$ (having hidden strange quark composition) resonances and the possibility for their reconstruction via the hadronic decay channels are considered within the integrated hydrokinetic model (iHKM) for the LHC and RHIC. It is found that the combined effect of the re-scattering and recombination of the daughter hadrons leads to the suppression of their registration up to 20% in the most central collisions. The most intensive re-scattering takes place at times $\tau < 15$ fm/c, where at least 70% of direct K^* resonances become unobservable due to their decay products scattering. At larger times the recombination effect becomes dominating leading to generation of a great amount of additional K^* 's, at least 50% of direct ones.

The effectiveness of experimental $K(892)^*$ identification is significantly better for essentially non-central collisions, where the transverse size of the system is small and particles easily escape from the hadronic medium. As for $\phi(1020)$ resonance, it seems that KK correlations at the afterburner stage lead to the excess of registered $\phi(1020)$. The calculations do not exclude that the full number of produced $\phi(1020)$ by up to 30% exceeds their number on hadronization/particization hypersurface. Such an effect, as well as additional $K(892)^*$ production at the final stage of the collision, can probably be explained by the regeneration of these resonances due to $K\bar{K}$ and $K^+\pi^-$ correlations at the afterburner stage through the UrQMD coalescence mechanism.

The iHKM model with $T_p = 163$ MeV reproduces the experimental data provided by the ALICE Collaboration on K^* and ϕ resonances transverse momentum spectra for $p_T < 1.8$ GeV/c in Pb + Pb collisions at the LHC energy $\sqrt{s_{NN}} = 2.76$ TeV. The results on ϕ/K and K^*/K particle number ratio dependence on particle multiplicity are described in iHKM sufficiently well, except for K^*/K ratio in the peripheral events. The corresponding K^*/K values for RHIC numerically agree with the results of STAR Collaboration. The results on particle number ratios, obtained with $T_p = 156$ MeV for the LHC are similar (only slightly worse), to these at $T_p = 163$ MeV. However, the comparison with the thermal model results points out that the corrections con-

nected with interactions at hadron afterburner stage of the matter evolution, are fairly noticeable. It supports the continuous freeze-out conception.

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2.2. Порівняння часів емісії піонів та каонів для енергії LHC

$$\sqrt{s_{NN}} = 2.76 \text{ TeV}$$



On m_T dependence of femtoscopy scales for meson and baryon pairs

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Abstract

The m_T -dependencies of the femto-scales, the so-called interferometry and source radii, are investigated within the hydrokinetic model for different types of particle pairs — pion–pion, kaon–kaon, proton–proton and proton–lambda, — produced in Pb+Pb and $p + p$ collisions at the LHC. In particular, such property of the femto-scales momentum behavior as m_T -scaling is studied for the systems with (w) and without (w/o) intensive transverse flow, and also w and w/o re-scattering at the final afterburner stage of the matter evolution. The detailed spatiotemporal description obtained within hydrokinetic model is compared with the simple analytical results for the spectra and longitudinal interferometry radii depending on the effective temperature on the hypersurface of maximal emission, proper time of such emission, and intensity of transverse flow. The derivation of the corresponding analytical formulas and discussion about a possibility for their utilization by the experimentalists for the simple femtoscopy data analysis is the main aim of this theoretical investigation.

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Keywords: Correlation femtoscopy; Scaling; Hadron rescattering; Transverse flow; Lead–lead collisions; LHC

1. Introduction

The spatiotemporal structures of particle emission in nucleus–nucleus, proton–(anti)proton and proton–nucleus collisions are essentially defined by the dynamics of the collision processes [1–4]. Therefore the correlation interferometry [5], that measures the femtoscopic

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scales by means of two- (or many-) particle correlations, allows one to study such processes experimentally. The corresponding femtoscopic patterns can be presented in different forms. One of them is the k_T -momentum dependence of the interferometry radii $R_i(k_T = |\mathbf{p}_{T1} + \mathbf{p}_{T2}|/2)$, that results from a 3D Gaussian fit in $q^i = p_1^i - p_2^i$ to the two-particle correlation function $C(\mathbf{q}, k_T)$, defined as a ratio of the two-particle spectrum to the product of the single-particle ones. The other one is the source function $S(\mathbf{r}^*)$ [6] reflecting the dependence of the pair production on the distance $\mathbf{r}^*$ between the two emitted particles in the rest frame of the pair. Both patterns supplement each other, and a reliable model should describe/predict all the mentioned types of the femtoscopic observables, if it contains a detailed space–time picture of the collision process.

It is important to note that the correlation function behavior depends also on the particle species. The detailed behavior of this dependence can be used to discriminate between different scenarios of matter evolution and particle emission in the collision processes. For example, the hydrodynamic picture of A+A collisions for the particular case of negligible transverse flow leads to the same $m_T^{-1/2}$ behavior¹ of the longitudinal radii $R_l(k_T)$ for the pairs of identical pions and kaons, and even leads to the complete m_T -scaling in the case of common freeze-out [2,3].

In Ref. [7] it was found that hydrokinetic model (HKM) [8–11] predicts strong violation of such a scaling between pions and kaons at the LHC, and predicts k_T -scaling (for $k_T > k_0 \approx 0.4 \text{ GeV}/c$) instead. In this letter we analyze in detail the physical reasons for m_T -scaling violation. As it turned out that the reasons were found to be quite general, there is a hope that the found m_T peculiarities will be confirmed in the LHC experimental comparative analysis of pion and kaon femtoscopy data. In addition we predict the m_T -behavior for femtoscopic scales in other cases, including the case of meson and baryon source function radii.

To simplify the theoretical study, we reconsider the analytical results, describing m_T (or k_T) behavior of the femtoscopy scales in a situation with strong transverse flows typical for RHIC and LHC energies. We derive a simple analytical formula for such a scenario, which fits well the complex HKM results for different hadron pairs. Since the HKM describes simultaneously the large number of bulk observables, it gives experimentalists a simple tool for estimation of the life-time of expanding fireball, related to the maximal emission of the specific hadron pair created in proton or nuclear collisions.

2. Analytical model for the interferometry radii

The hydrokinetic model (HKM) (see details in Ref. [11]) was developed to simulate the evolution of matter formed in relativistic heavy-ion collisions and describe/predict bulk observables at RHIC and LHC. In Ref. [11] a good description was reached for pion, kaon, (anti)proton and all charged particle spectra at different centralities, as well as for the elliptic flows. Concerning the femtoscopic scales, the confirmed HKM prediction [12] about a reduction of the $\frac{R_{out}}{R_{side}}$ ratio in A+A collisions at LHC as compared to RHIC was done²; the interferometry radii for pions and kaons at RHIC [10,11] and LHC [7,11] in A+A collisions were calculated; the source functions for kaons and pions at RHIC were described at the top RHIC energy and predicted for the

¹ Here, $m_T^2 = m^2 + (|\mathbf{p}_{T1} + \mathbf{p}_{T2}|/2)^2 = m^2 + k_T^2$ is the transverse mass of the particle pair.

² Here we discuss HKM, however the similar result was obtained earlier in hydrodynamics with crossover equation of state [13]. The main reason for the reduction of R_{out}/R_{side} ratio at the LHC as compared to RHIC is strengthening of positive r - τ correlations for points of surface emission with the increase of collision energy (see numerical details and analytical approximation in [12]). Note, however, that the effect is suppressed if the equation of state is soft, as it is at the 1st order phase transition.

LHC one [14]. In addition, HKM describes well pion interferometry radii in $p + p$ collisions at the LHC energy $\sqrt{s} = 7$ TeV if one incorporates the quantum uncertainty principle into a quasi-classical event generator [15].

All the results for A+A collisions are obtained at the same initial conditions at $\tau = 0.1$ fm/c, produced by MC Glauber generator GLISSANDO [16] (with just re-scaled maximal initial energy density at the transition from RHIC to LHC), the same type of hydrodynamics evolution (however with different baryochemical potentials at RHIC and LHC), the same lowest temperature when the local thermal and chemical equilibrium still exist in the hydrodynamically expanding medium, $T_{ch} = 165$ MeV, and with UrQMD cascade at the final afterburner stage. Such a self-consistent and reliable simulation of the data, including femtoscopy ones, indicates that the spatiotemporal picture of particle emission in the model can serve as a reference point for further studies, no matter how much advanced future models will be developed. Note, that in HKM and similar models many factors act simultaneously, the results can be obtained only by means of time-consuming numerical calculations, and it is not always clear, how each physical factor affects the complete space–time picture of the particle emission in the collision process.

Here we provide some analytical estimates for the spectra and femtoscopy scales, aiming to reveal the main parameters forming the particle emission distribution in space and time. In Refs. [9,17] the problem of spectra formation in hydrodynamic approach to A+A collisions is considered within the Boltzmann equations. It is shown analytically and illustrated by numerical calculations that the particle momentum spectra can be described by the Cooper–Frye prescription (CFp) [18] despite the fact that the freeze-out is not sharp and has a finite temporal width. The latter is equal to the inverse of the particle collision rate at 4-points $(t_\sigma(\mathbf{r}, p), \mathbf{r})$ of the maximal particle emission at a fixed momentum p . The set of these points forms the hypersurfaces $t_\sigma(\mathbf{r}, p)$ which depend on the values of p and typically do not enclose completely the initially dense matter. This is an important difference from the standard CFp, with a common freeze-out hypersurface (that is, typically, an isotherm) for all the momenta p . Also, the well known problem of CFp – negative contributions to the spectra from non-space-like parts of the freeze-out hypersurface – is naturally eliminated in this improved/generalized prescription: at each momentum p the hypersurface σ_p is always a space-like one.

In this letter we partially use the results of Refs. [19,20] and consider the hypersurface of constant Bjorken (proper) time τ , $\sigma_p: \tau = \text{const}$, as the hypersurface of maximal emission for soft enough particle momenta p . It is shown [17] that such a hypersurface, limited in transverse direction $\mathbf{r}_T$, is typical for soft particles. For the particles with hard momenta, e.g. $p_T > 0.8$ GeV/c, the hypersurface of maximal emission is different from $\tau \approx \text{const}$: there is the correlation between the transverse radius and the time of radiation (see details in [19,20]). To cut the hypersurface of maximal emission (m.e.) $\tau = \tau_{m.e.} = \text{const}$ in the transverse plane for soft quanta, and at the same time get analytical approximation, we use here the Gaussian cutoff factor $\rho(\mathbf{r}_T)$ in the way proposed first in Refs. [19,20]. Namely, the Wigner function for bosons in almost central events has the form:

$$f_{l.eq.}(x, p) = \frac{1}{(2\pi)^3} [\exp(\beta p \cdot u(\tau_{m.e.}, \mathbf{r}_T) - \beta\mu) - 1]^{-1} \rho(\mathbf{r}_T). \quad (1)$$

Here and below $\beta = 1/T$ is the inverse of temperature at $\tau_{m.e.} = \sqrt{t_{m.e.}^2 - x_{L,m.e.}^2}$, $\mathbf{r}_T \equiv \mathbf{x}_T$ is 2D transverse radius-vector, 4-coordinate is $x^\mu = (\tau \cosh \eta_L, \mathbf{r}_T, \tau \sinh \eta_L)$ with longitudinal rapidity $\eta_L = \text{arctanh } v_L = \frac{1}{2} \ln \frac{t+x_L}{t-x_L}$, transverse rapidity $\eta_T \equiv \eta_T(r_T) = \text{arctanh } v_T(r_T)$, the hydrodynamic velocity $u^\mu(x) = (\cosh \eta_L \cosh \eta_T, \frac{\mathbf{r}_T}{r_T} \sinh \eta_T, \sinh \eta_L \cosh \eta_T)$. The cutoff factor $\rho(\mathbf{r}_T)$ has the form [19,20]:

$$\rho(\mathbf{r}_T) = \exp[-\alpha(\cosh \eta_T(r_T) - 1)]. \quad (2)$$

Here, according to results of [19,20] and the interpretation corrected for the improved CFp [17], the parameter $\alpha = R_v^2/R_T^2$, where R_T is the transverse homogeneity length in r_T (near $r_T = 0$ for small k_T) along the hypersurface $\tau = \tau_{m.e.} = \text{const}$, and R_v is the hydrodynamic length, $R_v = (v'(r_T))^{-1}$, near the same r_T . The small α can be reached at very intensive flow when the hydrodynamic length R_v is much smaller than the homogeneity length R_T (which is, generally speaking, different for different particles), while the upper limit is reached in the case of no transverse flow, $R_v = \infty$. One can see that the contributions from particles with high momenta emitted from the hypersurface $\tau = \tau_{m.e.} = \text{const}$ are suppressed by the cutoff factor, since such particles radiate typically from rapidly moving fluid elements, $\cosh \eta_T(r_T) \gg 1$. Another $\tau(\mathbf{r}_T)$ hypersurface should be used for hard particles.

To get single particle spectra $p_0 d^3N/d^3p$ and the correlation function of bosons $C(p, q)$ in smoothness and mass shell approximation, we use the generalized Cooper–Frye method as it was discussed above:

$$p_0 \frac{d^3N}{d^3p} = \int_{\sigma_{m.e.}(p)} d\sigma_\mu p^\mu f_{l.eq.}(x, p), \quad (3)$$

$$C(p, q) \approx 1 + \frac{\left| \int_{\sigma_{m.e.}(k)} d\sigma_\mu k^\mu f_{l.eq.}(x, k) \exp(iqx) \right|^2}{\left(\int_{\sigma_{m.e.}(k)} d\sigma_\mu k^\mu f_{l.eq.}(x, k) \right)^2}, \quad (4)$$

where $q = p_1 - p_2$, $k^\mu = (\sqrt{m^2 + \left(\frac{\mathbf{p}_1 + \mathbf{p}_2}{2}\right)^2}, \frac{\mathbf{p}_1 + \mathbf{p}_2}{2})$. Note that in smoothness and mass shell approximation $k \approx p = (p_1 + p_2)/2$. Then 4-vector q has three independent components, which can be selected along the beam axis, $q_{long} \equiv q_l$, along the pair transverse momentum vector $\mathbf{k}_T$, $q_{out} \equiv q_o$, and along *side* direction, $q_{side} \equiv q_s$, being orthogonal to both *long* and *out* directions.

With all these notations, one can apply the saddle point method (in complex plane) to calculate the spectra and correlation function (4) in Boltzmann approximation for longitudinally boost-invariant expansion. The results were obtained in Ref. [21] on the base of the approach using the Wigner function (1) [19,20]. The remarkable fact is that the behavior of the correlation function in *long* direction depends solely on the parameter α and does not depend on the transverse velocity profile at the hypersurface of maximal emission, while, as expected, the behavior of the correlation function in *out* and *side* directions is model dependent and quite sensitive to the details of the velocity profile. Because of this, here we analyze and use only the longitudinal projection of the correlation function. Let us introduce the value λ , associated with the homogeneity length in longitudinal direction in a presence of transverse flow $\lambda_l = \tau \sqrt{\frac{T}{m_T}(1 - \bar{v}_T^2)}^{1/2}$ [19,20]:

$$\lambda^2 = \frac{\lambda_l^2}{\tau^2} = \frac{T}{m_T}(1 - \bar{v}_T^2)^{1/2}, \quad (5)$$

where the transverse velocity in the saddle point $\bar{v}_T = k_T/(m_T + \alpha T)$. Here $T = T_{m.e.}$ is the temperature at the hypersurface of maximal emission, $\tau = \tau_{m.e.}$. Then in LCMS [21]:

$$C(k, q_l, q_s = q_o = 0) = 1 + \frac{\exp\left[\frac{2}{\lambda^2} \left(1 - \sqrt{1 + \tau^2 \lambda^4 q_l^2}\right)\right]}{\left[1 + \tau^2 \lambda^4 q_l^2\right]^{3/2}} \xrightarrow{k_T \rightarrow \infty} 1 + \exp(-\lambda_l^2 q_l^2). \quad (6)$$

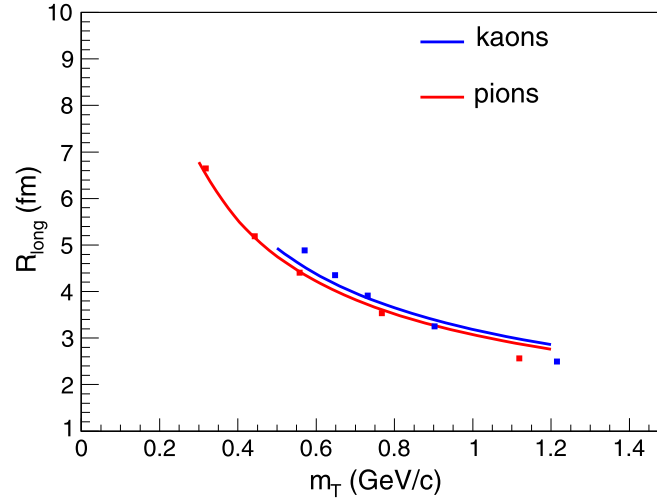


Fig. 1. The m_T dependencies of longitudinal femtoscopy radius R_l calculated in HKM model without re-scattering stage for $K^{ch}K^{ch}$ and $\pi^-\pi^-$ pairs (markers) together with the corresponding fits (lines) according to formula (8) where transverse flow is absent ($\alpha = \infty$). The temperature is $T = 165$ MeV. The results are related to $\sqrt{s_{NN}} = 2.76$ GeV Pb+Pb collisions at the LHC, $c = 0-5\%$, $|\eta| < 0.8$, $0.14 < p_T < 1.5$ GeV/ c .

At large $m_T/T \gg 1$ the correlation function has the Gaussian form, the interferometry radii coincide with homogeneity lengths in this approximation, that was first obtained for pure Bjorken expansion (without transverse flow: $\bar{v}_T = 0$) in Ref. [2]: $R_l = \lambda_l = \tau \sqrt{\frac{T}{m_T}}$. In Ref. [22] for the same case (no transverse flow) the correction was found to the interferometry radii at small $m_T/T \approx 1$, that should be applicable for pions:

$$R_l^2 = \lambda_l^2 \rightarrow \lambda_l^2 \frac{K_2(\frac{m_T}{T})}{K_1(\frac{m_T}{T})}, \quad (7)$$

where K_n are modified Bessel functions. As one can see from (6), at not large $m_T/T \approx 1$ the correlation function has non-Gaussian form in q_l . Correspondingly, at not large m_T/T such a radius describes only the peak of the correlation function, where the Gaussian approximation in the limit of small q_l , $C(q_l) = 1 + \exp(-R_l^2 q_l^2)$, can be obtained. The detailed analysis of different analytical approximations for interferometry radii is done in Ref. [21].

The approximation of the correlation function (6) at small q_l leads to the following analytical result for the interferometry radii in the case of boost-invariant longitudinal expansion and transverse flow with arbitrary velocity profile:

$$R_l^2(k_T) = \tau^2 \lambda^2 \left(1 + \frac{3}{2} \lambda^2 \right). \quad (8)$$

The λ is defined by the formula (5). The direct comparison with the formula (7) for pure Bjorken picture (in our case it corresponds to $\alpha \rightarrow \infty$) demonstrates only 1–3% deviation between the two results in all actual k_T interval for pions. An advantage of the formula (8) is that it is derived for the case of transverse flow of any intensity that is especially important for LHC energies.

3. Analytic fitting of HKM results

The idea of m_T -scaling for the interferometry radii of different bosons has been launched by the 1D hydrodynamic result [2]: $R_l \propto \sqrt{T/m_T}$. The formulas (7) and (8) extended for all m_T values in the case of 1D expansion (it means $\alpha = \infty \Rightarrow \bar{v}_T = 0$ in formula (8)) also lead to scaling: R_l is the function of τ and m_T/T only. In Fig. 1 we demonstrate the corresponding fit to the

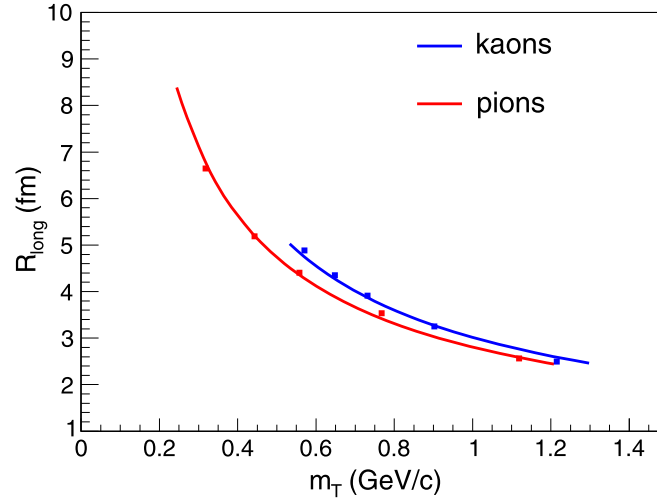


Fig. 2. The same as in Fig. 1 but fits account now for the intensive transverse flow, $\alpha = 2.8$. The time parameters for pions and kaons are $\tau_\pi = 7.41$ fm/c and $\tau_K = 7.56$ fm/c correspondingly, $T_\pi = T_K = 165$ MeV.

HKM results for pions and kaons with the standard Cooper–Frye prescription at the “freeze-out” hypersurface $T = 165$ MeV, where we artificially switched off the UrQMD re-scattering, leaving only resonance decays. As we see, the best fits are very bad: $\chi^2/\text{ndf} = 2562.31$ for pions and $\chi^2/\text{ndf} = 585.24$ for kaons, and there is no m_T -scaling. The absence of m_T scaling in full HKM (with re-scattering at the afterburner UrQMD stage) for R_l at LHC energies is noticed recently in Ref. [7].

One of the reasons for violation of scaling in HKM becomes clear from Fig. 2 where the same results of “succise” HKM (without re-scattering) were fitted by the full formula (8) that accounts for transverse flow. Here we also use the “freeze-out” temperature 165 MeV. The fits for both pions and kaons are quite satisfactory with the same value of temperature parameter and times $\tau_\pi = 7.41$ fm/c for pions and $\tau_K = 7.56$ fm/c for kaons. For both mesons $\alpha = 2.8$. These fits illustrate that, as one can see from (8), (5), the transverse flow destroys m_T scaling, since the interferometry radius depends now on both variables m_T/T and k_T/T .

The additional reason for m_T -scaling violation could be re-scattering at the afterburner stage that also can break the similarity between pion and kaon longitudinal interferometry radii. A non-Gaussian shape of the correlation functions also affects the m_T -behavior of the corresponding femtoscopy scale. To investigate in detail the role of these factors, we should analyze the results of full HKM calculation (including the re-scattering stage). In order to define the effective T and α parameter values for an analysis of the interferometry radii behavior in full HKM, one can study the corresponding momentum spectra. These parameters can be extracted from the combined fit to pion and kaon spectra calculated in HKM, accounting for transverse flow. Such a fit can be performed using the formula obtained in the same approximation as the result (6) [20]:

$$p_0 \frac{d^3 N}{d^3 p} \propto \exp[-(m_T/T + \alpha)(1 - \bar{v}_T^2)^{1/2}]. \quad (9)$$

In Fig. 3 one can see the experimental [23] and HKM spectra together with the combined pion–kaon fit to HKM points. The fit with a common temperature is performed according to (9) in p_T -range 0.5–1.0 GeV/c. The extracted parameter values are the following: $T = 144 \pm 3$ MeV, $\alpha_\pi = 5.0 \pm 3.5$ and $\alpha_K = 2.2 \pm 0.7$. Since the analytic formulas (7), (8) describe the Gaussian radii, which correspond to the region of small q in the case of non-Gaussian correlation function, we restrict the fitting range to $q = 0$ –0.04 GeV/c when extracting the interferometry radii from

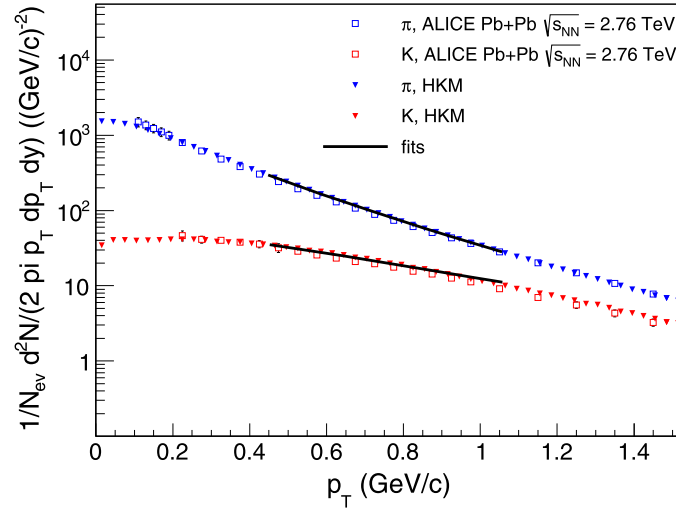


Fig. 3. Pion (blue [dark gray]) and kaon (red [light gray]) momentum spectra in Pb+Pb collisions at the LHC, $\sqrt{s_{NN}} = 2.76$ GeV. Open squares show experimental values [23], triangles show HKM results, black lines correspond to combined (with the same temperature T) pion and kaon spectra fit according to (9). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

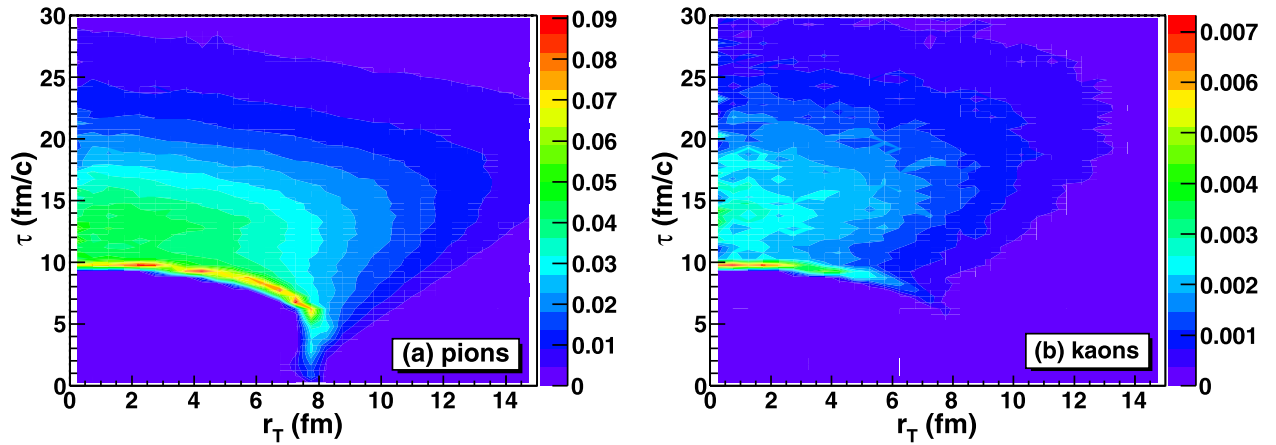


Fig. 4. The momentum angle averaged emission functions per units of space–time and momentum rapidities $g(\tau, r_T, p_T)$ [fm^{-3}] (see body text) for pions (a) and kaons (b) obtained from the HKM simulations of Pb+Pb collisions at the LHC $\sqrt{s_{NN}} = 2.76$ GeV, $0.2 < p_T < 0.3$ GeV/c, $|y| < 0.5$, $c = 0$ –5%.

full HKM. One can see the m_T -dependence of the radii extracted in such a way in Fig. 5 together with the analytical fit according to Eq. (8), where T, α are constrained according to the combined spectra fit. The maximal emission times extracted from the fit for pions and kaons are $\tau_\pi = 9.44 \pm 0.02$ fm/c and $\tau_K = 12.40 \pm 0.04$ fm/c respectively while the rest of parameters take values $T_\pi = 147$ MeV, $T_K = 141$ MeV, $\alpha_\pi = 8.5$ and $\alpha_K = 1.5$. For comparison, let us present the pion and kaon emission pictures in HKM based on the invariant emission functions $G(x, p) = p^0 \frac{d^7 N}{d^4 x d^3 p}$. In Fig. 4, we demonstrate the reduced emission function averaged over all momentum angles $g(r_T, \tau; p_T) = \frac{p^0 d^6 N}{dr_o dr_s d\eta d\tau dp_T dy} \Big|_{r_s=0}$ for pions and kaons at $0.2 < p_T < 0.3$ GeV/c in the central rapidity bins for both space–time η and momentum y rapidities. Here r_o is the component of $\mathbf{r}_T$ along the vector $\mathbf{p}_T$, and r_s is its component in the transverse direction orthogonal to $\mathbf{p}_T$. Such a type of the emission function (in r_s and r_o variables) was used in [19–21] to derive the analytical approximation for the correlation function (see Section 2). The fit results for the

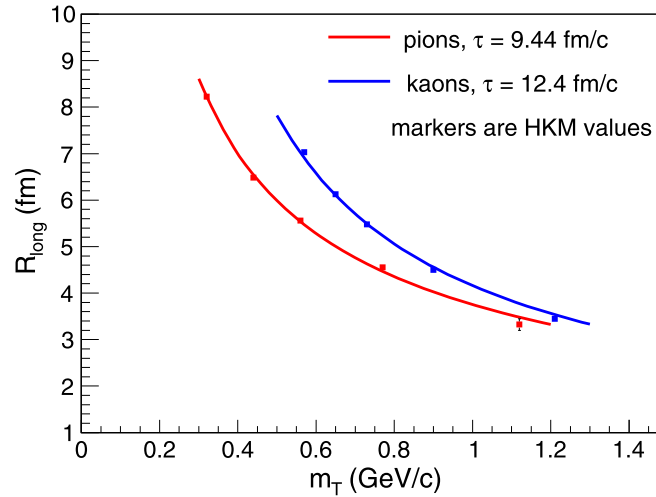


Fig. 5. The same as in Figs. 1, 2 but radii are calculated in the full HKM model including re-scattering stage. To reduce the effect of the non-Gaussian correlation functions, we take more narrow fitting range for them, $q = 0\text{--}0.04$ GeV/ c . The fit parameters T and α correspond to combined pion and kaon spectra fitting. At $T_\pi = 147$ MeV, $T_K = 141$ MeV, $\alpha_\pi = 8.5$ and $\alpha_K = 1.5$ extracted maximal emission times are $\tau_\pi = 9.44 \pm 0.02$ fm/ c and $\tau_K = 12.40 \pm 0.04$ fm/ c .

times of maximal emission are in good agreement with the HKM emission picture. One can notice that the kaon radiation has two maxima: one happens near $\tau = 10$ fm/ c and the second, broader and less pronounced, is at $\tau \approx 14\text{--}15$ fm/ c . This second local maximum must be due to decays of $K^*(892)$, having life-time 4–5 fm/ c , into kaons $K^\pm(493.7)$. In such a situation the single extracted $\tau_K \approx 12$ fm/ c value can be understood as some effective (or mean) maximal emission time for kaons, corresponding to the mentioned actual two maximal emission times.³ It is worthy noting that the difference between this “mean” time and time of pion maximal emission is significantly larger than in the case when only resonance decays are taken into account, but not re-scattering. The reason is that a free streaming of fast K^* with subsequent decay into $K^\pm$ leads to the additional large source, which contribution to the correlation function results mostly in non-Gaussian behavior of it. At the same time the re-scattering of K^* involves these mesons in some kind of collective motion producing thereby the direct link between the time of maximal emission and the value of the longitudinal radii. One can also note that the results of fitting the femto-scales extracted from full HKM (accounting for the re-scattering and K^* decays) as well as combined fitting of the HKM spectra give different T and α parameters for pions and kaons, and so one can conclude that in addition to intensive transverse flows m_T scaling is violated because of the re-scattering and resonance decays. The influence of the rescattering on violation of m_T -scaling was also noted in [24].

Next, if we apply Eq. (8) to description of the femtoscopy radii obtained from the fits to pion and kaon HKM correlation functions in a broader range (e.g. $q = 0\text{--}0.2$ GeV/ c) [7], covering not only their bell-like parts, but also the flatter ones,⁴ we will get unsatisfactory model data description for kaons using the fit parameters corresponding to spectra combined fit (see Fig. 6, blue (dark gray) solid line) with $T_K = 141$ MeV, $\alpha_K = 1.5$, and $\tau_K = 11.09 \pm 0.02$ fm/ c . As for pions, they are described well with parameters restricted by combined spectra fit: $T_\pi = 141$ MeV,

³ One can also notice that this extracted effective τ_K appears to be close to the arithmetical mean value of the two maximal emission times seen in Fig. 4.

⁴ Such wide fitting q range, covering the full interferometry region, is typically utilized in the experimental analysis, see e.g. [25].

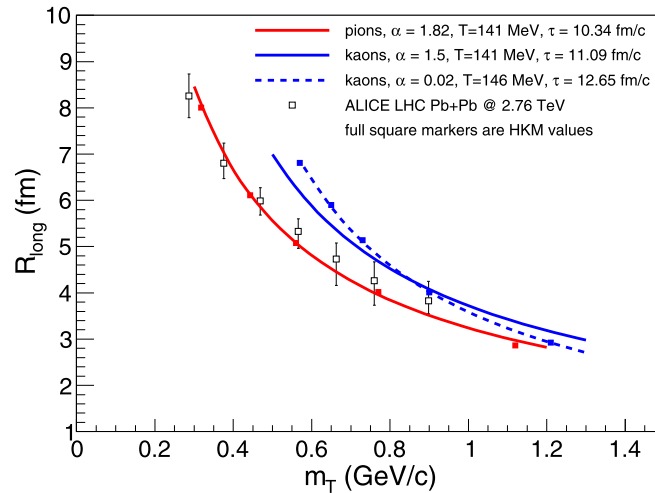


Fig. 6. The same as in Fig. 5 but the radii are extracted from the fits to full HKM correlation functions in a wide q range, $q = 0\text{--}0.2$ GeV/ c . Thus, the significant deviation of the CF shape from the Gaussian one leads to a distorted radii m_T -behavior. At fitting it can be compensated by decreasing α_K as compared to that obtained from the spectra fit. Experimental data [25] for pions are demonstrated for comparison.

$\alpha_\pi = 1.82$, $\tau_\pi = 10.34 \pm 0.06$ fm/ c . To get more or less adequate fit for the kaon longitudinal radius corresponding to wide q -region, one should remove constraints on α_K parameter. The dashed blue line in Fig. 6 represents such a fit with still constrained temperature, $T = 144 \pm 3$ MeV, and free α_K and τ_K . The resulting fit at $T = 146$ MeV and $\alpha = 0.02$ provides good data description and gives $\tau_K = 12.65 \pm 1.58$ fm/ c , that is in good agreement with the previous results for radii extracted at $q = 0\text{--}0.04$ GeV/ c and the emission picture in Fig. 4. Therefore, the combined fit (with common temperature) for the pion and kaon spectra leads to the reliable estimate of the maximal particle emission times in the case of fitting only the peak of the non-Gaussian correlation function. The same good estimates can be extracted also from the Gaussian fit in large q range, but for kaons α_K parameter has to be fitted without restrictions coming from common pion and kaon spectra fitting.

The difference as compared to the previous results for “without re-scattering” regime as well as an essential difference between the main fit parameters for pions and kaons mean that the re-scattering plays an important role in violation of m_T scaling, in addition to transverse flow. The very small α value in kaon fit can indicate that the re-scattering stage affects seriously kaon emission picture and the re-scattering contribution to the femtoscopy scale cannot be described basing on the pure hydrodynamic (plus the resonance decays near freeze-out hypersurface) approach. The peculiarities of the re-scattering for kaons leads to more pronounced non-Gaussian character of the correlation function shape than for pions and also to a distortion of the femtoscopy scales m_T -dependence. It is found that for compensation of the influence of such a factor on the extracted effective time of maximal emission, in the case of wide q -fitting range for kaons, one can eliminate restriction for the parameter α coming from combined pion and kaon spectra fit.

In some sense a complementary method of obtaining the information about the space–time structure of the system from the correlation measurements, is known as the source imaging [26–28] aiming to extract the source function $S(\mathbf{r}^*)$, that represents the time-integrated distribution of particle emission points separation $\mathbf{r}^*$ in the pair rest frame. The analysis of the experimental data for baryon correlations, often based on the Lednický–Lyuboshitz formula [29], supposes isotropic Gaussian separation distribution described by single width parameter r_0 . This

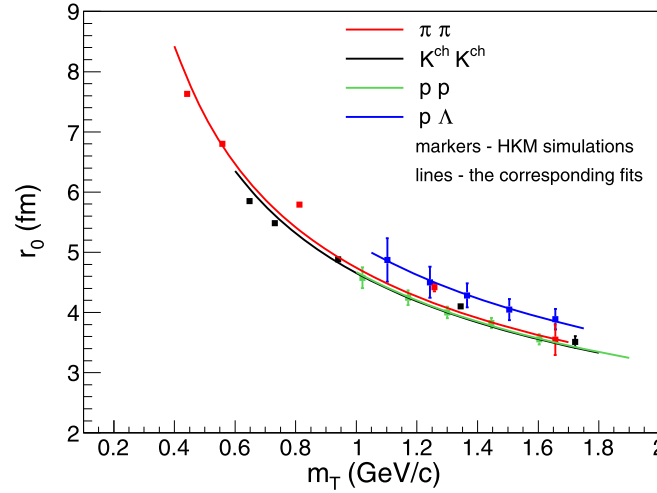


Fig. 7. The m_T dependencies of $\pi\pi$, $K^{ch}K^{ch}$, pp and $p\Lambda$ source radii r_0 extracted from corresponding angle-averaged source functions calculated in HKM for $\sqrt{s_{NN}} = 2.76$ GeV Pb+Pb collisions at the LHC, $c = 0-5\%$, $|\eta| < 0.8$. Transverse momentum ranges are $0.14 < p_T < 1.5$ GeV/c for pions and kaons, $0.7 < p_T < 4.0$ GeV/c for protons and $0.7 < p_T < 5.0$ GeV/c for Lambdas.

value can be extracted from the Gaussian fit to the angle-averaged source function $S(r) = 1/(4\pi) \int_0^{2\pi} \int_0^\pi S(r, \theta, \phi) \sin\theta d\theta d\phi$. In this paper we also try to apply the fitting function (8) to pion, kaon, proton and proton- Λ pairs to describe the m_T dependence of source radii r_0 extracted from the corresponding angle-averaged source functions $S(\mathbf{r}^*)$ in HKM by means of Gaussian fits to them, see details in Ref. [14]. The results are presented in Fig. 7. At this fitting the temperature T was constrained according to the combined pion and kaon spectra fit result $T = 144 \pm 3$ MeV, while α and τ were left free. The fits are unexpectedly good with the parameters $T = 141$ MeV for all the particle pairs, $\tau_{\pi\pi} = 11.47 \pm 0.03$ fm/c, $\tau_{KK} = 11.26 \pm 0.04$ fm/c, $\tau_{pp} = 11.30 \pm 0.13$ fm/c, $\tau_{p\Lambda} = 12.44 \pm 0.29$ fm/c. The parameter α for all the pairs is about 10^4 and more, i.e. actually $\alpha \rightarrow \infty$. The latter means that there are no transverse flows in the pair rest frame, saddle point in transverse velocity is zero for fluid elements in rest, that is natural. Found m_T -scaling (for proton- Λ pairs $m_T = \sqrt{\left(\frac{m_p + m_\Lambda}{2}\right)^2 + k_T^2}$) with slightly variative multiplier τ looks very surprising.

Let us also apply the result (8) to $p + p$ collisions at LHC energy $\sqrt{s} = 7$ TeV [30]. The theoretical results for the pion interferometry radii in $p + p$ collisions with different particle multiplicity accounting for uncertainty principle in small systems were presented in Ref. [15]. The results without re-scattering are almost the same as in full HKM. Therefore we assume the temperature of chemical freeze-out to be 165 MeV. In Figs. 8, 9 we fit by formula (8) these results obtained in the full HKM model, including re-scattering at the afterburner stage and corrections accounting for quantum uncertainty principle. One can see that, as it is expected, the values of τ in fitting function are much smaller than in Pb+Pb collisions, namely, in the multiplicity bin with mean charged particle multiplicity 17.9 we obtained $\tau = 1.83 \pm 0.02$ fm/c, $\alpha = 13.9 \pm 0.22$. At the same time for smaller mean multiplicity 9.2 both values are smaller: $\tau = 1.60 \pm 0.04$ fm/c, $\alpha = 9.52 \pm 0.44$.

4. Conclusions

The analytical formula for the longitudinal interferometry radii that allows one to extract the time of maximal emission for the certain hadron pairs in the case of strong transverse flows in

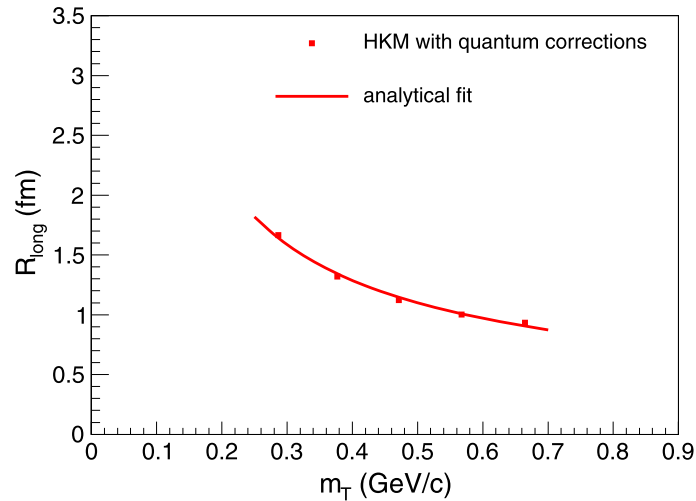


Fig. 8. The m_T dependence of pion longitudinal femtoscopy radius R_l calculated in the full HKM model with corrections accounting for quantum uncertainty principle (red markers) together with the corresponding fit (8) (red line). The temperature is $T = 165$ MeV, $\tau_{\text{HKM}} = 1.60 \pm 0.04$ fm/c and $\alpha = 9.52 \pm 0.44$. The results are related to $\sqrt{s} = 7$ TeV pp collisions at the LHC, $\langle dN_{ch}/d\eta \rangle = 9.2$.

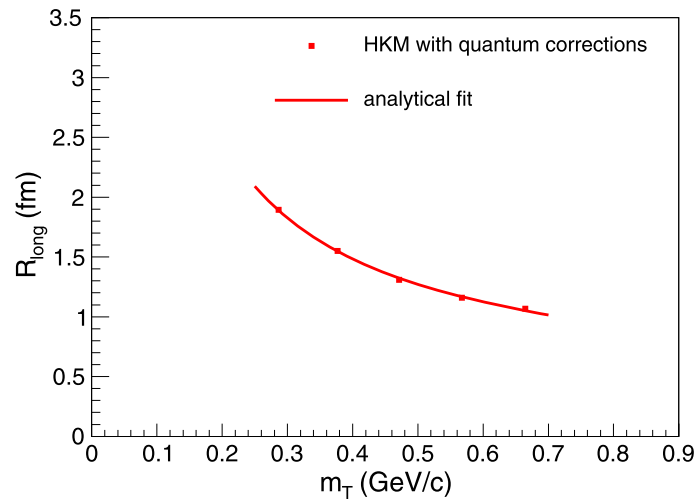


Fig. 9. The same as in Fig. 8 for collision events with $\langle dN_{ch}/d\eta \rangle = 17.9$. The values of the fit parameters are $\tau_{\text{HKM}} = 1.83 \pm 0.02$ fm/c, $\alpha = 13.9 \pm 0.22$.

A+A and $p + p$ collision processes is proposed. It is compared with detailed calculations and spatiotemporal picture of the particle emission in hydrokinetic model that already described well the large number of the bulk observables. Within such an analysis the factors were clarified affecting the interferometry radii and leading to violation of m_T -scaling between pions and kaons. They are: strong transverse flows, re-scattering and resonance decays at the afterburner stage and non-Gaussian form of the correlation function. It is found that for compensation of the influence of the latter factor on the extracted effective time of maximal emission for kaons, one may eliminate restriction on the “flow intensity” parameter α for kaons which follows from combined pion and kaon spectra fit.

As for the pion interferometry radii in $p + p$ collisions, the analytical fits to the corresponding HKM calculations give quite reasonable estimates for the times of the maximal emission for the collisions with different multiplicities.

The fitting formula has been applied also to the so-called source radii obtained from the Gaussian fit to the source function describing the dependence of the pair production on the distance

between the two emitted particles in the rest frame of the pair. The pion–pion, kaon–kaon, proton–proton and proton– Λ pairs produced in Pb+Pb collisions at the LHC were analyzed within HKM. Very surprisingly, it was found that analytical fit with slightly different τ for different pairs and with the other common adequate parameters — no transverse flow in the pair rest frame and the same temperature taken from combined pion and kaon spectra fit — gives very good result and leads to the m_T scaling behavior of angle-averaged source radii for all mentioned baryon and meson pairs.

We hope that the results will be especially interesting for the experimentalists for simple fitting of the femtoscopy data and extracting the time of maximal emission in continuously emitting fireballs.

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2.3. Часи емісії піонів та каонів для енергії LHC $\sqrt{s_{NN}} = 5.02$ TeV

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Kaon and pion maximal emission times extraction from the femtoscopy analysis of 5.02 A TeV LHC collisions within the integrated hydrokinetic model

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Abstract

A simple method for the extraction of the times of maximal emission for kaons and pions using the combined fitting of their transverse momentum spectra and the longitudinal interferometry radii dependencies on the pair transverse mass m_T is applied to Pb+Pb collisions at the LHC energy $\sqrt{s_{NN}} = 5.02$ TeV. The method is based on the analytical formulas, that were earlier successfully utilized in the studies of Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV. To test the method, the spectra, radii and particle radiation picture are calculated within the integrated hydrokinetic model (iHKM), that includes all the stages of the matter evolution in high-energy A+A collisions: the system's formation, its thermalization, viscous hydrodynamics evolution, particlization and subsequent hadronic cascade. The model describes and predicts well already published LHC data in "soft physics" kinematic region. Thus, the fitting results for maximal emission times of kaons and pions are compared to the approximate maximal emission time values, estimated based on the emission function plots, obtained in iHKM. The developed simple method was successfully applied to the data analysis in the case of lower LHC energy and can be used in further experimental studies of femtoscopy data in relativistic A+A collisions.

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Keywords: LHC; Kaon; Pion; p_T spectra; Femtoscopy scales; Maximal emission time

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1. Introduction

The well-developed experimental technique of correlation femtoscopy measurements in the field of high-energy heavy-ion collision physics makes it possible to investigate the spatio-temporal structure of the systems produced in such collisions, as well as the peculiarities of the process of their evolution (see, e.g. review [1]). The femtoscopy, or interferometry radii, extracted from the Gaussian fits to the measured two-particle correlation functions at given pair momentum k_T , are generally associated with the homogeneity lengths of a rapidly expanding system, or the three-dimensional sizes of the fragment of the system, where the particles with momentum k_T are mainly emitted from [2,3]. The detailed structure of homogeneity lengths contains also the information on, e.g., the strength of the developed collective flow and the lifetime of the created fireball [4] and on the space-time correlations between emitted particles [5,6]. The femtoscopy analysis can provide even such advanced information about the dynamics of the system's expansion as the effect of the resonance decays and hadron-hadron scatterings at the afterburner stage of the collision on the formation of bulk observables, or the times at which the particles of different species are mostly emitted from the system [7,8]. The study of the correlation functions for pairs of non-identical particles can help to find out particles of which sort are emitted earlier [9,10].

In the paper [7] we proposed a method for the estimation of the times of maximal emission for pions and kaons in the LHC Pb+Pb collisions at 2.76A TeV based on a simple analytical formulas allowing the simultaneous fitting of transverse momentum spectra for both considered particle sorts followed by fitting of the corresponding *long* interferometry radii dependencies on pair m_T (the *long* radii correspond to the projection of three-dimensional correlation function in the direction of the beam axis). The formulas were obtained as a result of analytical approximation for single-particle and two-particle momentum spectra in A+A collisions [3,7]. This method gave us the estimates for the effective pion and kaon emission times within the hydrokinetic model (HKM) [11,12], which was used to calculate the particle spectra in our study. The obtained estimates were also in agreement with the corresponding emission function plots obtained in HKM. In addition, the first application of the method to the RHIC BES energies [13], where the UrQMD [14,15] was used as the evolutionary model, was quite satisfactory.

The method was successfully used by the ALICE Collaboration [16] for the estimation of pion and kaon maximal emission times within their experimental analysis. Both studies showed that the effective emission time for kaons is larger than for pions. This fact, together with observed essentially non-Gaussian shape of the considered correlation functions and the absence of scaling between $R_{\text{long}}(m_T)$ dependencies for kaons and pions, suggested that the secondary kaons, coming from the $K^*(892)$ resonance decays and involved into intensive interaction with the hadronic medium at the final stage of the collision, give an important contribution to the total kaon yield. Similar results were obtained later within the more advanced integrated hydrokinetic model (iHKM) [17,18] for the case of Au+Au collisions at the top RHIC energy in our recent paper [19].

In [20] we presented the results of our systematic study of Pb+Pb collisions at the LHC energy 5.02A TeV within the iHKM model concerning the most of the bulk observables, including the predictions for femtoscopy radii in the three centrality classes ($c = 0 - 5\%$, $c = 20 - 30\%$, and $c = 40 - 50\%$, where e.g. $c = 0 - 5\%$ class includes 5% of the most central events, and the classes with larger c correspond to less central events; in the collision data analysis one usually defines the collision centrality based on the measured charged particle multiplicity, assuming that the latter is higher for central events and lower for non-central ones). However, our previous

study did not include the analysis of the maximal emission times for kaons and pions. That is why in the present work we are going to close this gap and to apply the developed technique to the case of Pb+Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV in order to get more details about the character of the matter evolution at this LHC energy and to compare it with the case of $\sqrt{s_{NN}} = 2.76$ TeV collisions.

2. Analytical model

In this section we briefly explain the idea of the method utilized (see [7] for details) and the origination of the analytical formulas for spectra and radii fitting.

The formation of particle spectra measured in heavy-ion collision experiments can be described using a modified Cooper-Frye prescription (CFp) for particlization hypersurface σ , which consists of all space-time points $(t_\sigma(\mathbf{r}, p), \mathbf{r})$, where the maximal emission of quanta with momentum p takes place [21,22]. In this approach, by contrast with the standard CFp, where the same particlization hypersurface (typically an isotherm) is used for all the momenta p , one does not have any problems with the negative contributions from non-spacelike elements of the switching hypersurface, since for each specified momentum p the corresponding fragment of the hypersurface is spacelike by its construction.

Here, similarly to [7], we base our consideration on the approximation [23,24] of such generalized CFp, where we suppose the hypersurface σ to be of constant proper time τ (equal to the time of maximal emission for soft quanta, $\tau = \tau_{m.e.} = \text{const}$) and to be limited in the direction $\mathbf{r}_T$, transverse to the beam axis. Such an assumption corresponds to the emission of soft particles with momenta $p_T \approx 0.2 - 0.4$ GeV/c, whereas for particles with $p_T > 0.8$ GeV/c strong space-time correlations between particle radiation events take place, and the related hypersurface parts essentially differ from $\tau = \text{const}$ [23,24]. Thus, we utilize the following analytical representation for the bosonic Wigner function for soft enough quanta:

$$f_{l.eq.}(x, p) = \frac{1}{(2\pi)^3} [\exp(\beta p \cdot u(\tau_{m.e.}, \mathbf{r}_T) - \beta\mu) - 1]^{-1} \rho(\mathbf{r}_T), \quad (1)$$

where β denotes the inverse temperature, $u^\mu(x) = (\cosh \eta_L \cosh \eta_T, \frac{\mathbf{r}_T}{r_T} \sinh \eta_T, \sinh \eta_L \cosh \eta_T)$ is hydrodynamic velocity, $\eta_L = \text{arctanh } v_L$ and $\eta_T = \text{arctanh } v_T(r_T)$ are longitudinal and transverse rapidities, corresponding to respective longitudinal and transverse collective velocities v_L and v_T , and $\rho(\mathbf{r}_T)$ is the Gaussian cutoff factor

$$\rho(\mathbf{r}_T) = \exp[-\alpha(\cosh \eta_T(r_T) - 1)], \quad (2)$$

where the parameter α is defined as $\alpha = R_v^2/R_T^2$, and in the latter ratio R_T denotes the homogeneity length in transverse direction $\mathbf{r}_T$ (for r_T close to zero and at small momenta k_T), while $R_v = (v'(r_T))^{-1}$ is the hydrodynamic length near $r_T = 0$.

The α parameter characterizes the strength of the collective flow: strong flow corresponds to small values of α , since in this case one has $R_v \ll R_T$, and for the case of absent flow $R_v \rightarrow \infty$, so that $\alpha \rightarrow \infty$ as well. The factor $\rho(\mathbf{r}_T)$ effectively limits the particlization hypersurface in transverse direction and removes the contributions from hard quanta, for which $\cosh \eta_T(r_T) \gg 1$.

According to the improved CFp utilized in our analysis one can calculate the single-particle spectra $p_0(d^3N/d^3p)$ and two-particle correlation functions $C(p, q)$ as follows:

$$p_0 \frac{d^3N}{d^3p} = \int_{\sigma_{m.e.}(p)} d\sigma_\mu p^\mu f_{l.eq.}(x, p), \quad (3)$$

$$C(p, q) \approx 1 + \frac{\left| \int_{\sigma_{m.e.}(k)} d\sigma_\mu k^\mu f_{l.eq.}(x, k) \exp(iqx) \right|^2}{\left(\int_{\sigma_{m.e.}(k)} d\sigma_\mu k^\mu f_{l.eq.}(x, k) \right)^2}. \quad (4)$$

Here we use the conventional denotations of the 4-vectors $k^\mu = \left(\sqrt{m^2 + \left(\frac{\mathbf{p}_1 + \mathbf{p}_2}{2} \right)^2}, \frac{\mathbf{p}_1 + \mathbf{p}_2}{2} \right)$ and $q = p_1 - p_2$, where $\mathbf{p}_j$ and p_j are particle momentum vectors and 4-vectors correspondingly ($j = 1, 2$), and m is particle mass. When one works in both smoothness and mass shell approximations, one has $k \approx p = (p_1 + p_2)/2$ and 4-vector q having only three independent components. In femtoscopy analysis one usually chooses them to be q_{long} — along the beam axis direction, q_{out} — along the pair transverse momentum $\mathbf{k}_T$ direction, and q_{side} — orthogonal to both *long* and *out* directions.

To obtain some simple analytical expressions for spectrum and correlation function, which would be convenient for fitting and interpretation of the experimental or the realistic simulation results, one can approximately calculate Eq. (3) and Eq. (4), substituting the Wigner function $f_{l.eq.}(x, p)$ by (1) and using the saddle point method, as it was done in [25] within the Boltzmann approximation for the case of longitudinally boost-invariant expansion.

It is interesting, that within the obtained in [25] approximation the behavior of the correlation function $C(p, q)$ in *long* direction is defined only by the α parameter value and does not depend on the profile of transverse velocity v_T at the particlization hypersurface (unfortunately, this is not the case for the two transverse directions). This allows to simplify further analysis significantly, if in what follows we restrict ourselves to the analysis of longitudinal direction only.

Introducing the longitudinal homogeneity length at non-zero transverse flow, $\lambda_l = \tau \sqrt{\frac{T}{m_T} (1 - \bar{v}_T^2)^{1/2}}$ [23,24] and denoting its ratio to τ as λ , one has

$$\lambda^2 = \frac{\lambda_l^2}{\tau^2} = \frac{T}{m_T} (1 - \bar{v}_T^2)^{1/2}. \quad (5)$$

Here $\bar{v}_T = k_T / (m_T + \alpha T)$ is the transverse flow velocity in the saddle point, and $T = T_{m.e.}$ is the temperature at the hypersurface of maximal emission, $\tau = \tau_{m.e.}$. Then in LCMS frame one can write for the correlation function in $l \equiv \text{long}$ direction (at $q_{\text{out}} = q_{\text{side}} = 0$) [25]:

$$C(k, q_l) = 1 + \frac{\exp \left[\frac{2}{\lambda^2} \left(1 - \sqrt{1 + \tau^2 \lambda^4 q_l^2} \right) \right]}{[1 + \tau^2 \lambda^4 q_l^2]^{3/2}} \xrightarrow{k_T \rightarrow \infty} 1 + \exp(-\lambda_l^2 q_l^2). \quad (6)$$

The obtained expression implies that in general case the correlation function is not Gaussian, which means that one can obtain different analytical approximations for the Gaussian interferometry radii based on this function, depending on the considered physical situation (see [25] for more details).

In particular, when one considers the case of longitudinally boost-invariant matter expansion with a transverse flow having arbitrary velocity profile $v_T(\mathbf{r}_T)$, then for small q_{long} (the peak of the correlation function) one has for $R_{\text{long}}(m_T)$:

$$R_{\text{long}}^2(m_T) = \tau^2 \lambda^2 \left(1 + \frac{3}{2} \lambda^2 \right), \quad (7)$$

where $m_T = \sqrt{m^2 + k_T^2}$. The formula (7) can be applied in case of transverse flow of any intensity, which is especially important for the LHC energies.

Having obtained the formula (7) for the femtoscopy radii fitting from the approximation for the correlation function (4), one can further use a similar approach to get the formula for momentum spectrum starting from the Eq. (3) [24]. As a result one will have

$$p_0 \frac{d^3 N}{d^3 p} \propto \exp[-(m_T/T + \alpha)(1 - \bar{v}_T^2)^{1/2}]. \quad (8)$$

This formula allows one to approximate the slope of the transverse momentum spectrum at not very high p_T in the presence of transverse collective flow under the assumption that the shape of the spectrum is close to the exponential one.

The procedure, proposed in [7] for the estimation of the time of maximal emission for pions and kaons in the LHC Pb+Pb collisions, suggests at first to determine the parameters T and α based on the results of combined fitting of pion and kaon p_T spectra with Eq. (8) and then use the found parameter values to fit the corresponding $R_{\text{long}}(m_T)$ dependencies with Eq. (7). This latter fitting gives one the desired time of maximal emission values τ_π and τ_K .

3. Results and discussion

In the current study we follow the same algorithm as described in [7] and perform fitting of the p_T spectra and *long* femtoscopy radii obtained from the iHKM realistic simulations of the relativistic heavy-ion collisions. The model consists of several modules, each describing one of the collision stages (initial state of the system right after the nuclei have passed through each other, pre-thermal expansion of far-from-equilibrium system, hydrodynamical expansion of nearly thermalized matter, particlization and hadron cascade stage — see [17,18] for details). In [20] the model was tuned for the simulation of Pb+Pb collisions at the LHC energy 5.02 A TeV and showed good agreement with the experimental results on different particle production observables for this energy. The predictions for interferometry radii in the three centrality classes ($c = 0 - 5\%$, $c = 20 - 30\%$, and $c = 40 - 50\%$) were also made (see Figs. 1, 2 and 3). Thus, we have all the necessary data to apply our method and try to extract the maximal emission times.

In Fig. 4 one can see the plot demonstrating pion and kaon p_T spectra for the most central collisions ($c = 0 - 5\%$) calculated in iHKM together with the ALICE Collaboration experimental data [26] and fitting curves according to Eq. (8). The fitting is carried out in the momentum range $0.45 < p_T < 1.0$ GeV/c. The temperature T is fixed to be a common parameter for both pion and kaon spectrum, while the α and normalizing constant parameters are supposed to be different for pions and kaons. It is worth noting that as compared to the case of 2.76 A TeV collisions, the fitting results are not so stable and fluctuate depending on the used p_T range and the initial constraints put on the parameters. This can mean that the shape of the spectra for 5.02 A TeV Pb+Pb collisions is not so close to the exponential one as for the lower LHC energy. In such a situation we had to compare different fitting results and choose one of them. We finally settled on the result with the common temperature $T = 138 \pm 16$ MeV, $\alpha_\pi = 4.8 \pm 1.1$ and $\alpha_K = 2.4 \pm 0.6$, based on several considerations, such as that the effective temperature should not differ much from the value $T = 144$ MeV, obtained in the case of $\sqrt{s_{NN}} = 2.76$ TeV, where the fit was more stable, and also that the parameter errors and the fit's χ^2 should be as small as possible.

Fixing then the T and α parameters at the values found during the spectra fitting, we performed the fitting of *long* radii dependency on the pair m_T (see Fig. 5). Similarly to [7] we readily obtained a good fit for pion radii and extracted the corresponding pion maximal emission time $\tau_\pi = 8.97 \pm 0.04$ fm/c, while for kaons we had to make α parameter free again to get satisfactory description of the iHKM points. As a result, we obtained $\alpha_K = 0.062 \pm 0.049$ and the

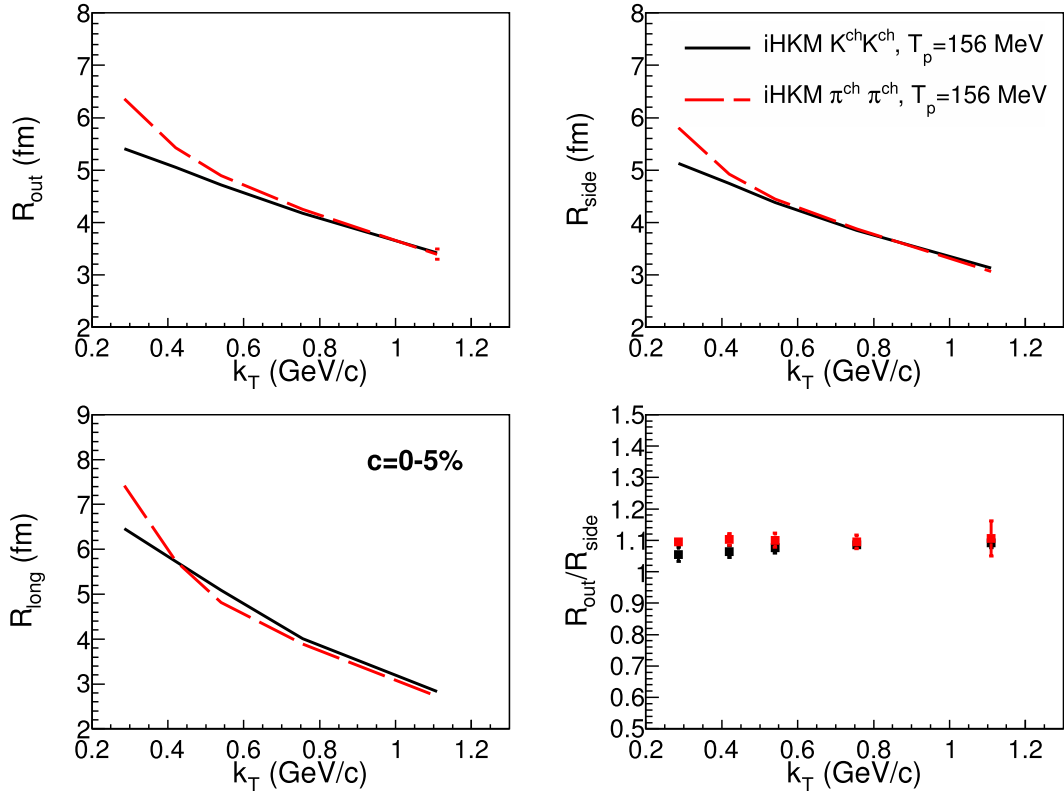


Fig. 1. The iHKM results for pion and kaon interferometry radii in the LHC Pb+Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV, $c = 0 - 5\%$.

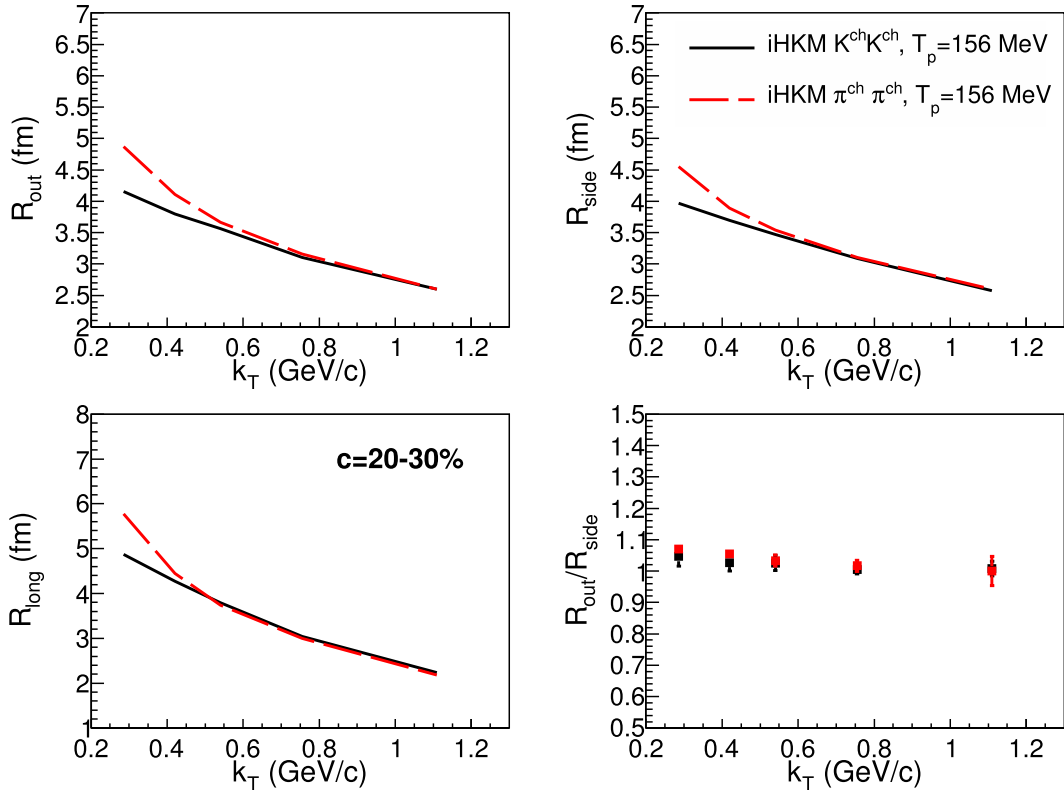


Fig. 2. The same as in Fig. 1 for $c = 20 - 30\%$.

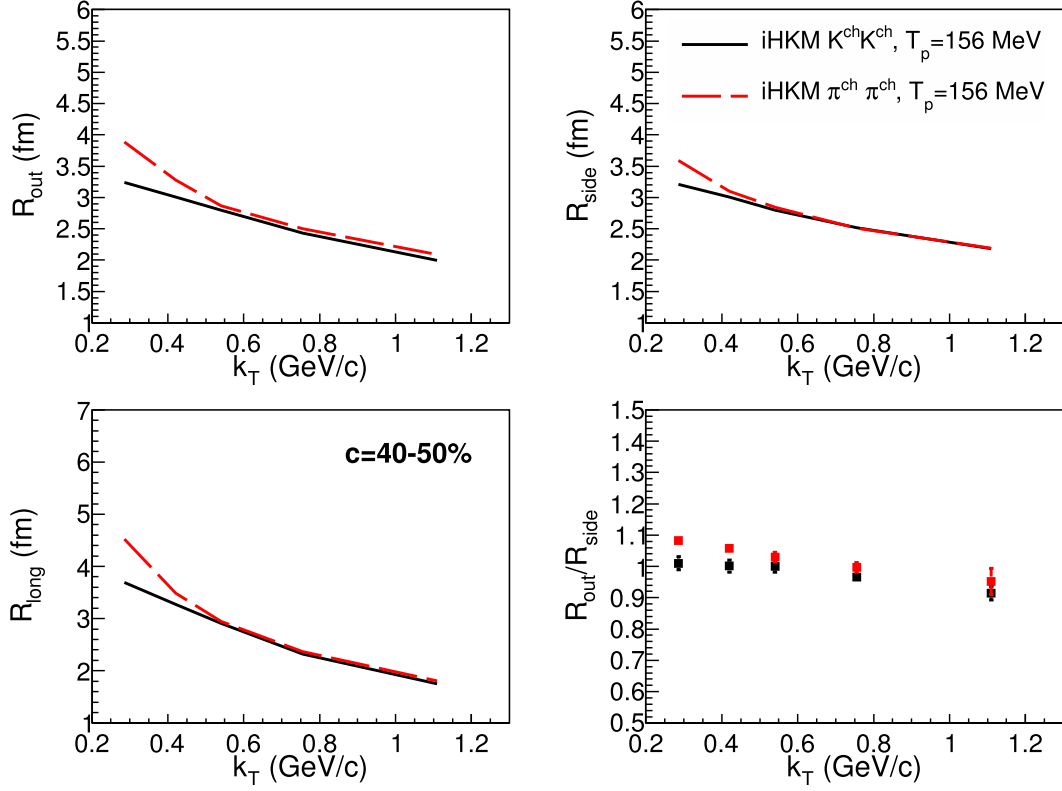


Fig. 3. The same as in Fig. 1 for $c = 40 - 50\%$.

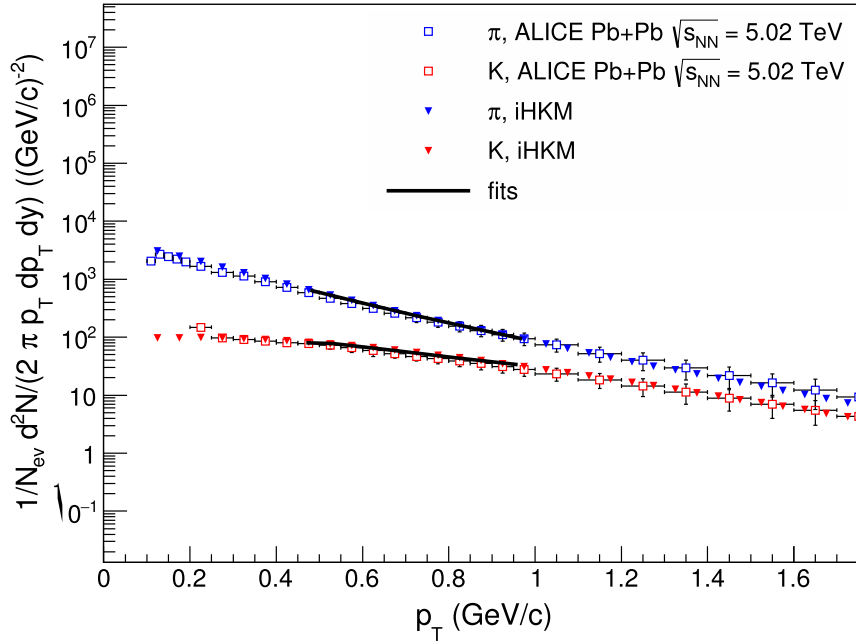


Fig. 4. The iHKM results for pion and kaon p_T spectra compared to the ALICE data [26] for the LHC Pb+Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV ($c = 0 - 5\%$) together with the lines, representing a combined fit to the iHKM spectra using (8) with the same effective temperature T for pions and kaons.

maximal emission time $\tau_K = 12.73 \pm 0.12$ fm/c. One can also see that the m_T scaling between pion and kaon radii is violated, as well as in $\sqrt{s_{NN}} = 2.76$ TeV collisions, apparently due to the presence of strong transverse flow and intensive hadron-hadron interactions at the afterburner

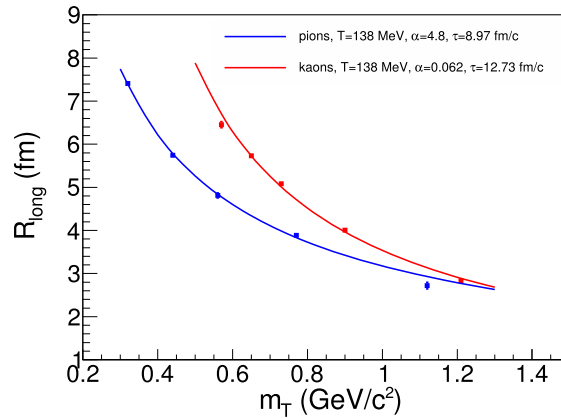


Fig. 5. The fitting of pion (blue squares) and kaon (red squares) femtoscropy radii, calculated in iHKM for $c = 0 - 5\%$ with the lines corresponding to formula (7). The common effective temperature $T = 138$ MeV and the value $\alpha_\pi = 4.8$ are taken from the combined pion and kaon p_T spectra fit with Eq. (8). The α value for kaons as well as the maximal emission times τ_π and τ_K are free parameters. Their values extracted from the best fit are: $\alpha_K = 0.062 \pm 0.049$, $\tau_\pi = 8.97 \pm 0.04$ fm/c and $\tau_K = 12.73 \pm 0.12$ fm/c. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

stage of the collision, as it was advocated in [7]. At the same time, k_T scaling takes place starting from $k_T \approx 0.5$ GeV/c for all femtoscropy scales. Such a scaling for pion and kaon radii at the LHC was predicted in Ref. [27] and confirmed by the ALICE Collaboration [16].

A slight overestimation of pion radius at the highest considered m_T value, $m_T = 1.12$ GeV/c², in fitting curve can be possibly connected with the fact that we use the approximation of $\tau = \tau_{m.e.} = \text{const}$ at hadronization hypersurface, applicable for soft particles with not very high p_T . However quanta with m_T close to 1 GeV/c² are emitted from the side part of the overall hadronization hypersurface, where τ values are smaller than $\tau_{m.e.}$ for soft particles. That is why, since fitted radii values are proportional to τ (see (7)), the fitting curve goes somewhat higher than the iHKM point for $m_T = 1.12$ GeV/c².

The reason for redefining the α parameter for kaons for the radii fitting is non-Gaussian shape of the correlation function. The formula (7) was derived under the assumption of small q_{long} , and thus it describes well only the radii corresponding to the peak part of the non-Gaussian correlation function. The $R_{\text{long}}(m_T)$ fit with α_K fixed to the value extracted from the combined spectra fitting would describe the iHKM points well if the latter were obtained from the Gaussian fits to the model correlation functions in a narrow range for q , as it was demonstrated in [7] (see Figs. 5, 6 from [7] and the related text) for the radii extracted using the q range $|q| < 0.04$ GeV/c. And to describe the femtoscropy radii obtained from the correlation function fits in a wider q interval (we used the range $|q| < 0.2$ GeV/c to extract the radii presented here) one should use a separate α_K value, different from that ensuring the kaon spectrum description.

We also present the results on pion and kaon maximal emission times for the two other centrality classes, $c = 20 - 30\%$ and $c = 40 - 50\%$, for which the iHKM predictions on femtoscropy radii were made in [20]. The corresponding plots are shown in Figs. 6 and 7. For the $c = 20 - 30\%$ case the temperature parameter extracted from the combined pion and kaon p_T spectra fit is $T = 125 \pm 12$ MeV and the corresponding $\alpha_\pi = 3.94 \pm 0.81$. The $\alpha_K = 0.35 \pm 0.11$ value is again extracted from the radii fit, and the maximal emission times are $\tau_\pi = 7.59 \pm 0.03$ fm/c and $\tau_K = 9.87 \pm 0.14$ fm/c. Finally, in the $c = 40 - 50\%$ case the values $T = 127 \pm 15$ MeV and $\alpha_\pi = 4.20 \pm 1.18$ were found from the spectra fit, and the *long* radii fit gave $\alpha_K = 0.53 \pm 0.11$, together with the times $\tau_\pi = 5.88 \pm 0.02$ fm/c and $\tau_K = 7.35 \pm 0.08$ fm/c.

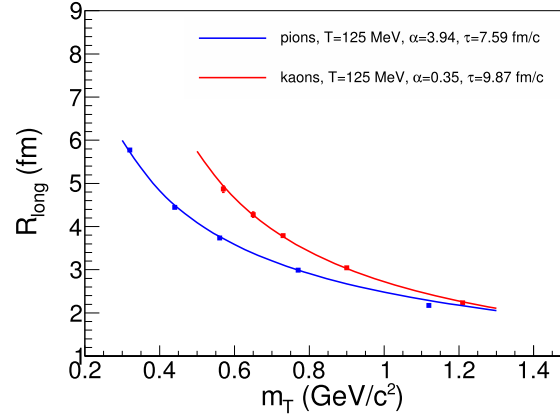


Fig. 6. The fitting of pion (blue squares) and kaon (red squares) femtoscopy radii, calculated in iHKM for $c = 20 - 30\%$ with the lines corresponding to formula (7). The common effective temperature $T = 125$ MeV and the value $\alpha_\pi = 3.94$ are taken from the combined pion and kaon p_T spectra fit with Eq. (8). The α value for kaons as well as the maximal emission times τ_π and τ_K are free parameters. Their values extracted from the best fit are: $\alpha_K = 0.35 \pm 0.11$, $\tau_\pi = 7.59 \pm 0.03$ fm/c and $\tau_K = 9.87 \pm 0.14$ fm/c.

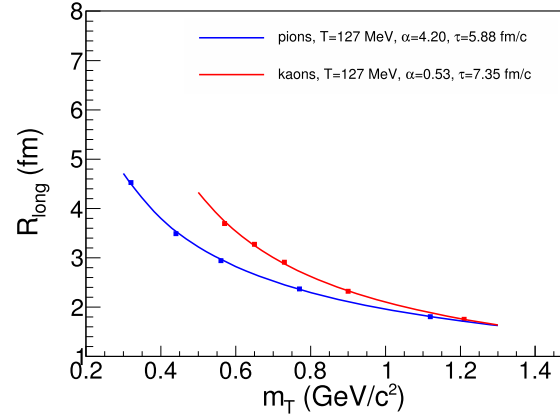


Fig. 7. The fitting of pion (blue squares) and kaon (red squares) femtoscopy radii, calculated in iHKM for $c = 40 - 50\%$ with the lines corresponding to formula (7). The common effective temperature $T = 127$ MeV and the value $\alpha_\pi = 4.20$ are taken from the combined pion and kaon p_T spectra fit with Eq. (8). The α value for kaons as well as the maximal emission times τ_π and τ_K are free parameters. Their values extracted from the best fit are: $\alpha_K = 0.53 \pm 0.11$, $\tau_\pi = 5.88 \pm 0.02$ fm/c and $\tau_K = 7.35 \pm 0.08$ fm/c.

One can note that the obtained effective temperature in case of non-central events is lower than that for the most central collisions, as well as the corresponding maximal emission times. The values of α , on the contrary, are higher for the non-central collisions. Such interrelation between the fit parameter values reflects the actual physical difference between the systems, formed in central and non-central collisions, namely that those created in non-central collisions live shorter, cool-down faster and develop less intensive collective flows during their evolution.

Additionally, in Figs. 8-10 we demonstrate the plots for the averaged emission functions of pions and kaons for the three considered centrality classes, which allows to qualitatively identify the regions of the maximal emission for particles of each species (with $0.2 < p_T < 0.3$ GeV/c) and in such a way to approximately estimate the corresponding effective maximal emission times τ .

One can see, that the maximal emission time values, previously obtained from the fits, e.g. for $c = 0 - 5\%$ events, $\tau_\pi = 8.97$ fm/c and $\tau_K = 12.73$ fm/c, are in agreement with the presented emission pictures, since according to the plot, the maximum of pion emission should be close to

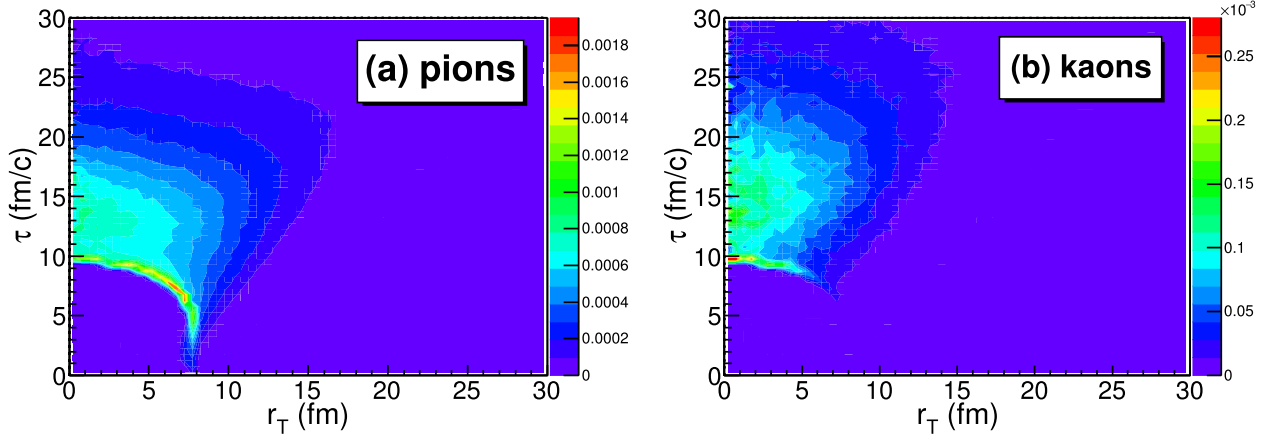


Fig. 8. The emission functions per units of space-time and momentum rapidities averaged over momentum angles $g(\tau, r_T, p_T)$ [fm^{-3}] for pions (a) and kaons (b) obtained from the iHKM simulations of the LHC Pb+Pb collisions at the energy $\sqrt{s_{NN}} = 5.02$ TeV, $0.2 < p_T < 0.3$ GeV/c, $|y| < 0.5$, $c = 0 - 5\%$.

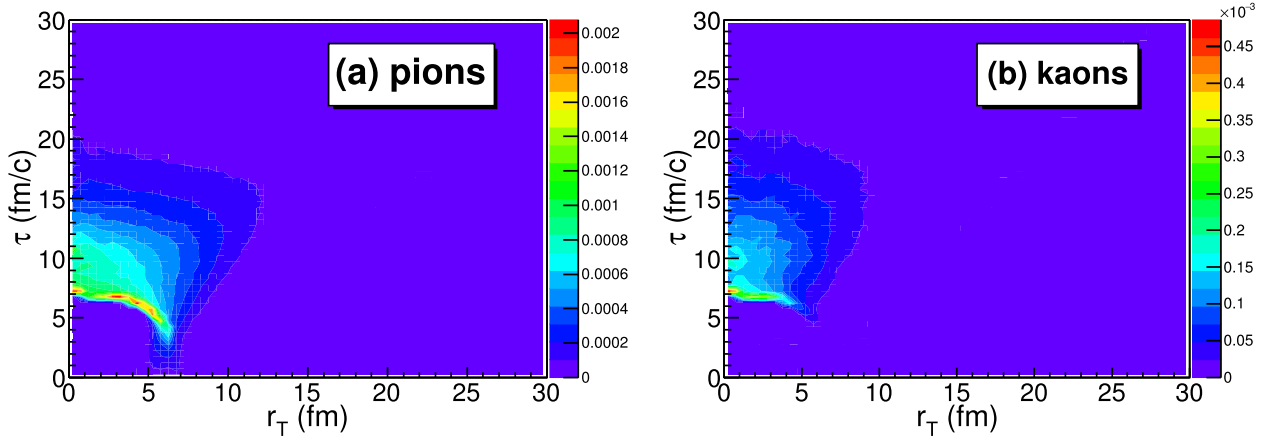


Fig. 9. The same as in Fig. 8 for the events from $c = 20 - 30\%$ centrality class.

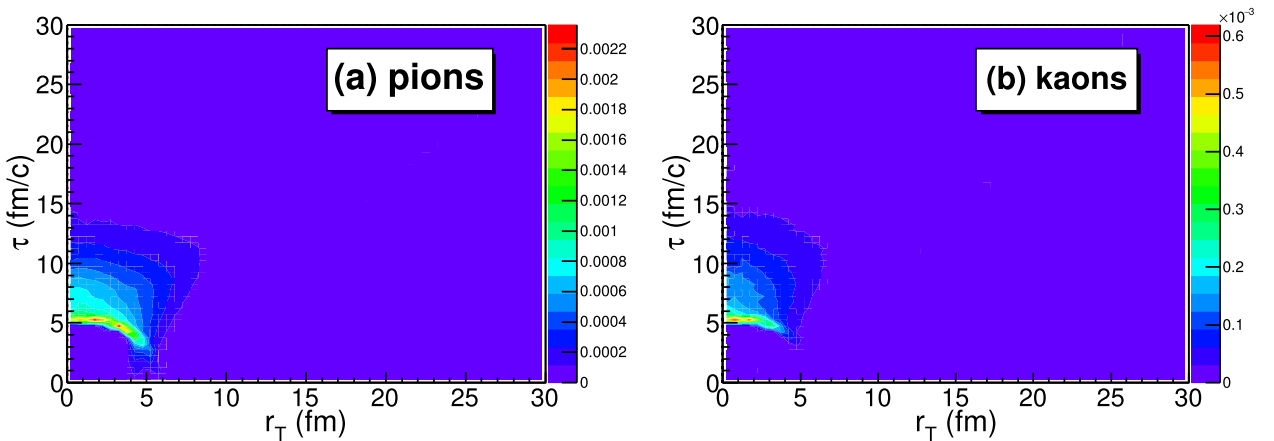


Fig. 10. The same as in Fig. 8 for the events from $c = 40 - 50\%$ centrality class.

the particlization time in the center of the system, $\tau \approx 10$ fm/c, and for kaons the effective τ_K value should be between the two emission maxima, seen on the plot (more clearly in numerical representation), namely between $\tau \approx 10$ fm/c and $\tau \approx 15$ fm/c. The second in time maximum is

conditioned by $K^*(892) \rightarrow \pi + K$ decays, as it was earlier explained in Ref. [7] and in fact was confirmed by the results of the ALICE Collaboration [16].

Quite similar situation takes place also for non-central collisions (see Figs. 9, 10): the times of maximal emission for pions, $\tau_\pi = 7.59$ fm/c and $\tau_\pi = 5.88$ fm/c, extracted from the fits are close to the central parts' particlization times, $\tau \approx 7.5$ fm/c and $\tau \approx 5$ fm/c, following from the emission pictures. For kaons the obtained times of maximal emission, $\tau_K = 9.87$ fm/c and $\tau_K = 7.35$ fm/c, are, similarly to the central collision case, higher than those of pions by about 2 – 3 fm/c, i.e. are between the particlization time and the time of K^* resonance decay, whose lifetime is about 4 – 5 fm/c and which forms a second kaon emission maximum on the radiation plots (however, for non-central collisions, especially for the $c = 40 - 50\%$ case, this second maximum is less pronounced than in central events, maybe due to smaller number of produced particles in non-central collisions).

4. Conclusions

The times of the maximal emission of kaons and pions in the Pb+Pb collisions at the LHC energy 5.02A TeV were estimated based on the transverse momentum spectra and *long* femtoscopy radii fitting with the analytical formulas accounting for the presence of transverse collective flow. Comparing the fitting results for different centrality classes, one can observe that the maximal emission times, effective temperatures, and flow intensities are smaller in non-central events, than in central ones. The fitting results are in a qualitative agreement with the regions of the most intensive particle emission seen on the averaged emission function plots. The stability of the spectra fits is worse as compared to the case of the lower LHC energy 2.76A TeV, which may be due to noticeable deviations of the spectrum shape from the exponential.

In this note, we found that, similarly to LHC Pb+Pb collisions at the energy 2.76A TeV, kaons radiate later than pions at all centralities also at the energy 5.02A TeV. The reason, again, is mostly decays of K^* resonance. The intensive hadron-hadron scatterings at the afterburner stage of the collision along with very intensive transverse flow results in breaking of m_T scaling between pion and kaon interferometry radii. At the same time, we again, as for the smaller LHC energy, predict k_T scaling at not very small k_T for pion and kaon femto-scales also for Pb+Pb collisions at currently the highest energy. We are looking forward to the corresponding results of femtosopic analysis from the LHC Collaborations for the energy 5.02A TeV.

CRediT authorship contribution statement

V.M. Shapoval: Data curation, Investigation, Visualization, Writing – original draft. **Yu.M. Sinyukov:** Conceptualization, Funding acquisition, Methodology, Project administration, Resources, Supervision.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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2.4. Спектри, кореляції та часи емісії частинок у зіткненнях ядер золота при найвищій енергії RHIC



Description of bulk observables in Au+Au collisions at top RHIC energy in the integrated hydrokinetic model

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Abstract

The results on the main bulk observables obtained in the simulations within the integrated hydrokinetic model (iHKM) of Au+Au collisions at the RHIC energy $\sqrt{s_{NN}} = 200$ GeV are presented along with the corresponding experimental data from the STAR and the PHENIX collaborations. The simulations include all the stages of the collision process: formation of the initial state, its gradual thermalization and hydrodynamization, viscous relativistic hydro-evolution, system's hadronization and particlization, and, finally, an expansion of the interacting hadron-resonance gas. The model gives a satisfactory description of charged-particle multiplicities, particle number ratios, transverse momentum spectra for pions, kaons, (anti)protons, lambdas, xi, and omega, charged-particle v_2 coefficients, and femtoscopy radii at all collision centralities. It is demonstrated how one can estimate the times of the pion and kaon maximal emission from the femto-scales.

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Keywords: Gold-gold collisions; RHIC; Multiplicity; Momentum spectra; Interferometry radii

1. Introduction

The comprehensive studies of ultrarelativistic heavy ion collisions allow researchers gradually, step by step reveal new properties of rather interesting and unusual form of matter, created in these processes, and construct more and more clear and full picture of evolution of such super

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dense and super hot systems. As it became clear after the thorough analysis of bulk observables at RHIC and LHC, such as particle multiplicities, transverse momentum spectra, and femtoscopy scales, the strongly interacting quark-gluon matter, formed in a collision at high energy, at some stage of its evolution undergoes collective expansion and behaves like a nearly thermalized, (quasi)macroscopic system. This fact justified the application of hydrodynamical and statistical mechanics approximations for the theoretical description of this stage. However, the pre-thermal dynamics, leading to the system's thermalization, as well as the “afterburner” stage of its evolution, also play an important role in the formation of final observables. That is why a realistic model, allowing to successfully describe and predict various experimental data and helping to understand the reasons and mechanisms for the specific experiment results, should be complex and include an adequate simulation of all the stages of the collision process.

In this work, we present the results of our study, devoted to the description of different bulk observables in Au+Au collisions at the RHIC energy $\sqrt{s_{NN}} = 200$ GeV within such type of hybrid approach — the integrated hydrokinetic model [1] (iHKM).

Despite the experiments at the top RHIC energy were performed quite a long time ago, and the most recent results concern heavy ion collisions at the LHC, the datasets, collected at RHIC, are still in use and still are of interest for the analysis, in particular, for the studies dealing with kaon femtoscopy [2]. Additionally, although the time has passed since the first papers, presenting the results of certain measurements at RHIC (e.g., two-pion femtoscopy), were published, the STAR and PHENIX collaborations continue to issue new articles, containing results on the same topic, but with increased accuracy, in a wider region, with new cuts applied, etc. This fact also motivates one not to forget about the RHIC data.

Previously, the Au+Au collisions at the top RHIC energy were successfully simulated in the hydrokinetic model [3–5], the model-predecessor of the modern, more developed iHKM, which proved to be good in describing observables at the LHC energies [1,6,7]. Here we aim to adjust the iHKM to the description of yields, p_T spectra, interferometry radii, etc. at RHIC and see what differences in the model parameters and tuning will it require as compared to the LHC case.

2. Model description

In iHKM the evolution of the system, formed in the relativistic nuclear collision starts with the pre-thermal stage, which simulates the process of gradual transformation of the initially not thermalized system to a nearly thermal one, close to local thermal and chemical equilibrium, that can be further described using viscous hydrodynamics approximation. At this stage an energy-momentum transport approach in the relaxation time approximation is utilized (see [1,8] for details).

The initial distribution of energy density in the transverse plane for the pre-thermal stage is chosen to be a linear combination of wounded nucleons and binary collision contributions in GLISSANDO [9] (version 2.702) Monte Carlo Glauber model:

$$\epsilon(b, \mathbf{r}_T) = \epsilon_0(\tau_0) \frac{(1 - \alpha)N_w(b, \mathbf{r}_T)/2 + \alpha N_{bin}(b, \mathbf{r}_T)}{(1 - \alpha)N_w(b=0, \mathbf{r}_T=0)/2 + \alpha N_{bin}(b=0, \mathbf{r}_T=0)}, \quad (1)$$

where the parameters α (defining the proportion between the two contributions to $\epsilon(b, \mathbf{r}_T)$) and $\epsilon_0(\tau_0)$ (defining the maximal initial energy density in the center of the system for the most central collisions) are adjusted to provide the best fit to experimental dependence of mean charged particle multiplicity in the pseudorapidity region $|\eta| < 0.5$ on centrality, and the value of the starting time τ_0 ensures the best description of pion p_T spectrum slope in

the most central events. For the current study we obtained $\alpha = 0.18$ and $\epsilon_0 = 235 \text{ GeV/fm}^3$ at $\tau_0 = 0.1 \text{ fm}/c$. At the LHC energies, the coefficient $\alpha = 0.24$, and at the same initial time $\tau_0 = 0.1 \text{ fm}/c$ one has $\epsilon_0 = 679 \text{ GeV/fm}^3$ for $\sqrt{s_{NN}} = 2.76 \text{ TeV}$ [1], and $\epsilon_0 = 1067 \text{ GeV/fm}^3$ for $\sqrt{s_{NN}} = 5.02 \text{ TeV}$ [7]. The other parameters, such as the initial momentum anisotropy, viscosity-to-entropy ratios, relaxation and thermalization times, are the same at all the mentioned collision energies at RHIC and LHC and at the same Laine-Schroeder equation of state [10] (the latter is only corrected to take into account a small chemical potential at the top RHIC energy as discussed below).

The initial momentum distribution for the pre-equilibrium dynamics is of the following “Color-Glass-Condensate-like” form:

$$f_0(p) = g \exp \left(-\sqrt{\frac{(p \cdot U)^2 - (p \cdot V)^2}{\lambda_{\perp}^2}} + \frac{(p \cdot V)^2}{\lambda_{\parallel}^2} \right), \quad (2)$$

where $U^\mu = (\cosh \eta, 0, 0, \sinh \eta)$, $V^\mu = (\sinh \eta, 0, 0, \cosh \eta)$, η is space-time rapidity, and initial momentum anisotropy $\Lambda = \frac{\lambda_{\perp}}{\lambda_{\parallel}} = 100$ [1].

After the pre-thermal evolution in iHKM follows the viscous hydrodynamics stage, realized within the Israel-Stewart formalism. Here we describe the collisions at RHIC, so, in contrast to the LHC case, we need to account for a small, but still non-zero chemical potential (baryon and strange) in the equation of state (EoS) for the quark-gluon phase. In order to do this, we modify the EoS at zero chemical potential according to [11]:

$$\frac{p(T, \mu_B, \mu_S)}{T^4} = \frac{p(T, 0, 0)}{T^4} + \frac{1}{2} \frac{\chi_B}{T^2} \left(\frac{\mu_B}{T} \right)^2 + \frac{1}{2} \frac{\chi_S}{T^2} \left(\frac{\mu_S}{T} \right)^2, \quad (3)$$

where $p(T, 0, 0)$ is the Laine-Schroeder [10] equation of state at zero chemical potential and

$$\frac{\chi_i}{T^2} = \frac{1}{VT^3} \frac{\partial^2 \ln Z}{\partial (\mu_i/T)^2}, \quad i = B, S. \quad (4)$$

Here the “mixed” terms with χ_{BS} , as well as the terms proportional to electric chemical potential μ_E are neglected due to their smallness. In this paper we take $\mu_B = 21 \text{ MeV}$ at the hadronization/particlization hypersurface $T = 165 \text{ MeV}$ for the best description of the ratio of proton yield to that of antiproton for the centrality $c = 0 - 5\%$. And the value of $\mu_S = 5 \text{ MeV}$ is obtained using the variation method as satisfying the condition of zero strangeness on the hadronization (particlization) hypersurface:

$$S|_{\sigma_p} = 0, \quad S = \sum_i (N(i) - \bar{N}(i)) \mu_{S,i}, \quad (5)$$

where the sum is taken over all particle species, $N(i)$ is the number of particles of the i th sort, $\bar{N}(i)$ is the number of corresponding antiparticles on the hadronization hypersurface, and $\mu_{S,i}$ is the strange chemical potential of the particle species i . The evolution equations with chemical potential depending on T are considered in the same way as in Refs. [3,4].

We assume that at the top RHIC energy the strange quarks do not have enough time to reach the chemical equilibrium in *non-central collisions*, so that the kaon spectra get down. The same concerns proton spectra, since about a half of produced protons come from the decays of strange resonances (such as Λ , Σ , Ξ). To take this into account we introduce an effective downscaling factor $\gamma_S(\tau_p)$, depending on the characteristic particlization time τ_p (roughly it is life-time of quark-gluon plasma) for each given centrality. This time is calculated in iHKM. Each hadron

Table 1
The values of γ_S for different centrality classes.

centrality [%]	τ_p [fm/c]	γ_S
0-10	7.55	0.989
10-20	6.50	0.973
20-30	5.60	0.953
30-40	4.85	0.933
40-50	4.20	0.909
50-60	3.60	0.881
60-70	3.00	0.843
70-80	2.30	0.777

yield is multiplied by $\gamma_S^{S_i}$, where S_i is the strangeness of the hadron species i . We assume the dependence

$$\gamma_S(\tau_p) = A \exp(-b/\tau_p), \quad (6)$$

with $A = 1.1$ and $b = 0.8$ fm/c. This choice guarantees $\gamma_S = 1$ for the most central events and a good description of kaon and proton spectra, together with K/π ratio. Table 1 shows the gamma factors, obtained for different centralities based on the corresponding particlization times.

After the hydrodynamic stage in iHKM we have the particlization stage, when we switch from the description of the system's evolution in terms of continuous medium to the description in terms of particles. In the current analysis this switching is performed at the hadronization hypersurface $T = 165$ MeV. After that particles pass the stage of the hadronic cascade, realized in iHKM within the UrQMD model (version 3.3p1) [12], which simulates resonance decays, as well as numerous elastic and inelastic hadron scatterings, taking place in expanding and initially dense hadronic gas.

A common problem of so-called “hybrid approaches” to relativistic heavy ion collision simulation that use Cooper-Frye prescription is the necessity to account for the formally negative contributions to particle numbers from non-spacelike parts of the hadronization hypersurface. We made an estimation of such contributions in our model and found that for the currently used model setup, adjusted to the description of observables at top RHIC energy, their fraction is quite small, namely about 2%. We think this can be explained by high hydrodynamic velocities of the fluid elements, crossing the non-spacelike parts of the particlization hypersurface. Therefore, the corresponding particles belong to the tail of the relativistic (Boltzmann) spectrum, and their negative contributions to the Cooper-Frye formula are negligible.

Currently a lot of research groups use various kinds of hybrid approaches to the simulation and study of high-energy nuclear collisions [13–18] (see [19] for a review) and obtain more or less successful description of particular observables. Typically these approaches describe expansion of locally equilibrated matter within certain hydrodynamic formalism (ideal or viscous; (1+1)-, (2+1)- or (3+1)-dimensional) starting with some initial energy/entropy density profile, and at the final stage of system's evolution, when it decouples into hadrons, switching to some hadron cascade model (UrQMD, JAM [20], etc.) is done. Such switching is usually performed at some isothermal hypersurface, corresponding to temperature close to 150–170 MeV. The different models of the system's initial state can be applied, e.g. Monte Carlo Kharzeev-Levin-Nardi (MC KLN) model [21], based on Color Glass Condensate (CGC) [22] effective gluon field theory, and mentioned Monte Carlo Glauber (MC Glauber) model are most commonly used for generating averaged smooth initial density profile. As for event-by-event simulations,

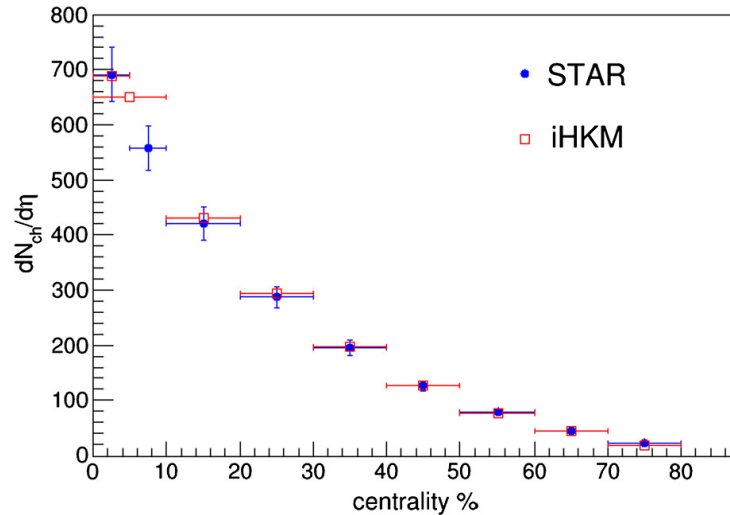


Fig. 1. The iHKM results on the mean charged particle density $\langle dN_{ch}/d\eta \rangle$ at $|\eta| < 0.5$ for different centrality classes, compared with the STAR data [28].

a CGC-based IP Glesma [23], NeXuS [24] and EPOS [25], based on Gribov-Regge approach, and HIJING model [26], based on Lund jet fragmentation scheme and perturbative QCD, are generally utilized. There also are other models, which can be applied, including various parameterizations of initial conditions. Additionally, different equations of state can be assumed for the expanding dense quark-gluon matter within different hybrid approaches (lattice QCD EoS, s95p parametrization [27], etc.).

The iHKM model, used in the present study, is favorably different from other approaches in that it includes the description of *prethermal* dynamics of the system and, starting from non-equilibrium state at the initial time $\tau_0 \sim 0.1$ fm/c, gradually brings the system to the state of local equilibrium, to which a hydrodynamic approach can be further applied. The other models typically lack such mechanism of matter thermalization, beginning description of the system's evolution from already equilibrated state. This state is typically attributed to comparably large times, $\tau \sim 0.6 - 1$ fm/c, when the thermalization can be considered achieved. However, as it was shown in [8], collective flows and other effects develop quite close to the hydrodynamics rate already at the prethermal stage, when there are no pressure gradients, just due to the finiteness and azimuthal asymmetry of the system. Accordingly, in iHKM the thermalization time parameter τ_{th} has little effect on the final calculation results, while one of the main model parameters, defining the results' behavior, becomes the initial time τ_0 . That is why, for the successful simultaneous description of a large set of observables, one needs to start matter expansion at early times, when matter is still non-equilibrated, as it is realized in iHKM.

3. Results and discussion

Having adjusted the model parameters and run enough simulations to collect good statistics, we can immediately calculate a great variety of observables, basing on the model output.

In Fig. 1 and Table 2 one can see the iHKM results on the mean multiplicity of all charged particles $\langle dN_{ch}/d\eta \rangle$ in pseudorapidity range $|\eta| < 0.5$ for eight centrality classes — 0-5%, 10-20%, 20-30%, 30-40%, 40-50%, 50-60%, 60-70%, 70-80% — in comparison with the STAR experimental data [28]. The model results are in good agreement with the experiment.

In Fig. 2 we present our results on K^-/π^- and $\bar{p}/\pi^-$ particle number ratios for different centrality classes in comparison with the experiment [29]. The hadrons with rapidity from the

Table 2

Mean charged particle density $\langle dN_{\text{ch}}/d\eta \rangle$ at $|\eta| < 0.5$ for different centrality classes.

centrality [%]	$\langle dN_{\text{ch}}/d\eta \rangle$ in iHKM	$\langle dN_{\text{ch}}/d\eta \rangle$ from STAR
0-5	688	691 ± 49
10-20	431	421 ± 30
20-30	294	287 ± 20
30-40	197	195 ± 14
40-50	127	126 ± 9
50-60	77	78 ± 6
60-70	45	45 ± 3
70-80	19	22 ± 2

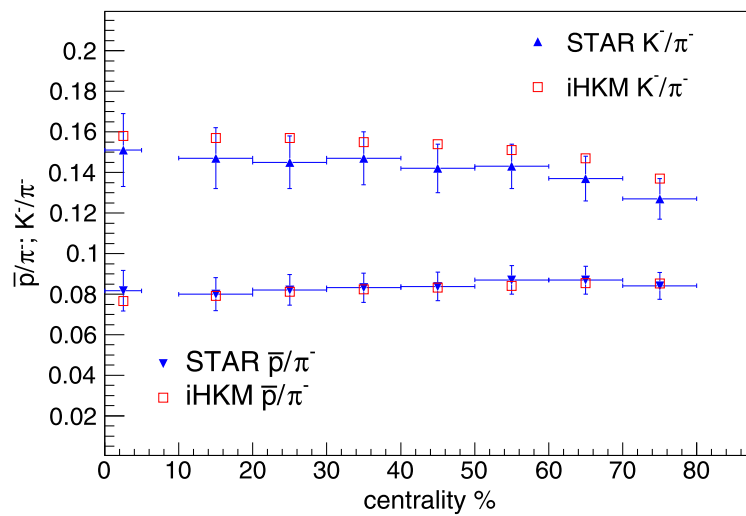


Fig. 2. The ratios of K^- (upper points) and $\bar{p}$ (lower points) yields to that of π^- , obtained in iHKM in comparison with the STAR data [29]. Particles with $|y| < 0.1$ were selected for the analysis.

range $|y| < 0.1$ were selected to construct all the ratios. As one can see from the figure, the iHKM describes well both ratios at all centralities within the errors, however K/π model points go through the upper edges of the experimental error bars.

Figs. 3-6 demonstrate the iHKM description of transverse momentum spectra for negatively charged pions and kaons, as well as for protons and antiprotons at different centralities in the rapidity range $|y| < 0.1$. The simulation results are compared with the STAR experimental data [29]. One can say that the model reproduces the measured spectra quite well for all the mentioned particle species and all the centrality classes. In Figs. 7, 8 and 9 one can find also the iHKM results on strange baryon p_T spectra, namely Λ , Ξ and Ω , in comparison with the STAR Collaboration experimental points [30]. Slight deviations of the iHKM lines from the data can be noticed in K and $\bar{p}$ spectra at low p_T for the very peripheral collisions with $c = 70 - 80\%$ only. Note however, that such a good agreement with data is achieved here using a “strangeness suppression” γ_s factor (6), which helps to lower the “raw” model kaon, (anti)proton, lambda, xi and omega spectra for non-central events. In STAR paper [30] similar suppression factors γ_s are extracted from the Boltzmann fits to the measured strange baryon p_T spectra. In Fig. 10 we compare these values, obtained by STAR, with our γ_s , calculated according to (6), based on the characteristic particlization times τ_p . As a result, we see that our γ_s factors match with those calculated by STAR within the error bars.

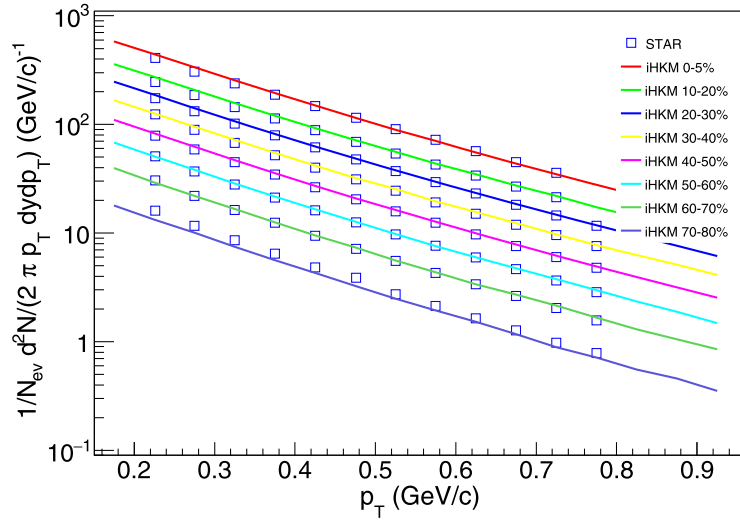


Fig. 3. The p_T spectra of π^- in comparison with the STAR data [29] at midrapidity, $|y| < 0.1$. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

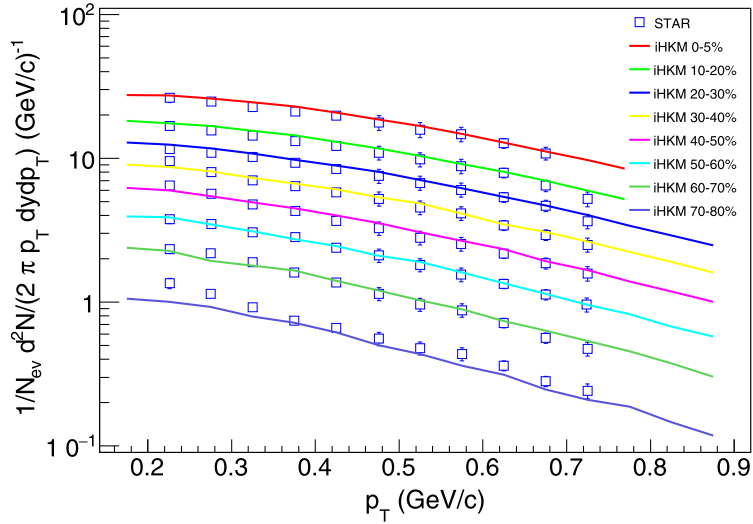


Fig. 4. The p_T spectra of K^- in comparison with the STAR data [29] at midrapidity, $|y| < 0.1$.

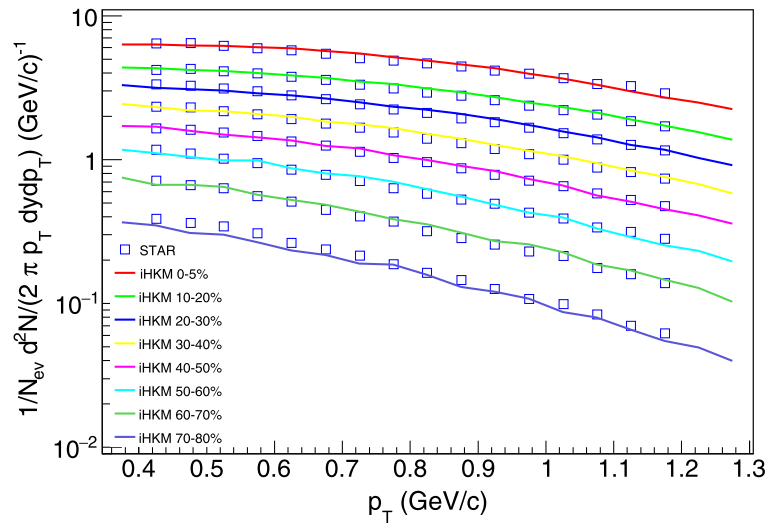


Fig. 5. The proton p_T spectra in comparison with the STAR data [29] at midrapidity, $|y| < 0.1$.

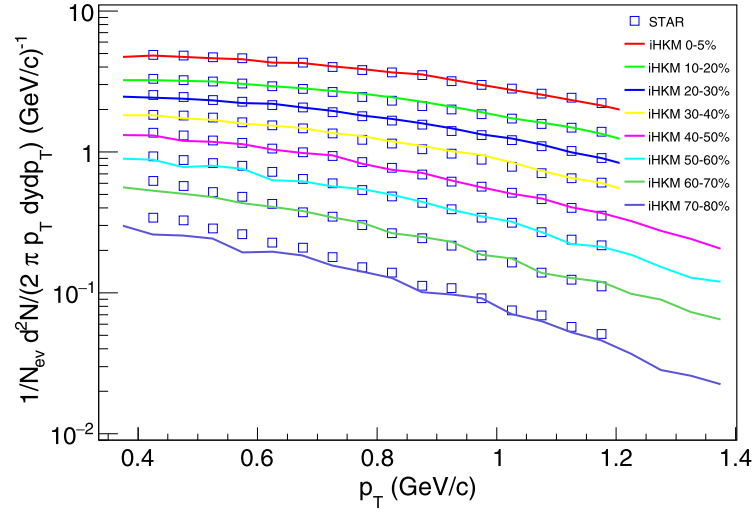


Fig. 6. The antiproton p_T spectra in comparison with the STAR data [29] at midrapidity, $|y| < 0.1$.

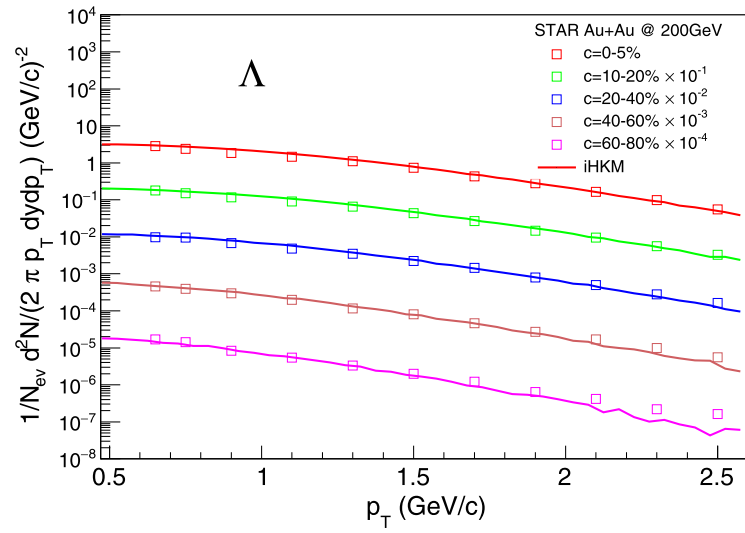


Fig. 7. The p_T spectra of Λ for different centralities, compared with the STAR Collaboration data [30], $|y| < 1$.

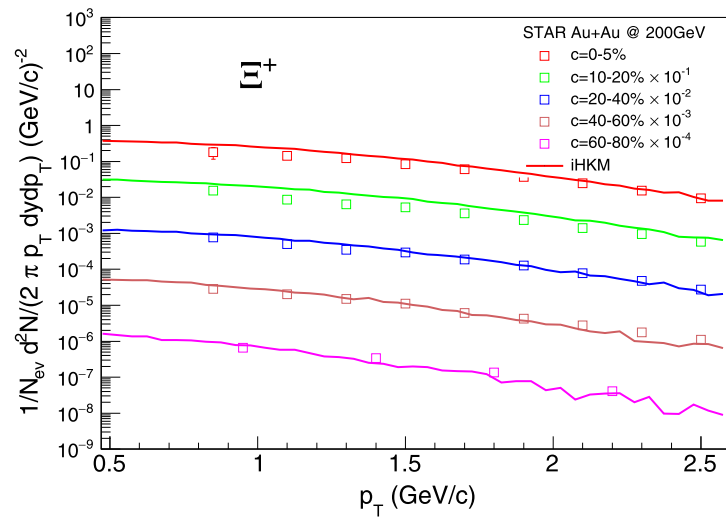


Fig. 8. The Ξ^+ transverse momentum spectra for different centralities, compared with the STAR Collaboration data [30], $|y| < 0.75$.

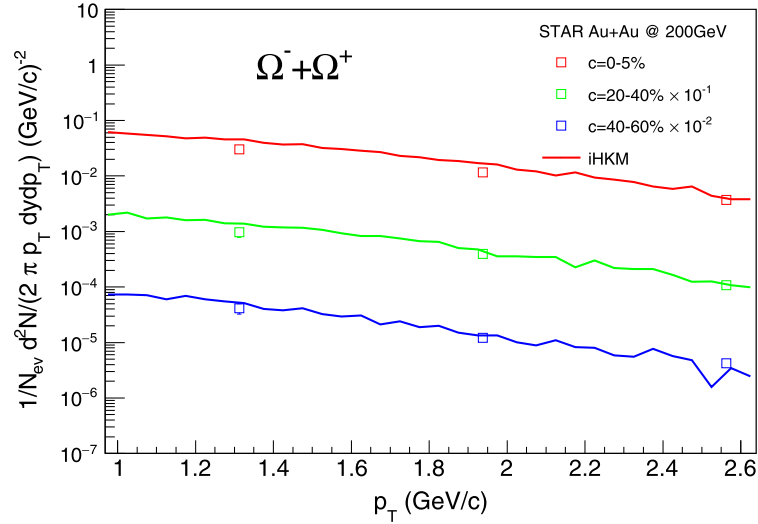


Fig. 9. The Ω transverse momentum spectra for different centralities, compared with the STAR Collaboration data [30], $|y| < 0.75$.

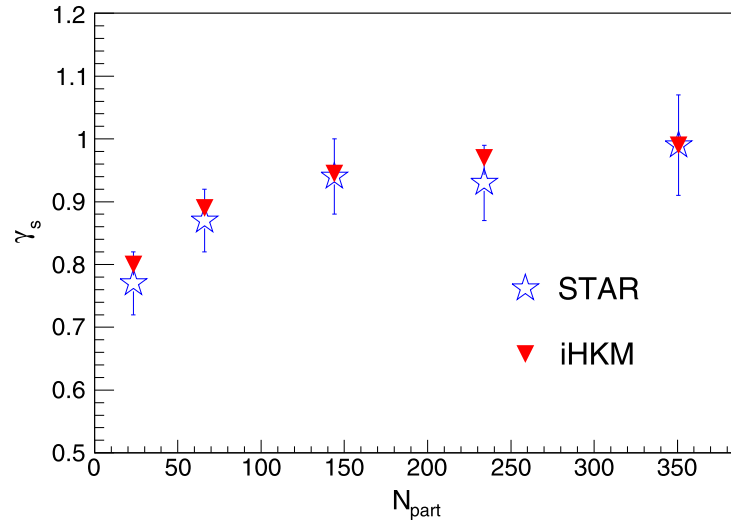


Fig. 10. The comparison of γ_s factors (for different centralities), utilized in the current analysis within iHKM and those used in the STAR paper [30].

In the next Fig. 11 for three centrality classes, $c = 10 - 20\%$, $c = 20 - 30\%$, and $c = 30 - 40\%$, we show the iHKM results on p_T -dependence of the elliptic flow, or v_2 coefficients, characterizing the anisotropy of the all-charged-particles transverse momentum spectra. The model lines go in agreement with data for not very high p_T .

In Figs. 12-15 one can find our results on the femtoscopy radii R_{out} , R_{side} , and R_{long} , extracted from the Gaussian fits to $\pi^-\pi^-$ and K^-K^- momentum correlation functions (CF). The dependencies of the interferometry radii on the mean pair transverse momentum k_T are presented for the four centrality classes: $c = 0 - 10\%$, $c = 10 - 20\%$, $c = 30 - 40\%$, and $c = 60 - 70\%$. All the correlation functions are built considering the specially selected particles with $0.15 < p_T < 1.55$ GeV/c and $|\eta| < 1$.

In the first two figures, the iHKM results for the two centralities, $c = 0 - 10\%$ and $c = 10 - 20\%$, are compared with the experimental data on pion and kaon femtoscopy scales from the PHENIX [5] and the STAR [2,32] collaborations. The STAR data for KK pairs [2] are pre-

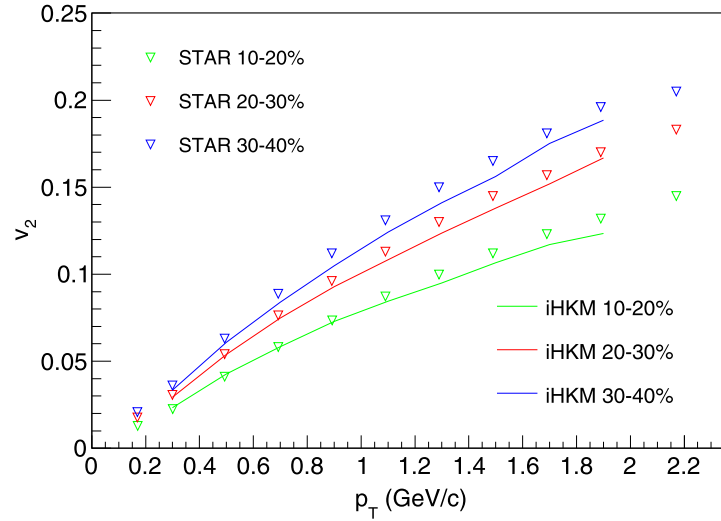


Fig. 11. The v_2 coefficients vs. p_T for the charged particles, calculated in iHKM (lines), together with the STAR data [31] (triangles). The results for the three centralities are shown: $c = 10 - 20\%$, $c = 20 - 30\%$, and $c = 30 - 40\%$.

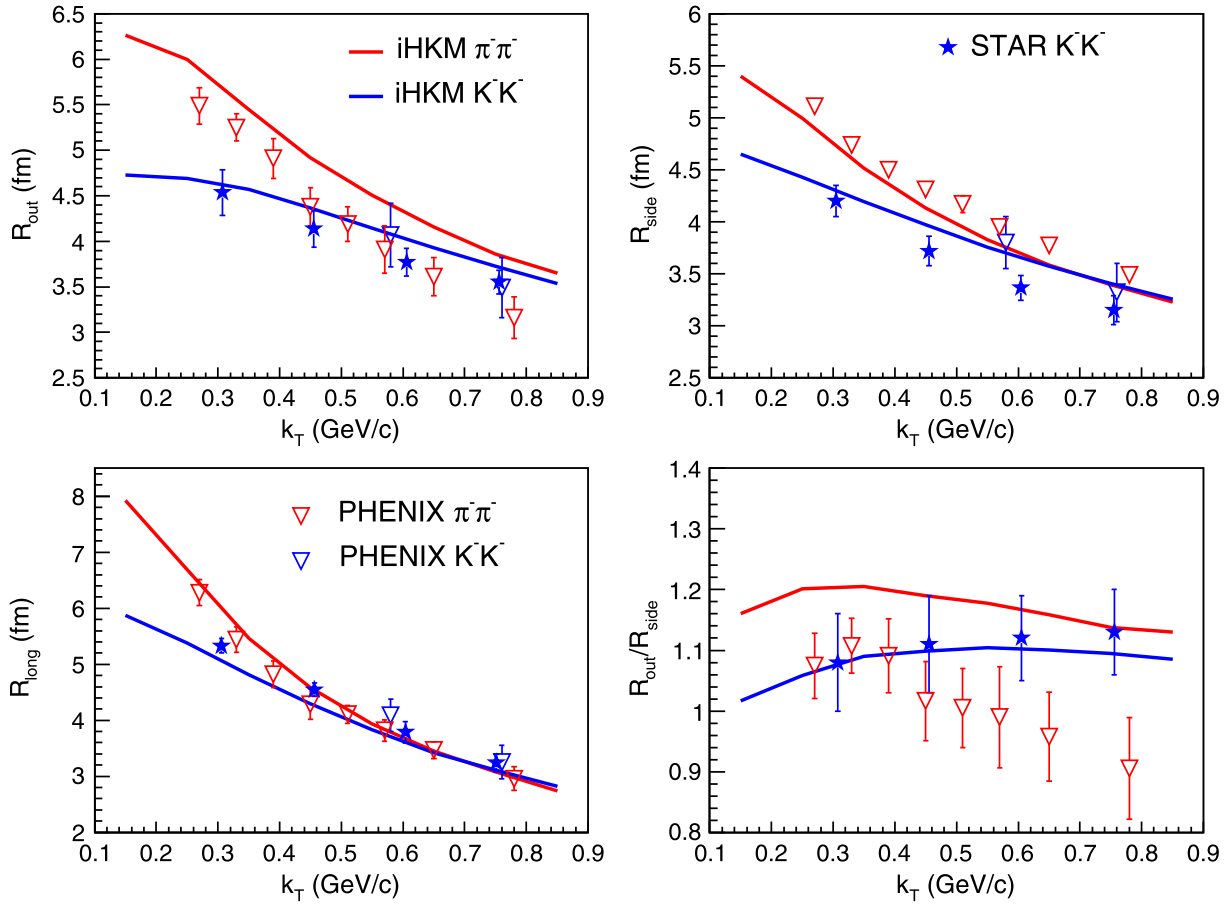


Fig. 12. The iHKM results on $\pi^-\pi^-$ and K^-K^- femtoscopy scales vs. pair k_T (lines) in comparison with the experimental data from PHENIX [5] (triangles) and STAR [2,32] (stars) for the events from the centrality class $c = 0 - 10\%$. Red color is related to pions, blue color is related to kaons.

liminary. From the plots it is clear, that iHKM gives a good description of $R_{\text{long}}(k_T)$ dependency both in $\pi\pi$ and in KK case. As for R_{out} and R_{side} radii, the description quality is worse. The model lines for KK look well, but for $\pi\pi$ pairs we see that R_{out} values are overestimated, es-

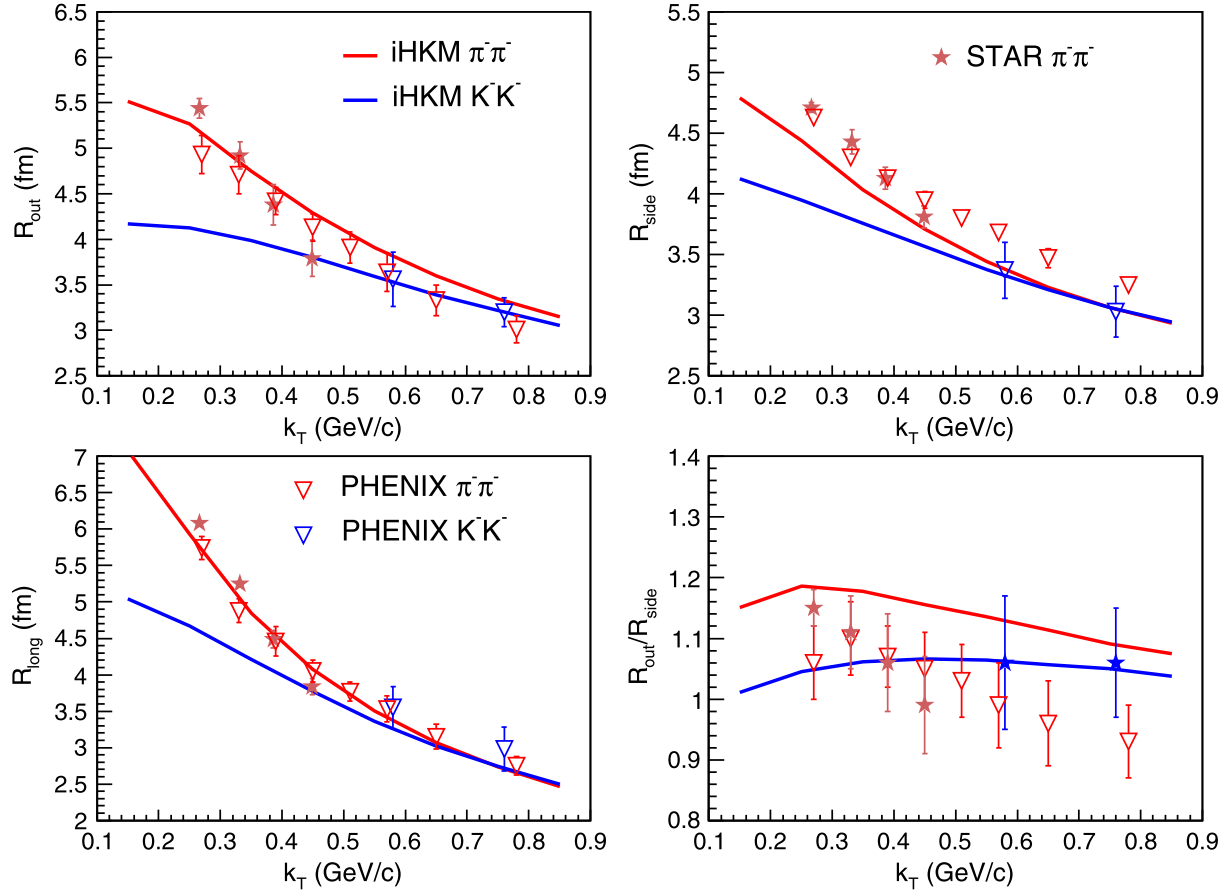


Fig. 13. The same as in Fig. 12 for the centrality class $c = 10 - 20\%$.

pecially for $c = 0 - 10\%$, while R_{side} values are, conversely, underestimated, most noticeably at high k_T . As a result, we obtain a rather overestimated $R_{\text{out}}/R_{\text{side}}$ ratio in pion case.

Also, looking at the figures, one could say that at least R_{side} and R_{long} iHKM curves exhibit scaling between pions and kaons at high k_T . For the LHC energies such k_T -scaling at $k_T > 0.4$ GeV/c for all radii was previously noticed in iHKM simulations [7,33] and then for $\sqrt{s_{NN}} = 2.76$ TeV Pb+Pb collisions confirmed in the ALICE experimental paper [34].

Comparing the presented results with those obtained by other theoretical groups, we can say that generally iHKM performance is comparable to other existing hybrid models. However, we consider the advantage of iHKM to be the fact it allows to describe simultaneously a wide class of bulk observables, including the femtoscopy radii, using the same tuning of the model parameters. Another advantage is small number of main model parameters, which adjust it to the description of particular type of high-energy collision.¹

In addition, in this paper for top RHIC energy collisions ($c = 0 - 10\%$) we apply the method, proposed for the LHC case in [35] and successfully applied in [34], that allows one to extract pion and kaon maximal emission times, τ_π and τ_K , having their p_T spectra and $R_{\text{long}}(m_T)$ dependencies.

At first we perform a combined fit to pion and kaon transverse momentum spectra, using the analytical formula from [35]:

¹ As a matter of fact, in practice such adjustment involves only tuning of ϵ_0 , α and τ_0 , while other parameters, i.e. τ_{rel} , τ_{th} , Λ , η/s , can be considered fixed for relativistic collisions at the top RHIC and LHC energies, see [1,8] for details.

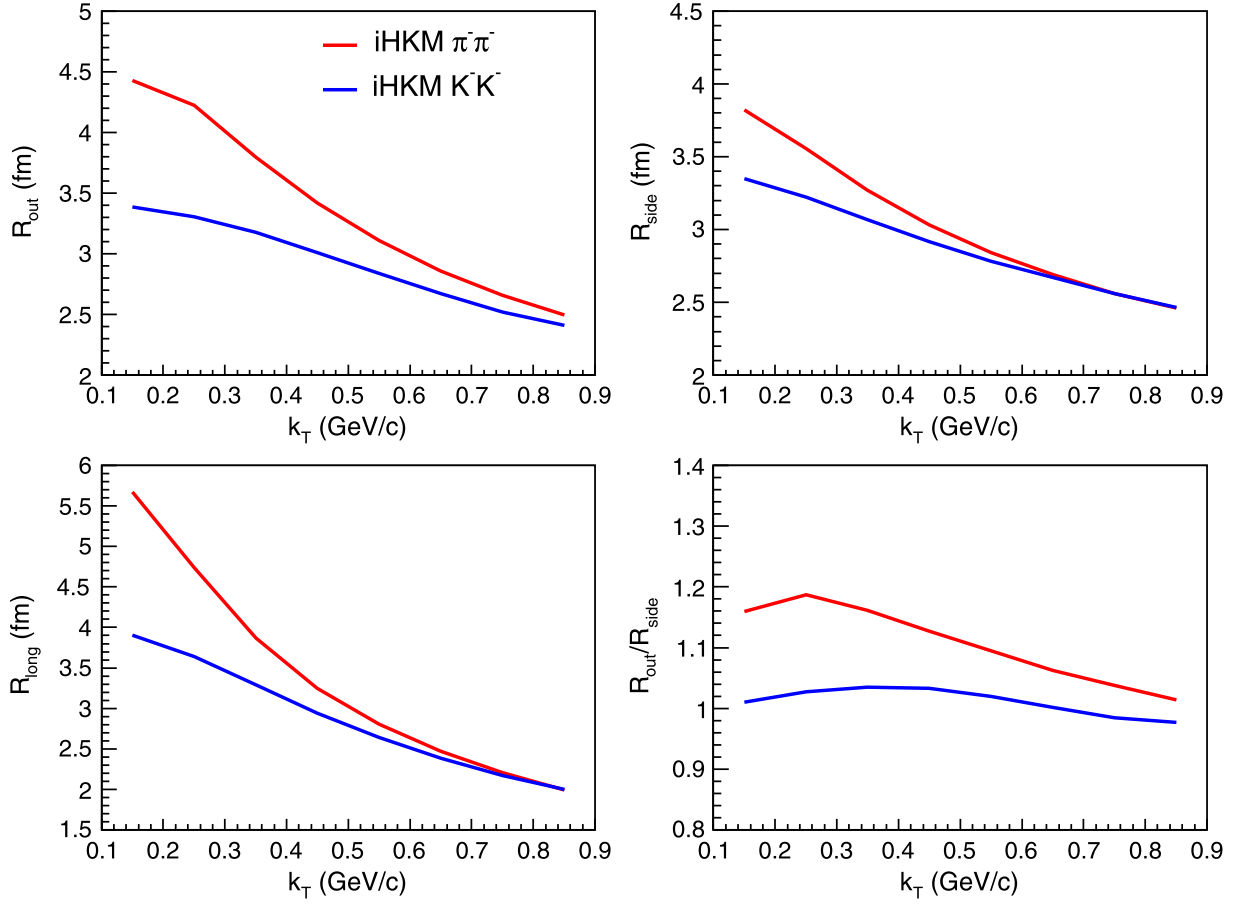


Fig. 14. The same as in Fig. 12 for the centrality class $c = 30 - 40\%$.

$$p_0 \frac{d^3 N}{d^3 p} \propto \exp[-(m_T/T + \alpha)(1 - \bar{v}_T^2)^{1/2}]. \quad (7)$$

Here T is the effective temperature, α is a parameter, characterizing the intensity of collective flow (the infinite α means absent flow, while small α values mean strong flow), and $\bar{v}_T$ is the flow transverse velocity at the saddle point, $\bar{v}_T = k_T/(m_T + \alpha T)$ (see [35] for details). The spectra fitting is done in p_T range $0.45 < p_T < 1.0$ GeV/c. As a result, we obtain $T = 141$ MeV as a common temperature value for both pions and kaons, and two α values, $\alpha_\pi = 7.86 \pm 2.11$ and $\alpha_K = 5.54 \pm 2.61$, for each hadron species respectively.

After that we use another formula from [35] to fit kaon and pion $R_{\text{long}}(m_T)$ dependencies:

$$R_{\text{long}}^2(m_T) = \tau^2 \lambda^2 \left(1 + \frac{3}{2} \lambda^2\right), \quad (8)$$

where λ depends on the system's homogeneity length in longitudinal direction λ_l , namely $\lambda^2 = (\lambda_l/\tau)^2 = T/m_T \cdot (1 - \bar{v}_T^2)^{1/2}$, and τ is the corresponding maximal emission time. Our values of R_{long} radii were obtained from Gaussian fits to the corresponding correlation functions in the particle momentum difference q interval $|q| < 0.3$ GeV/c.

Then, having constrained the T and α parameters according to the results of combined p_T -spectra fitting, we extract the desired maximal emission time $\tau_\pi = 7.12 \pm 0.01$ fm/c for pions from the fit to pion $R_{\text{long}}(m_T)$ dependency using the formula (8). To obtain the τ value for kaons, similarly to the LHC case, described in [35], we have to set α parameter free at fitting kaon R_{long} points. Eventually, we obtain maximal emission time $\tau_K = 9.71 \pm 0.02$ fm/c, and the

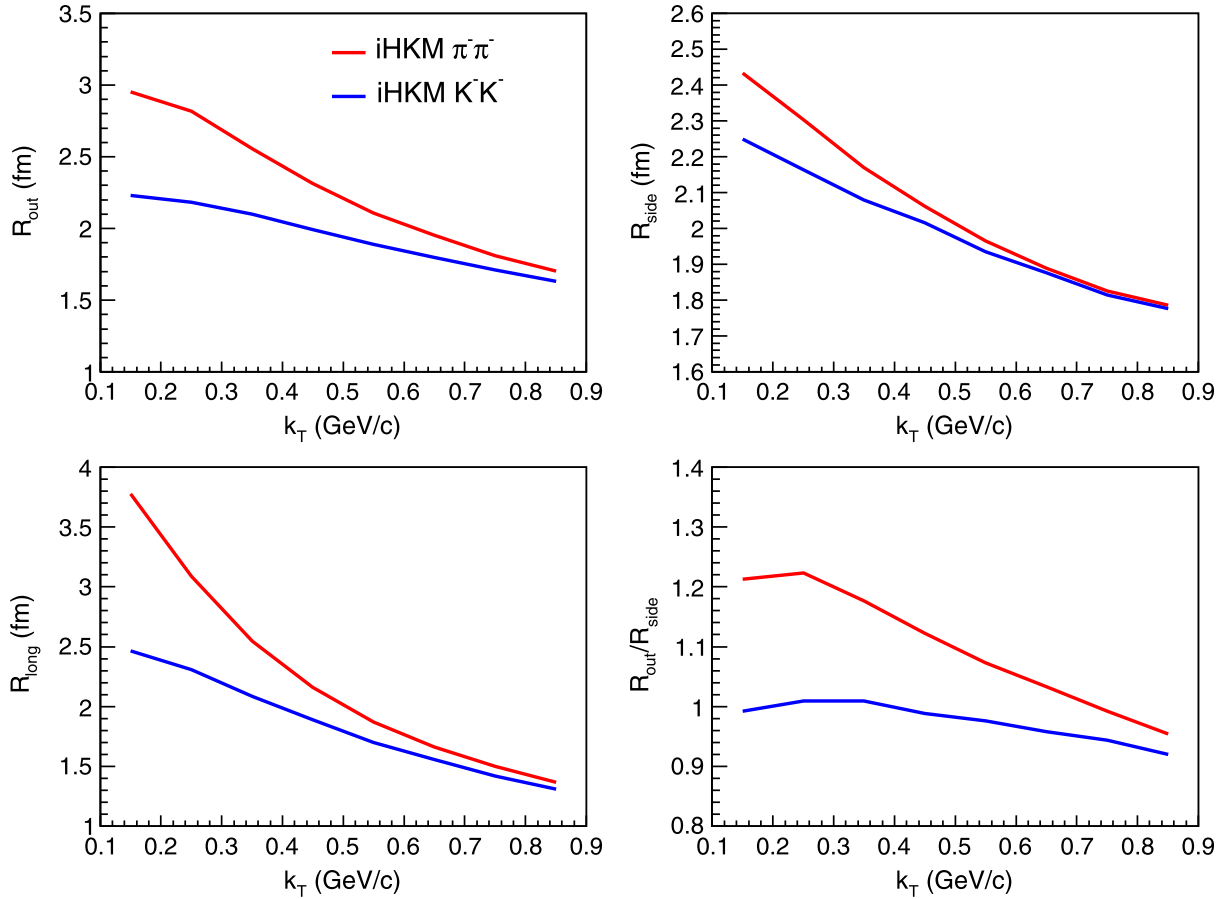


Fig. 15. The same as in Fig. 12 for the centrality class $c = 60 - 70\%$.

kaon α value $\alpha_K = 0.12 \pm 0.02$. One can see both iHKM $R_{\text{long}}(m_T)$ dependencies together with fits to them in Figs. 16, 17.

Finally, in Fig. 18 we present the iHKM pion and kaon averaged emission functions $g(\tau, r_T, p_T)$, which reveal the space-time picture of radiation of these particles. The maximal emission times can be approximately found from these plots, if one attributes some τ values to the regions, where each $g(\tau, r_T, p_T)$ has maximum. As it is readily seen, maximal emission time values obtained in this way are close to those accurately extracted from fits. More detailed analysis, provided in Ref. [36], shows that the reason for a larger time of maximal emission, obtained for kaons, as compared with pions, is in intensive decays and recombinations of K^* mesons (having life-time near 4 fm/c), which take place at the afterburner stage of the collision. This effect was also found in the ALICE experimental analysis [34].

4. Conclusions

The integrated hydrokinetic model showed itself not less successful in describing the variety of bulk observables in Au+Au collisions at the top RHIC energy, than in Pb+Pb ones at the available LHC energies. After adjusting the main model parameters, namely the maximal initial energy density $\epsilon_0(\tau_0)$ and the binary collision contribution to the initial transverse energy-density profile α , the iHKM was able to describe simultaneously the experimental particle yields and their ratios, p_T spectra for pions, kaons, (anti)protons, lambdas, xi, and omega, as well as v_2 of the charged particles.

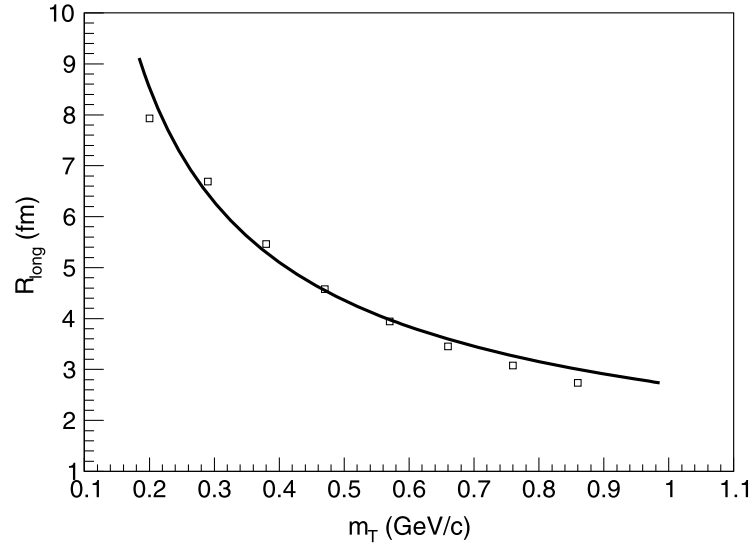


Fig. 16. The R_{long} dependency on pair m_T for the negatively charged pion pairs together with the fit to it according to formula (8) for top RHIC energy collisions with $c = 0 - 10\%$.

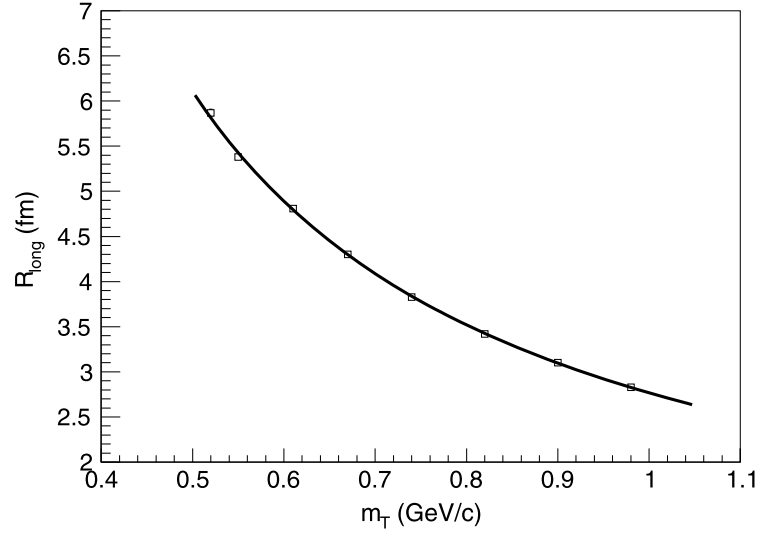


Fig. 17. The same as in Fig. 16 for negatively charged kaon pairs.

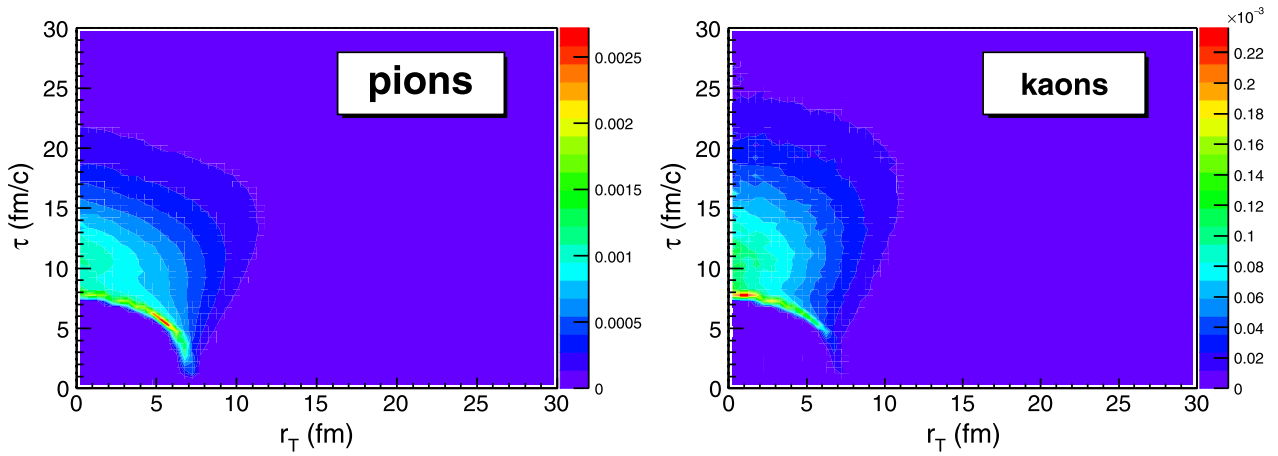


Fig. 18. The pion and kaon emission functions $g(\tau, r_T, p_T)$ [fm^{-3}], averaged over complementary variables, obtained from iHKM for the centrality class $c = 0 - 10\%$. Particles with $0.2 < p_T < 0.3$ GeV/c and $|\eta| < 1$ were chosen for the analysis.

Femtoscopes scales of the kaon pairs are described well in our model, and in the pion case we observe some overestimation of R_{out} and underestimation of R_{side} radius, more pronounced at high pair k_T .

Experimentally measured longitudinal radii R_{long} are perfectly described by the model at different centralities. So, the model results on R_{long} are used to extract the times of the maximal emission for pions and kaons according to the procedure, proposed for the LHC case in the paper [35]. We found that both corresponding times are about 2 fm/c less than at the LHC energy $\sqrt{s_{NN}} = 2.76$ TeV. Similarly as at the LHC, the maximal emission time for kaons is larger than for pions. And again, we explain the latter fact by the intensive decays and recombinations of K^* resonances at the final stage of the collision.

Acknowledgements

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2.5. Загальний час життя системи, утвореної в релятивістському зіткненні ядер

Article

Space–Time Structure of Particle Emission and Femtoscopy Scales in Ultrarelativistic Heavy-Ion Collisions

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Abstract: The analysis of the spatiotemporal picture of particle radiation in relativistic heavy-ion collisions in terms of correlation femtoscopy scales, emission, and source functions allows one to probe the character of the evolution of the system created in the collision. Realistic models, such as the integrated hydrokinetic model (iHKM), used in the present work, are able to simulate the entire evolution process of strongly interacting matter produced in high-energy nuclear collisions. The mentioned model describes all the stages of the system's evolution, including thermalisation and hydrodynamisation, which can help researchers figure out the specific details of the process and better understand the formation mechanisms of certain observables. In the current paper, we investigated the behaviour of the pion and kaon interferometry radii and their connection with emission functions in ultrarelativistic heavy-ion collisions at the Large Hadron Collider within iHKM. We focused on the study of the emission time scales at different energies for both particle species (pions and kaons) aiming to gain deeper insight into relation of these scales and the peculiarities of the mentioned system's collective expansion and decay with the experimentally observed femtoscopy radii. One of our main interests was the problem of the total system's lifetime estimation based on the femtoscopy analysis.

Keywords: kaon; pion; femtoscopy radius; emission function; emission time; particlisation



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1. Introduction

The investigation of the evolution process character of the extremely small, hot, dense, and rapidly expanding systems created in relativistic heavy-ion collisions has always been of great interest to researchers since the first studies of these phenomena started. The comprehensive analysis demonstrates that, soon after the smashing of the two colliding nuclei (each one of them can be initially thought of as a set of nucleons), the strongly coupled quark–gluon system is formed. Being at first non-equilibrium, this system some time later becomes thermalised and continues expanding as a whole liquid-like piece of matter [1–4]. Its expansion at this stage can be described within a relativistic hydrodynamics approach. As the system expands, it gradually cools down, becomes dedensified, and begins losing equilibrium. Below some temperature or energy density (usually the temperatures close to 150–160 MeV are spoken of), the system disintegrates, and its parts one by one are transformed into sets of hadrons. However, the produced hadrons do not likely become free immediately after the fireball decay. The latter would imply a sudden switching from a near-zero mean free path length to a near-infinity one, which seems hardly feasible. The particles would rather continue interacting intensively and evolving collectively, as single hadronic medium for some time. Therefore, to estimate the overall lifetime of the system created after the collision, one should in theory account for the hadronic phase of the matter evolution and try to find out the time scale (at least approximate), during which the hadronic medium stays connected enough to be considered a unified system.

In the literature, however, there are two different approaches to the stated problem. The first one supposes that, right after the hadronisation (and maybe a very short hadronic stage), the so-called “freeze-out” takes place, so that the chemical composition of the system and, almost simultaneously, the particle momentum spectra are frozen and no longer change, except for decays of resonances. Such an assumption goes back to the pioneering works by Fermi, Pomeranchuk, and Landau [5–7] and is typical for thermal models with statistical hadronisation [8–12], hydro-inspired freeze-out parametrisations, e.g., CROW [13,14], Buda–Lund [15], and blast wave [16–18], and hydrodynamics models with statistical hadronisation [19–32]. The latter usually employ the Cooper–Frye formalism [33] or its generalisations [34] to hadronise the system at the freeze-out hypersurface. There are various computer codes, developed for the calculations of statistical hadronisation in different models, e.g., SHARE [35,36], THERMUS [37], used for the extraction of thermodynamic parameters from the fits to the ratios of particle yields, or THERMINATOR [38,39], used as an event generator, producing particles based on the thermodynamic parameter values specified at a certain freeze-out hypersurface.

The second approach suggests the existence of a long “afterburner” stage of the collision (lasting about 5–10 fm/c or until the temperature drops to 80–120 MeV), which plays an important role in the measured observables’ formation. The corresponding theoretical description of heavy-ion collisions in this case is carried out within various hybrid (hydro+cascade) and full evolutionary models, aiming to simulate the entire process of the created system’s evolution, from the initial state formation to the final hadron cascade stage [40–51]. The study presented in this paper was carried out in the most-developed complete model—the integrated hydrokinetic model (iHKM)—which includes as one of the stages of the system’s evolution also a description of the process of the thermalisation/hydrodynamisation of initially non-thermal matter [52].

The existing experimental data on p_T spectra do not allow giving a clear preference to one of these approaches and yield the estimates for the kinetic freeze-out temperature in a very wide range (such as 100–140 MeV [29]), depending on the particle species selected for the analysis, momentum cuts, the inclusion/exclusion of resonance decays, and so on. The results on K^* resonance yields and K^*/K yields’ ratios [53–55] speak in favour of the afterburner scenario; the experimental K^*/K ratio values decrease when one goes from peripheral to central collisions, in contrast with almost flat dependency (keeping close to the measured values for peripheral events) obtained in the models, which do not account for the hadron re-scattering stage of collision (e.g., thermal model, blast wave model). These models also overestimate the measured K^* yield values. A nearly flat dependency on centrality is actually observed in experiments for the $\phi(1020)/K$ ratio, as the ϕ ’s decay products almost do not interact with the dense hadronic medium due to a large lifetime of this resonance. The observed suppression of the short-lived K^* yield ratio to kaon yield is likely due to re-scattering of K^* decay products during the afterburner collision stage, and this effect is stronger in central events because of the higher number of produced particles and longer hadronic stage duration in this case. However, the system’s lifetime determined based on these results depends on the cross-sections and reaction channels used in the model for the hadronic stage. The correlation femtoscopy data are also considered as a potential source of information on the duration of the matter collective motion in high-energy A + A collisions. For instance, in the ALICE Collaboration’s experimental analysis [56], the “freeze-out time” value for the 2.76 TeV Pb + Pb collisions at the LHC is defined from the fit to the measured longitudinal femtoscopy radii dependency on m_T (the result is 10–11 fm/c), but the fitting formula does not account for the strong transverse collective flow that should develop in such collisions.

In our previous papers [57–59], a simple method for the estimation of the times of pion and kaon maximal emission, based on the particle p_T spectra and the dependencies of longitudinal femtoscopy radii R_{long} on the pair transverse mass m_T , was proposed and tested in simulations within the different versions of the (integrated) hydrokinetic model [52,60]. Soon after the first of the mentioned publications, the proposed prescription

was used by the ALICE Collaboration in their experimental analysis of kaon femtoscopy for the 2.76 TeV Pb + Pb collisions at the LHC [61]. It provided estimates for the kinetic freeze-out temperature, $T_{kin} = 144$ MeV, and the maximal emission times, $\tau_\pi = 9.5 \pm 0.2$ fm/c and $\tau_K = 11.6 \pm 0.1$ fm/c, close to those obtained in our paper [60] (see the details below).

A clear influence of the post-hydrodynamic phase on the soft physics results was also noted in our analysis of different particle number ratios [62,63] at the LHC energies. These studies include a comparison of the values calculated in the full iHKM simulation regime with those obtained in a reduced regime without a hadronic cascade stage and also with the thermal model results [8,9].

According to simulation analysis presented in [64,65], a large fraction (about 60% or even more) of identified $K^*(892)$ resonances at the LHC are produced in the hadronic phase of collision, while primary particles coming from the QGP hadronisation cannot be observed due to rescattering of their decay daughters. By contrast, primary $\phi(1020)$ mesons with a lifetime of ≈ 50 fm/c can be identified without problems, apparently because they decay after the finishing of the intensive hadron rescattering phase (however, the iHKM simulation results in [64] do not exclude that the hadronic stage may lead to an additional increase in the identified $\phi(1020)$ number due to the KK correlation effect). The recent ALICE Collaboration data on the K^*/K and ϕ/K yield ratios in 5.02 TeV Pb + Pb collisions at the LHC confirm “the dominance of the rescattering effect in the hadronic phase” [55] for K^* results and its small significance in the ϕ case.

All these results seem to support the conception of continuous (rather than sudden) freeze-out and a prolonged afterburner stage. In accordance with this evolution picture, particles possessing different momenta and originating from different sources (primary particles and those leaving the system after elastic/inelastic reactions with the hadronic medium at the afterburner stage of collision) can be expected to radiate from the system at different times.

In the current paper, we tried to investigate the process of pion and kaon emission considered in [57,59] in more detail, clarify the relationship between the extracted emission times and the femtoscopy radii, and if possible, propose a way for the estimation of the full system’s lifetime, including the afterburner stage, based on our analysis of ultrarelativistic heavy-ion collisions within the integrated hydrokinetic model.

2. Research Motivation

In our studies [57–59], the transverse momentum spectra and femtoscopy radii for the A + A collisions at the top RHIC and the two LHC energies were fit with simple analytical formulas, containing parameters describing the system’s effective temperature, the strength of the collective flow, and the time of maximal emission for particles of each sort. The fitting formulas were obtained in approximation, suggesting that all the particles with comparably low momenta ($p_T \lesssim 0.5$ GeV/c) are emitted from the system at the hypersurface of constant proper time $\tau_{m.e.}$, limited in the transverse direction. Of course, the real emission picture for both pion and kaon mesons is much more complicated, so that the extracted maximal emission times correspond only to some effective values, approximately reflecting the time scales when the emission process for a given particle species is the most intensive.

To cross-check the fitting results and to obtain a more-detailed picture of the particle emission process in [57–59], we additionally built plots of averaged emission functions in coordinates (r_T, τ) for particles with small p_T (in accordance with the earlier applied approximation). The figures show that the emission function maximum for pions should be close to the particlisation time for the centre of the expanding system, which is in agreement with the previously obtained fitting result. As for the kaon emission function, it had two apparent maxima (one close to the pionic one and the other lying about 4 fm/c higher), such that the $\tau_{m.e.}$ value obtained from the fit was lying between them and could be associated with the “mean” maximal emission time with respect to the two maxima on the plot. The second maximum of the kaon emission function, causing the extracted maximal emission times for kaons to be about 2–3 fm/c larger than those for pions, is likely connected with

the decays of $K^*(892)$ resonance (its lifetime is about 4 fm/c), producing an additional noticeable portion of kaons after particlisation of the created fireball.

To investigate the emission process in more detail, we constructed the emission time distributions for both considered particle sorts (see Figures 1 and 2), which can be obtained by integrating the previously analysed emission function histograms over transverse radius r_T . An interesting thing one can notice about these distributions is that most particles of both species seem to be produced in the late, hadronic stage of the collision, so that the total number of pions and kaons emitted close to the hadronisation hypersurface is noticeably smaller than that of mesons radiated later. Near the time of hadronisation in the centre of the system (9–10 fm/c), one can see only local τ distribution maxima on the presented plots.

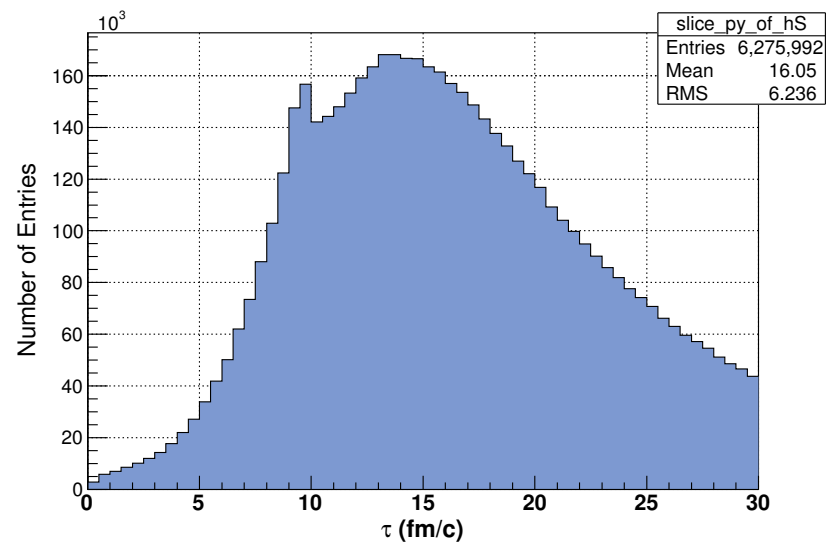


Figure 1. The time of emission distribution for pions in central Pb + Pb collisions ($c = 0$ –5%) at the LHC energy $\sqrt{s_{NN}} = 2.76$ TeV simulated within iHKM, $0.2 < p_T < 0.3$ GeV/c, $|y| < 0.5$.

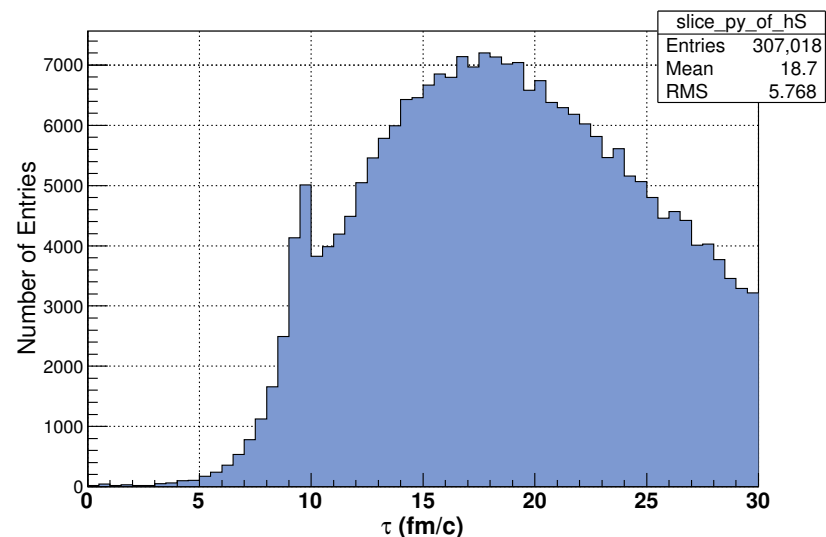


Figure 2. The same as in Figure 1 for kaons.

The found peculiarity of τ distributions is in agreement with our previous conclusions about the continuous character of particle spectra freeze-out; however, it is not clear why the extracted values of $\tau_{m.e.}$, obtained from the fits to single-particle p_T spectra and femtoscopy scales, do not reflect the apparent prevalence of the late post-hydrodynamic stage contribution in the total amount of emitted particles, following from the τ distribution figures.

Another interesting fact about the extracted maximal emission times is that their values are very close for the two considered LHC collision energies, 2.76 TeV and 5.02 TeV (the same applies to the times of particlisation of the corresponding systems). At first glance, it seems that, at higher collision energy, the created system should live longer and produce more particles, so that the corresponding times of maximal emission should be larger. Instead, at the two energies in our analysis for the most-central collisions ($c = 0\text{--}5\%$), we have comparable $\tau_{\text{m.e.}}$ for both pions and kaons, $\tau_\pi = 8.97 \pm 0.04 \text{ fm}/c$ and $\tau_K = 12.73 \pm 0.12 \text{ fm}/c$ at $\sqrt{s_{NN}} = 5.02 \text{ TeV}$ and $\tau_\pi = 10.34 \pm 0.06 \text{ fm}/c$ (or even $\tau_\pi = 9.44 \pm 0.02 \text{ fm}/c$; see [57] for details) and $\tau_K = 12.65 \pm 1.58 \text{ fm}/c$ at $\sqrt{s_{NN}} = 2.76 \text{ TeV}$. The corresponding temperatures, entering the fitting formulas, are $T = 138 \text{ MeV}$ and $T = 144 \text{ MeV}$, respectively.

In Section 4, we try to investigate in detail the above-stated issues.

3. Materials and Methods

As was already mentioned, the analysis of relativistic heavy-ion collisions for this work was carried out based on the results of computer simulations within the integrated hydrokinetic model (iHKM) [52,60,66,67]. The model successfully reproduces the experimental data on bulk hadronic observables (including femtoscopy radii) for the high-energy collisions at the LHC and RHIC [52,58,63,68]. It also allows achieving a good agreement with the data on direct photon production for the two mentioned collider experiments [69–71].

One of the most-important distinct features of the iHKM, as compared to other hybrid models used for the simulation of A + A collisions, is the presence of the pre-thermal dynamics description for the strongly interacting matter created as a result of the collision (it is implemented using the energy–momentum transport approach based on the Boltzmann equation in the relaxation time approximation—see the papers [52,66,67] for the details). During the corresponding evolution stage, the system is gradually transformed from an essentially non-equilibrium state it has at the very initial times right after the overlapping of the two colliding nuclei to a nearly equilibrated state, which can be further described in the approximation of relativistic viscous hydrodynamics [72,73]. The initial energy density distribution, serving as a starting point for the pre-thermal stage, is generated with the GLISSANDO code [74]:

$$\epsilon(b, \tau_0, \mathbf{r}_T) = \epsilon_0(\tau_0) \frac{(1 - \alpha)N_w(b, \mathbf{r}_T)/2 + \alpha N_{bin}(b, \mathbf{r}_T)}{(1 - \alpha)N_w(b = 0, \mathbf{r}_T = 0)/2 + \alpha N_{bin}(b = 0, \mathbf{r}_T = 0)}. \quad (1)$$

Here, the parameter ϵ_0 defines the initial energy density in the centre of the system, and the parameter α regulates the proportion between the contributions from the binary collision and wounded nucleon models to the $\epsilon(b, \tau_0, \mathbf{r}_T)$ in (1). These two parameters are used to adjust the model to the simulation of the concrete collision experiment.

For the initial momentum distribution, we take the anisotropic momentum distribution, inspired by the colour glass condensate effective gluon field theory:

$$f_0(p) = g \exp \left(- \sqrt{\frac{(p \cdot U)^2 - (p \cdot V)^2}{\lambda_\perp^2} + \frac{(p \cdot V)^2}{\lambda_\parallel^2}} \right), \quad (2)$$

where $U^\mu = (\cosh \eta, 0, 0, \sinh \eta)$, $V^\mu = (\sinh \eta, 0, 0, \cosh \eta)$, η is space–time rapidity, and $\Lambda = \lambda_\perp / \lambda_\parallel = 100$ is the model parameter, describing the initial momentum anisotropy (see [52,66,67] for the details).

The hydrodynamic stage of the matter expansion lasts until the temperature in the system drops to the particlisation value T_p (depending on the equation of state (EoS) utilised for the continuous quark–gluon matter; in the iHKM, we use the Laine–Schroeder [75] and the HotQCD Collaboration [76] EoSs). Below the particlisation temperature, the continuous medium transforms into the system of hadrons, which keeps on expanding, while the hadrons interact intensively with each other, so that numerous particle creation and

annihilation processes take place. This post-hydrodynamic stage of the matter evolution is described with the help of the UrQMD hadron cascade model [77,78].

The analytical formulas for spectra and femtoscopy radii fitting in [57–59] are obtained from the following considerations.

One assumes that the particlisation occurs at the hypersurface $\tau = \text{const} = \tau_{\text{m.e.}}$, limited in the direction transverse to the beam axis ($r_T \leq r_T^{\text{max}}$), which should be reasonable at least for the particles with not very high momenta ($p_T \lesssim 0.5 \text{ GeV}/c$). Accordingly, the Wigner function for soft bosons can be written as follows:

$$f_{l.eq.}(x, p) = \frac{1}{(2\pi)^3} [\exp(\beta p \cdot u(\tau_{\text{m.e.}}, \mathbf{r}_T) - \beta\mu) - 1]^{-1} \rho(\mathbf{r}_T), \quad (3)$$

where β is the inverse temperature, $u^\mu(x) = (\cosh \eta_L \cosh \eta_T, \frac{\mathbf{r}_T}{r_T} \sinh \eta_T, \sinh \eta_L \cosh \eta_T)$ is the collective four-velocity (here, in the u^μ definition, $\eta_L = \text{arctanh } v_L$ and $\eta_T = \text{arctanh } v_T(r_T)$ are longitudinal and transverse rapidities, which correspond to the respective velocities v_L and v_T), and $\rho(\mathbf{r}_T)$ is the cutoff factor, narrowing the particlisation hypersurface in the transverse direction:

$$\rho(\mathbf{r}_T) = \exp[-\alpha(\cosh \eta_T(r_T) - 1)]. \quad (4)$$

The parameter α here describes the collective flow intensity, in such a way that the lower is the value of α , the stronger is the flow (in the case of absent flow, $\alpha \rightarrow \infty$; for more details, see [57,59]).

Given Equation (3) for the Wigner function, one can obtain the approximate formulas for the momentum spectra and correlation functions. Applying the modified Cooper–Frye prescription (it suggests that the overall particlisation hypersurface is built as a set of points $(t(\mathbf{r}, p); \mathbf{r})$, corresponding to the maximal emission of particles with momentum p ; see [60] for the details) at the previously defined particlisation hypersurface and using the saddle point method, one arrives at the following expressions for the p_T spectra and the longitudinal femtoscopy radii [79]:

$$p_0 \frac{d^3 N}{d^3 p} \propto \exp[-(m_T/T + \alpha)(1 - \bar{v}_T^2)^{1/2}], \quad (5)$$

$$R_{\text{long}}^2(m_T) = \tau^2 \lambda^2 \left(1 + \frac{3}{2} \lambda^2\right). \quad (6)$$

Here, $m_T = \sqrt{m^2 + k_T^2}$, k_T is the pair mean transverse momentum, $T = T_{\text{m.e.}}$ is the temperature at the maximal emission hypersurface $\tau = \tau_{\text{m.e.}}$, $\bar{v}_T = k_T/(m_T + \alpha T)$ is the transverse collective velocity at the saddle point, and

$$\lambda^2 = \frac{T}{m_T} (1 - \bar{v}_T^2)^{1/2} \quad (7)$$

is the squared ratio of the longitudinal homogeneity length λ_{long} to $\tau_{\text{m.e.}}$ (as usual in femtoscopy studies, the *long* direction coincides with that of the beam axis, *out* corresponds to the pair transverse momentum vector, and *side* is orthogonal to the two others). Equation (6) is valid for the transverse flow of an arbitrary profile and strength; however, it was derived under the assumption of small q_{long} , corresponding to the top part of the correlation function peak.

According to the method, originally proposed in [57], in order to estimate the $\tau_{\text{m.e.}}$ values for pions and kaons, one needs to find the values of T and α first from the combined fit to the pion and kaon p_T spectra based on Equation (5) and, after that, determine the maximal emission times τ_π and τ_K from the femtoscopy radii $R_{\text{long}}(m_T)$ fitting (at already fixed T and α) using Equation (6).

4. Results and Discussion

4.1. Emission Time Distributions

As was already mentioned, the emission time distributions for both pions and kaons, shown in Figures 1 and 2 for the case of 2.76 TeV Pb + Pb central collisions at the LHC, suggest that a large number of particles leave the system noticeably later than the time of full particlisation, which is about 9–10 fm/c for the considered LHC energies (rather similar plots were obtained for 5.02 TeV collisions as well). The mean τ values that can be calculated based on the presented model histograms are 16.05 fm/c and 18.70 fm/c for pions and kaons, respectively, and an interesting question is why these time scales do not match with $\tau_{m.e.}$ estimated based on the femtoscopy radii m_T dependencies (i.e., 10.3 fm/c and 12.7 fm/c).

Here, one should consider several different factors that could possibly lead to such a result and try to find which of them played the major role in this situation.

One of the relevant factors could be the non-Gaussian shape of the corresponding particle emission functions and the correlation functions, which are used to determine the femtoscopy radii. In [80], the non-Gaussian character of pion and kaon emission was observed in our studies of the corresponding source functions (which can be defined as time-integrated distributions of distances between the particles forming pairs in femtoscopy analysis), calculated in the HKM (a previous version of the iHKM) for the case of Pb + Pb collisions at the LHC energy $\sqrt{s_{NN}} = 2.76$ TeV. In Figure 3, we demonstrate the projections of the mentioned meson source functions together with the Gaussian fits to them. As one can see, the actual shape of the source functions, especially in the *out* and *long* directions, includes noticeable non-Gaussian tails. The effect looks even more pronounced for pions. In our method for the maximal emission time estimation, we fit R_{long} interferometry radii dependencies on m_T , and as follows from Equation (6), the larger are the R_{long} values, the larger can be the resulting $\tau_{m.e.}$. The radii are extracted from the Gaussian fits to the correlation functions $C(\mathbf{q})$, which are connected with the source functions $S(\mathbf{r})$ through the so-called Koonin integral equation [81]:

$$C(\mathbf{q}^*) = 1 + \int d^3r^* S(\mathbf{r}^*) K(\mathbf{r}^*, \mathbf{q}^*), \quad (8)$$

where $K(\mathbf{r}^*, \mathbf{q}^*)$ is the integral transform kernel, reflecting the correlation mechanism(s) supposed to exist between the emitted particles, and the asterisk denotes the pair rest frame.

Accordingly, the pairs from the non-Gaussian tail of the source function, characterised by the large distances between emitted particles, will not contribute directly to the value of the femtoscopy radius and, eventually, to the $\tau_{m.e.}$ value. In such a way, some amount of particles that escape from the system at the late stage of the collision can be missed by the femtoscopy analysis, but still be visible in the overall τ distribution.

In the correlation function plots, such pairs, containing, e.g., particles coming from the long-lived resonance decays or liberated from the system after a series of elastic/inelastic reactions with other hadrons constituting the expanding hadronic medium (so that the distance between the two particles in the pair can be fairly large), typically form a practically invisible sharp and narrow peak [82], which can reduce the intercept λ value, determined from the fit to the correlation function, but does not really affect the radii.

In [57], we tried to reduce the effects stemming from the non-Gaussianity of the correlation functions by fitting them in a narrow q range ($0 < q_{long} < 0.04$ GeV/c), which also better corresponds to the assumptions made when deriving Formula (6). As a result, we observed the more-uniform behaviour of the pion and kaon radii, closer to m_T scaling (still broken because of the collective flow and afterburner rescattering effects), with common T and α parameters corresponding to the combined pion and kaon p_T spectra fitting; however, only a hardly noticeable increase of the femtoscopy radii (about 4%) could be reached.

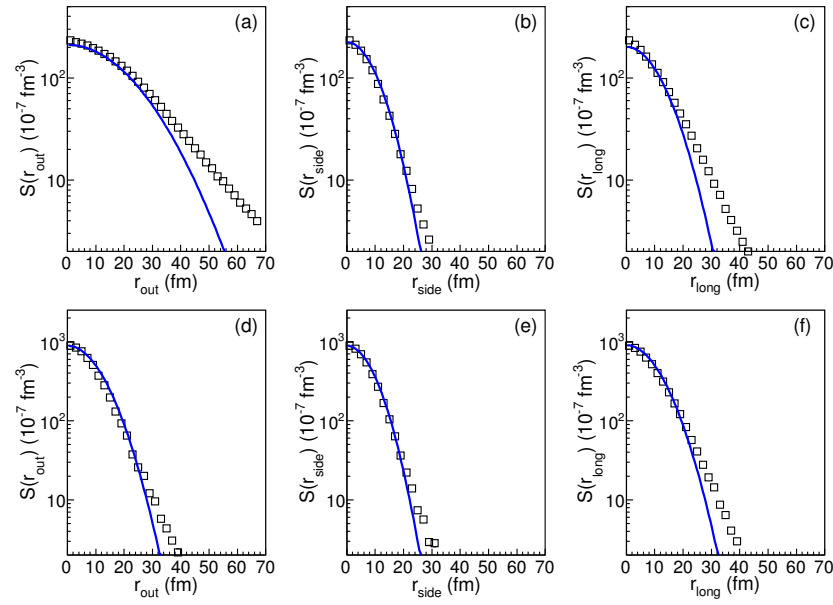


Figure 3. The source function projections calculated in the iHKM for pions (Panels (a–c)) and kaons (Panels (d–f)) produced in central Pb + Pb collisions at the LHC energy $\sqrt{s_{NN}} = 2.76$ TeV, $0.2 < p_T < 0.36$ GeV/c, $|y| < 0.5$. Squares represent the model output, and lines show the corresponding Gaussian fits.

The presence of a post-hydrodynamic hadronic cascade stage in the model certainly increases the resulting femtoscopy radii; in our analysis, carried out during the work in the papers [57,83], we found for the case of the LHC 2.76 TeV Pb + Pb collisions that the switching off of the cascade stage reduced the radii values by 12% for pions and by 25–30% for kaons. However, the maximal emission time extraction we are currently interested in was performed based on the full calculation mode results, i.e., with the hadron cascade turned on, so the above observation can hardly help us solve the stated problem.

A more-thorough analysis of the procedure we applied for the $\tau_{m.e.}$ extraction, however, suggests the following possible solution. Although the analytical Formulas (5) and (6), used for the spectra and the femtoscopy radii fitting, were derived under the assumption of isochronous particle emission from the hypersurface $\tau = const = \tau_{m.e.}$ fragment with $r_T < r_T^{max}$, typical for particles with not very high transverse momenta, the actual data fitting included also the region of comparably high p_T ($0.5 < p_T < 1.0$ GeV/c) and k_T ($0.3 < k_T < 1.1$ GeV/c). This apparently implies that the emission time distributions, actually corresponding to the applied fitting scheme, should as well include particles from a wide momentum region (still not exceeding, however, the region of hydrodynamics approximation applicability). Therefore, in Figures 4 and 5, we show the pion τ distributions constructed with a momentum cut $0.5 < p_T < 2.0$ GeV/c for central Pb + Pb collisions at the LHC energy $\sqrt{s_{NN}} = 2.76$ TeV (analogous distributions for kaons demonstrate a similar tendency as compared to the previous Figure 2). In Figure 5, an additional r_T constraint, $r_T < 10$ fm, was applied.

As one can see, the presented histograms correspond much better to the $\tau_{m.e.}$ estimates we previously obtained from the spectra and radii fitting, so that the distribution maxima are close to the full particlisation time (≈ 10 fm/c), the tails corresponding to the hadronic phase look much smaller and do not dominate in the overall distributions, and the mean emission time values are 12.5 fm/c and 10.4 fm/c for the r_T -unconstrained and r_T -constrained cases, respectively.

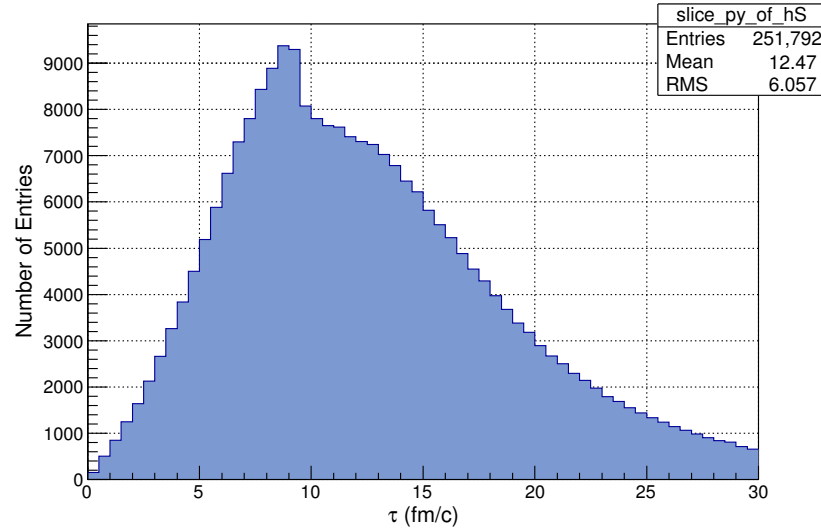


Figure 4. The same as in Figure 1, but for a wide p_T region, $0.5 < p_T < 2.0$ GeV/ c .

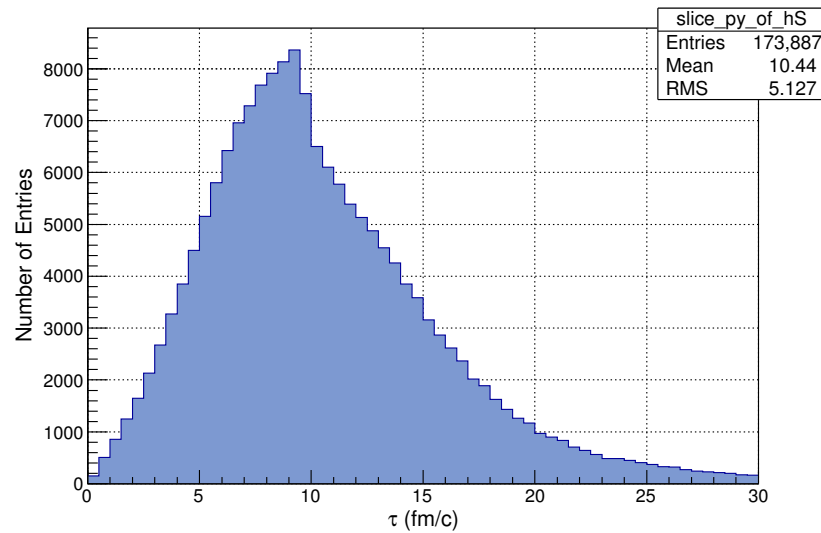


Figure 5. The same as in Figure 4, but with the constraint $r_T < 10$ fm.

The obtained wide-momentum-range τ distributions seem to be more relevant for characterising the emission process from the system as a whole. However, what about the low-momentum distributions shown before in Figures 1 and 2 and depicting a quite rich and prolonged hadronic stage of the matter evolution? Could we use these distributions to obtain estimates of the afterburner phase duration, corresponding to the “maximal emission times”, but now determined based on low-momentum spectra and radii? In principle, such an estimate would be in agreement with the modified Cooper–Frye prescription [60], used in [57,59] for the derivation of Formulas (5) and (6) and suggesting that the particles with different momenta p should be emitted from separate pieces σ_p of the complex hadronisation hypersurface, composed as a united set of all such fragments.

Therefore, to test such an approach, we used the described method for the $\tau_{m.e.}$ estimation, but applying the low-momentum cuts on p_T and k_T . The combined π and K spectra fitting was performed for the p_T range $0.25 < p_T < 0.55$ GeV/ c and gave the values $T = 106 \pm 16$ MeV, $\alpha_\pi = 2.8 \pm 0.5$, $\alpha_K = 1.2 \pm 0.7$, which were then fixed at the $R_{long}(m_T)$ fitting. The latter was performed for $k_T < 0.6$ GeV/ c and resulted in the maximal emission time values $\tau_\pi = 13.65 \pm 0.44$ fm/ c and $\tau_K = 16.73 \pm 0.49$ fm/ c .

Comparing these times with Figures 1 and 2, one can see that the new $\tau_{m.e.}$ values correspond to the maxima of the shown low-momentum emission time distributions and can be possibly interpreted as evidence of a prolonged afterburner stage and used as “upper-limit” estimates for the overall system’s lifetime (in the sense that they are defined based on low-momentum data and particles with low p_T are expected to leave the system later than those with high p_T). The new $\tau_{m.e.}$ value for kaons is also in agreement with the emission picture presented in Figure 2 of our paper [64], which implies that active K^* production at the hadronic stage of collision lasts until $\approx 15\text{--}20\text{ fm}/c$, while K^* resonance decays serve as an extra source of kaons (see also Figure 6 of this article).

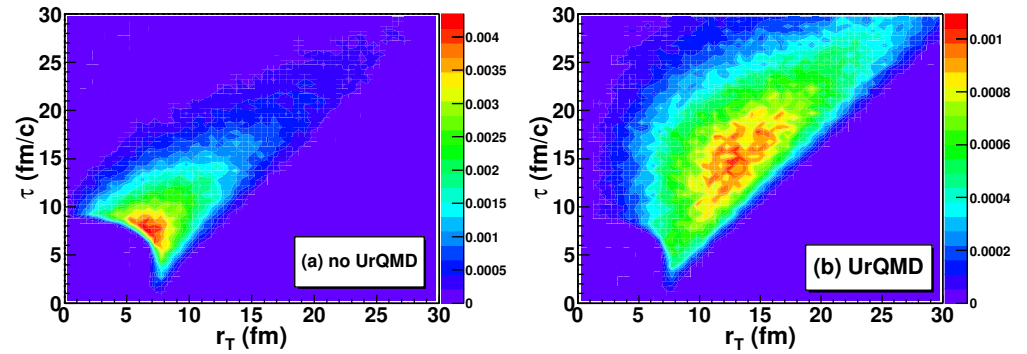


Figure 6. Emission functions $g(\tau, r_T)$ constructed in the iHKM for $K^+\pi^-$ pairs, coming from $K^*(892)$ decays [64], in two regimes: (a) free streaming of hadrons created at the particlisation stage (plus those coming from resonance decays), and (b) full iHKM calculation including UrQMD cascade as the final afterburner stage. The graphs correspond to the LHC Pb + Pb collisions at $\sqrt{s_{NN}} = 2.76\text{ TeV}$, $c = 5\text{--}10\%$; the transverse momentum and rapidity ranges are $0.3 < k_T < 5\text{ GeV}/c$ and $|y| < 0.5$, respectively.

4.2. Particlisation Times at Different Collision Energies

In the previous subsection, we focused on the particle emission picture including long and intensive emission from the hadronic stage of collision and on the estimation of the effective time scales characterising the emission duration. However, a detailed analysis of this problem cannot bypass the question of the hydrodynamic stage duration, defining the moment, when one should switch from the description of the system’s evolution in terms of a continuous medium to the description in terms of particles.

Conventionally, one assumes that the system’s hadronisation takes place at some critical temperature or energy density (whose concrete values depend on the applied equation of state for quark–gluon matter). As was mentioned, in the iHKM, we use two such equations of state, namely the Laine–Schroeder [75] EoS and the HotQCD Collaboration [76] one. The former is associated with the particlisation temperature $T_p = 165\text{ MeV}$ and the corresponding energy density $\epsilon_p = 0.5\text{ GeV}/\text{fm}^3$, while for the latter, the values $T_p = 156\text{ MeV}$ and $\epsilon_p = 0.27\text{ GeV}/\text{fm}^3$ are taken. The stated switching criterion implies that the particlisation process is not isochronous for different parts of the system, but starts quite early at the periphery and gradually reaches the centre. Thus, the system can be considered fully hadronised when the hadronisation has completed in its centre.

As we already remarked in the Introduction, an interesting fact about the particlisation times in ultrarelativistic heavy-ion collisions at the top RHIC and the LHC energies is that, in the iHKM, they are very close to each other (about $8\text{--}10\text{ fm}/c$), despite great differences in the collision energies (and, apparently, the corresponding initial energy densities). This feature is also reflected in the close maximal emission times obtained for pions in our studies related to the corresponding collision experiments [57–59].

To examine this issue, we built plots of the time evolution for the energy density in the centre of hydrodynamic grid $\epsilon(\tau)$ within iHKM (see Figures 7 and 8) in the case of the LHC Pb + Pb collisions at the two energies, $\sqrt{s_{NN}} = 2.76\text{ TeV}$ and $\sqrt{s_{NN}} = 5.02\text{ TeV}$. The

Laine–Schroeder EoS was used in the simulations for both energies with the critical energy density $\epsilon_p = 0.5 \text{ GeV/fm}^3$. For comparison, we also placed the curves corresponding to the ideal relativistic hydrodynamics three-dimensional Gubser solution [84]:

$$\epsilon(r_T, \tau) = \frac{\epsilon_0}{\tau^{4/3}} \frac{(2q)^{8/3}}{[1 + 2q^2(\tau^2 + r_T^2) + q^4(\tau^2 - r_T^2)^2]^{4/3}}, \quad (9)$$

and the Bjorken solution $\epsilon(\tau) = \epsilon_0 \tau^{-4/3}$ for purely longitudinal expansion, in the presented figures. The initial maximal ϵ values for the Gubser and the Bjorken flows correspond to those in the iHKM. Furthermore, in the Gubser flow case, the parameter q value, $q = 0.15 \text{ fm}^{-1}$, defining the transverse width of the energy density profile was chosen based on the fit to the initial iHKM $\epsilon(r_T)$ profile.

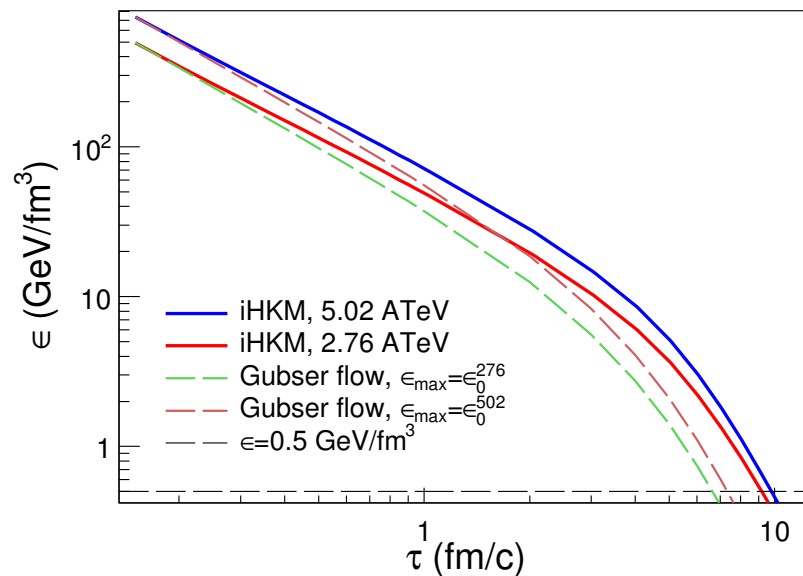


Figure 7. The energy density ϵ at the centre of the system depending on proper time τ . The iHKM curves for the two LHC energies, $\sqrt{s_{NN}} = 2.76 \text{ TeV}$ and $\sqrt{s_{NN}} = 5.02 \text{ TeV}$, together with the 3D Gubser solutions (9) of the relativistic hydrodynamics equations. The parameter q value, $q = 0.15 \text{ fm}^{-1}$, is defined from the Gubser fit to the initial iHKM $\epsilon(r_T)$ profiles; the initial ϵ values correspond to the iHKM ones.

From the comparison with the 1D Bjorken flow case (see Figure 8), one can conclude that the presence of strong transverse flow along with the longitudinal one is an important factor leading to the observed small difference in the particlisation times for the two collision energies (about 9.2 fm/c and 9.8 fm/c for the iHKM in the shown figure), defined as the moments when the respective energy densities drop to $\epsilon_p = 0.5 \text{ GeV/fm}^3$. Indeed, we see that, at the times after 3 fm/c, the Bjorken expansion is much slower than in the iHKM, the ratio between the two $\epsilon(\tau)$ (about 1.5) does not change with time, so that the respective particlisation times (about 26 fm/c and 36 fm/c) are much higher than in our model, the ratio between these times, $(\epsilon_{0,2}/\epsilon_{0,1})^{3/4}$, is about 1.35, and therefore, the difference between them is quite large (about 10 fm/c).

The Gubser flow, accounting for the transverse matter expansion, results in an $\epsilon(\tau)$ behaviour more similar to the iHKM curves at $\tau > 3 \text{ fm/c}$ and particlisation times much closer to those from the iHKM (about 6.7 fm/c and 7.3 fm/c), also with a small difference between the two values. The analysis of the particlisation time τ_p dependency on the initial energy density $\epsilon_0(\tau_0)$ in the iHKM shows that, for the considered RHIC and LHC energy range in central collisions at a fixed EoS and initial time τ_0 , the τ_p in the iHKM grows slowly with ϵ_0 , approximately as $\epsilon_0^{1/s}$, where s is close to 7. In our opinion, such a behaviour can

be explained by the intensive 3D expansion of the system, which takes place at large τ in high-energy A + A collisions and leads to a fast subsequent decay of the created fireball. The suggestion about the breakdown of the hydrodynamic description as a result of the intensive 3D expansion of the systems formed in high-energy hadron collisions was made already in 1953 by Landau in his pioneering paper concerning the hydrodynamic approach in high-energy physics [7].

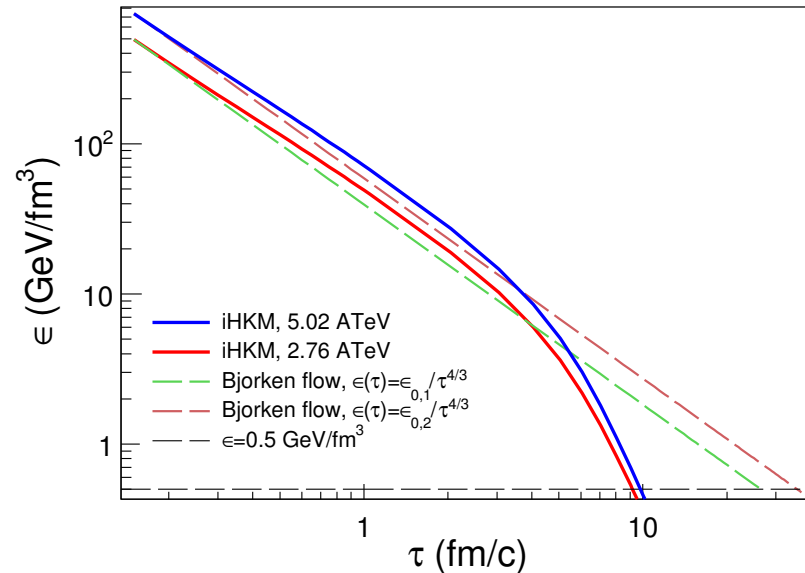


Figure 8. The same as in Figure 7, but 1D Bjorken solutions, $\epsilon(\tau) = \epsilon_0 \tau^{-4/3}$, are shown for comparison with the iHKM curves.

The energy density behaviour, somewhat similar to that shown in Figures 7 and 8, can be also observed in the following non-relativistic analytical solution of both ideal hydrodynamics and Boltzmann equations (see, e.g., [85,86]). The corresponding distribution function can be written as

$$f(t, \mathbf{x}, \mathbf{v}) = \frac{N}{(2\pi R_0)^3} \left(\frac{m}{T_0} \right)^{3/2} \exp \left(-\frac{m\mathbf{v}^2}{2T_0} - \frac{(\mathbf{x} - \mathbf{v}t)^2}{2R_0^2} \right), \quad (10)$$

where N is total particle number, m is the particle mass, R_0 is the initial Gaussian radius of the system, and T_0 is the initial temperature.

Given the distribution function (10), one easily finds the corresponding dependency of the energy density at $\mathbf{x} = 0$ on time:

$$\epsilon(\mathbf{x} = 0, t) = \frac{AT_0^4 (mR_0^2/T_0)^{5/2}}{(t^2 + mR_0^2/T_0)^{5/2}}, \quad (11)$$

where A is a constant.

In Figure 9, we present the illustration corresponding to the solution (11). The black line corresponds to a higher initial energy density, regulated by the T_0 parameter, which in this case has the value $T_0 = 350$ MeV. For the blue line, $T_0 = 300$ MeV, and the initial energy density is about two-times lower than for the black line. In both cases, the Gaussian radius of the initial fireball is $R_0 = 6$ fm, and the mass of the particles in the gas is $m = 0.5$ GeV/ c^2 . The black curve in the figure is shifted 1 fm/ c left, so that, after $t = 8$ fm/ c , we have the same energy density for both evolving systems. Therefore, similar to Figures 7 and 8, the same “particlisation” conditions are reached at nearly the same expansion times for the systems with the two different initial energy densities/temperatures, when the system’s expansion becomes essentially three-dimensional.

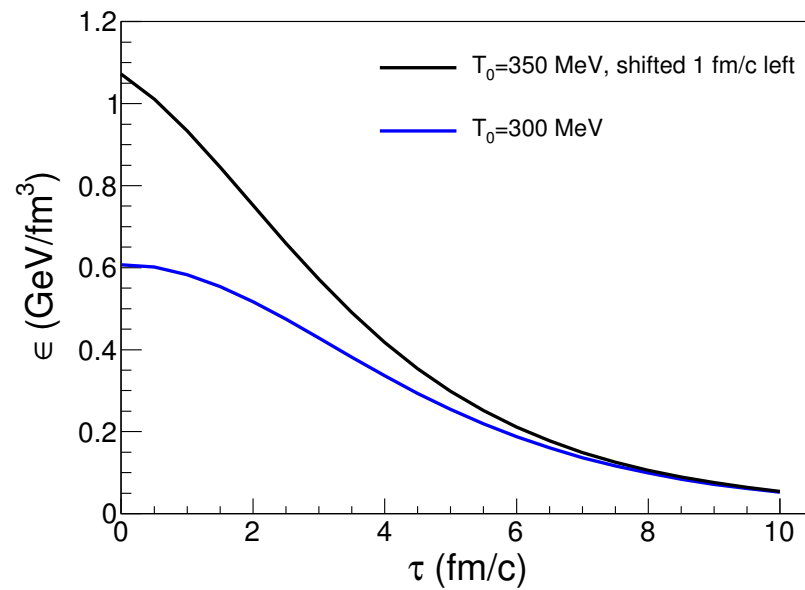


Figure 9. The energy density ϵ at the centre of the system dependencies on time τ in non-relativistic hydrodynamics and the Boltzmann equations' solution (11) with the two different initial ϵ values reaching the value $\epsilon = 0.5 \text{ GeV/fm}^3$ almost simultaneously (see the text for the details).

Although we consider the presented results quite interesting and worth attention in the context of heavy-ion collision studies, of course, there are certain aspects of the investigated phenomena left outside the scope of our research, so that additional analysis combined with further improvement of the utilised approaches should bring an even more-comprehensive and -detailed understanding of the collision dynamics. In particular, so far, we did not address the issue of light nuclei, molecules, and other loosely bound objects' production in A + A collisions, while the corresponding observables were actively analysed in recent experimental and theoretical studies. In our study of the K^* resonance production for 2.76 TeV LHC Pb + Pb collisions, we obtained underestimated values of the K^*/K ratio in peripheral events. This problem could be re-analysed in more detail, and the corresponding calculations for the 5.02 TeV case could be carried out. Other possible ways of extending our analysis could be accounting for high- p_T phenomena (like jets), consideration of the physics outside the midrapidity region, and adjusting the model for the description of planned ultra-high- and intermediate-energy (corresponding to the Beam Energy Scan program) experiments. The viscous hydrodynamics part of our model can be additionally improved by the implementation of the bulk viscosity and heat conductivity effects, as well as switching from boost-invariant to full-(3 + 1)D simulations.

5. Conclusions

We considered the two paradoxical femtoscopic observations in ultrarelativistic heavy-ion collisions. The first one is the closeness of the observed maximal emission times at the quite different collision energies. It can be explained by the *intensive* 3D (!) expansion of the system, which leads, starting from some time, to a fast decay of the formed continuous medium for a wide range of high collision energies. Therefore, all the corresponding systems reach the decay energy almost simultaneously.

Another paradoxical effect is that, despite the sufficient duration of the post-hydrodynamic/afterburner cascade stage, the observed times of the maximal emission are close to the particlisation times. The key to the answer lies in different pictures of particle radiation in the narrow soft p_T range and in a more wide one, typical for the femtoscopy analysis. For the latter, the times of maximal emission extracted from the corresponding fits nearly coincide with the particlisation times. For the former, the times are significantly larger and reflect the duration of the afterburner stage at this momentum interval.

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Data Availability Statement: All the necessary data are presented in the tables and figures within the article. The model results demonstrated in the graphs can also be obtained from the authors in tabular form.

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РОЗДІЛ 3

КОРЕЛЯЦІЙНИЙ АНАЛІЗ ВЗАЄМОДІЇ МІЖ БАРІОНАМИ У ЗІТКНЕННЯХ ВАЖКИХ ІОНІВ

3.1. Довжина розсіювання та радіус джерела $r_{\bar{\Lambda}}$ у зіткненнях на RHIC

Extracting $p\Lambda$ scattering lengths from heavy ion collisions

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The source radii previously extracted by the STAR Collaboration from the $p - \Lambda \oplus \bar{p} - \bar{\Lambda}$ and $\bar{p} - \Lambda \oplus p - \bar{\Lambda}$ correlation functions measured in 10% most central Au+Au collisions at top Relativistic Heavy Ion Collider (RHIC) energy, $\sqrt{s_{NN}} = 200$ GeV, differ by a factor of 2. The probable reason for this is the neglect of residual correlation effect in the STAR analysis. In the present paper we analyze baryon correlation functions within the Lednický and Lyuboshitz analytical model, extended to effectively account for the residual correlation contribution. Different analytical approximations for such a contribution are considered. We also use the averaged source radii extracted from hydrokinetic model (HKM) simulations to fit the experimental data. In contrast to the STAR experimental study, the calculations in HKM show both $p\Lambda$ and $p\bar{\Lambda}$ radii to be quite close, as expected from theoretical considerations. Using the effective Gaussian parametrization of residual correlations we obtain a satisfactory fit to the measured baryon-antibaryon correlation function with the HKM source radius value 3.28 fm. The baryon-antibaryon spin-averaged strong interaction scattering length is also extracted from the fit to the experimental correlation function.

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I. INTRODUCTION

Heavy ion collision experiments provide a good opportunity to study baryon-(anti)baryon strong interactions using the final state interaction (FSI) correlation technique [1–3]. It is based on the analysis of the momentum correlations caused by final state interactions between corresponding baryons produced in the collision. This activity is especially interesting in view of the ongoing nuclear collision experiments at the CERN Large Hadron Collider (LHC), which produce great numbers of various particles, including exotic multistrange, charmed, and beauty ones. It allows one to study the fundamental interactions between specific hadron species, which can hardly be achieved by other means. The extraction of this information makes it possible to check the correctness of hadron-hadron strong interaction models, constrain corresponding interaction potentials, and also improve existing cascade models (like UrQMD) by including into them the information about still unknown baryon-antibaryon annihilation cross sections.

In the paper [4] the experimental $p\Lambda$ and $p\bar{\Lambda}$ correlation functions measured by STAR at RHIC were fitted with Lednický and Lyuboshitz analytical model [1] allowing, in principle, to extract scattering lengths characterizing the two-particle strong interaction. However, apart from the interaction characteristics, the correlation function depends also on the source spatial structure, described in terms of function $S(\mathbf{r}^*)$, representing the time-integrated separation distribution of

particle emission points in the pair rest frame. This fact complicates the study of the particle interaction, as it increases the number of free parameters which enter the fit formula.

To simplify this study, one could calculate the corresponding source functions in realistic models of the collision process, which are known to describe well the experimental observables. The hydrokinetic model [5–7] provides successful simultaneous description of a wide class of bulk observables in the heavy ion collision experiments at the Relativistic Heavy Ion Collider (RHIC) and the LHC [8]. Moreover, it reproduces well [9] the pion and kaon source functions for semicentral Au+Au collisions at the top RHIC energy [10], including the specific non-Gaussian tails observed in the pair momentum and beam direction projections of the experimental source function. In this article we present the results of fitting the experimental data from [4] within the analytical model [1] where the Gaussian parametrization for the emission source function is utilized, and the corresponding Gaussian radii are extracted from hydrokinetic model (HKM) simulations.

II. MODELS DESCRIPTION

The STAR Collaboration studied [4] baryon-baryon $p - \Lambda \oplus \bar{p} - \bar{\Lambda}$ and baryon-antibaryon $p - \bar{\Lambda} \oplus \bar{p} - \Lambda$ correlation functions in 10% most central RHIC Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV. Protons and antiprotons in the transverse momentum range $0.4 < p_T < 1.1$ GeV/c with rapidity $|y| < 0.5$ and lambdas and antilambdas with $0.3 < p_T < 2.0$ GeV/c and $|y| < 1.5$ were selected for the analysis.

The experimental correlation function is constructed as the ratio of the distribution of particle momentum in the pair rest frame, k^* , in the same events to the analogous distribution in mixed events. Then the measured correlation function C_{meas} is corrected for the pair purity, defined as the fraction of correctly

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identified primary particle pairs among all the selected ones, to give the corrected function C_{corr} :

$$C_{\text{corr}}(k^*) = \frac{C_{\text{meas}}(k^*) - 1}{\lambda(k^*)} + 1, \quad (1)$$

where $\lambda(k^*)$ is the pair purity. The estimated mean pair purity in the experiment is $\lambda = 17.5 \pm 2.5\%$.

To fit the experimental correlation function the Lednický and Lyuboshitz analytical model [1] is used, which connects the two-particle correlation function $C(k^*)$ with the particle emission source size r_0 and the s -wave strong interaction scattering amplitudes $f^S(k^*)$ at a given total pair spin S . In the equal-time approximation, valid on condition $|t_1^* - t_2^*| \ll m_{2,1} r^{*2}$ for $\text{sign}(t_1^* - t_2^*) = \pm 1$ respectively, the correlation function can be calculated as a square of the wave function $\Psi_{-\mathbf{k}^*}^S$, representing the stationary solution of the scattering problem with the opposite sign of the vector $\mathbf{k}^*$, averaged over the total spin S and the distribution of the relative distances $S(\mathbf{r}^*)$:

$$C(k^*) = \langle |\Psi_{-\mathbf{k}^*}^S(\mathbf{r}^*)|^2 \rangle. \quad (2)$$

In typical nuclear collisions the source radius can be considered much larger than the range of the strong interaction potential, so $\Psi_{-\mathbf{k}^*}^S$ at small k^* can be approximated by the s -wave solution in the outer region:

$$\Psi_{-\mathbf{k}^*}^S(\mathbf{r}^*) = e^{-i\mathbf{k}^* \cdot \mathbf{r}^*} + \frac{f^S(k^*)}{r^*} e^{i\mathbf{k}^* \cdot \mathbf{r}^*}. \quad (3)$$

The effective range approximation for the s -wave scattering amplitude is utilized,

$$f^S(k^*) = \left(\frac{1}{f_0^S} + \frac{1}{2} d_0^S k^{*2} - i k^* \right)^{-1}, \quad (4)$$

where f_0^S is the scattering length and d_0^S is the effective radius for a given total spin $S = 1$ or $S = 0$.

The particles are assumed to be emitted unpolarized (i.e., with the polarization $P = 0$), so that the fraction of pairs in the singlet state $\rho_0 = 1/4(1 - P^2) = 1/4$, and in the triplet state $\rho_1 = 1/4(3 + P^2) = 3/4$.

The normalized separation distribution (source function) $S(\mathbf{r}^*) = N^{-1} d^3 N / d^3 \mathbf{r}^*$ is assumed to be Gaussian one,

$$S(\mathbf{r}^*) = (2\sqrt{\pi} r_0)^{-3} e^{-\frac{\mathbf{r}^{*2}}{4r_0^2}}, \quad (5)$$

where r_0 is considered as the effective radius of the source.

Under such assumptions the correlation function can be calculated analytically [1]:

$$C(k^*) = 1 + \sum_S \rho_S \left[\frac{1}{2} \left| \frac{f^S(k^*)}{r_0} \right|^2 \left(1 - \frac{d_0^S}{2\sqrt{\pi} r_0} \right) + \frac{2 \text{Re } f^S(k^*)}{\sqrt{\pi} r_0} F_1(2k^* r_0) - \frac{\text{Im } f^S(k^*)}{r_0} F_2(2k^* r_0) \right], \quad (6)$$

where $F_1(z) = \int_0^z dx e^{x^2 - z^2} / z$ and $F_2(z) = (1 - e^{-z^2}) / z$. The term $-\frac{d_0^S}{2\sqrt{\pi} r_0}$ in this expression corresponds to the correction accounting for a deviation of $\Psi_{-\mathbf{k}^*}^S$ from the true wave function

inside the range of the strong interaction potential. So, the model has quite a large number of parameters: the scattering lengths f_0^S and effective radii d_0^S , which may both be complex in general case, and the source radius r_0 . Although in principle all of them can be determined from the measured data, in each concrete situation the number of free parameters can be reduced by making certain reasonable assumptions about the values of some of them.

In our study the source radius r_0 is extracted from the Gaussian fit to the source functions calculated in hybrid HKM. The simulation of the full process of evolution of the system formed in nuclear or particle collision in hybrid HKM consists of two stages. The first one is hydrodynamical expansion of thermally and chemically equilibrated matter described within ideal hydrodynamics approximation with the lattice-QCD inspired equation of state [11] (corrected for small but nonzero chemical potentials), which is matched with the hadron-resonance gas in chemical equilibrium via crossover type transition. The second stage consists of gradual system decoupling after loosing chemical and thermal equilibrium. It can be described either within a hydrokinetic approach with a switch to a UrQMD cascade at some space-like hypersurface situated behind the hadronization one, or with a sudden switch to a UrQMD cascade at the hadronization hypersurface. In current study we choose the second variant of switching to cascade, based on [8], where the comparison of one- and two-particle spectra, calculated at both types of matching hydro and cascade stages for RHIC and LHC energies, showed a fairly small difference between them.

The model provides particle distribution functions $\frac{d^6 N}{d^3 x d^3 p}$ at the chosen switching hypersurface. Using the Monte-Carlo procedure, one generates particle momenta and coordinates according to these distributions, which serve as the input for the UrQMD hadronic cascade.

To perform a specific calculation one should specify the initial conditions for the hydrodynamics stage attributed to the starting proper time τ_0 . These conditions are the initial energy density (or entropy) profile $\epsilon(\mathbf{r})$ and the initial rapidity profile (initial flow) $y(\mathbf{r})$. Here we suppose longitudinal boost invariance and use $\epsilon(\mathbf{r}_T)$ corresponding to the Monte Carlo (MC) Glauber model calculated with GLISSANDO code [12]. The maximal energy density ϵ_0 is chosen to reproduce the experimental mean charged particle multiplicity, and the initial flow is supposed to be $y_T = \alpha \frac{r_T}{R^2(\phi)}$, with $\alpha = 0.45$ fm for top RHIC energy. Thus, model has only two free parameters, ϵ_0 and α . We start the hydrodynamic evolution at $\tau_0 = 0.1$ fm/c. The sudden switch from hydrodynamics to UrQMD is performed at the isotherm $T = 165$ MeV. The hadron distribution functions (for each hadron sort i) at the switching hypersurface σ_{sw} are calculated according to the Cooper-Frye formula

$$p_0 \frac{d^3 N_i}{p_T dp_T d\phi_p dy} = \int_{\sigma_{sw}} p^\mu d\sigma_\mu f_i^{eq}(p \cdot u(x), T(x), \mu_i(x)). \quad (7)$$

The procedure of filling the histograms for source functions $S(\mathbf{r}^*)$ (here $\mathbf{r}^*$ is the particle spatial separation in the pair rest

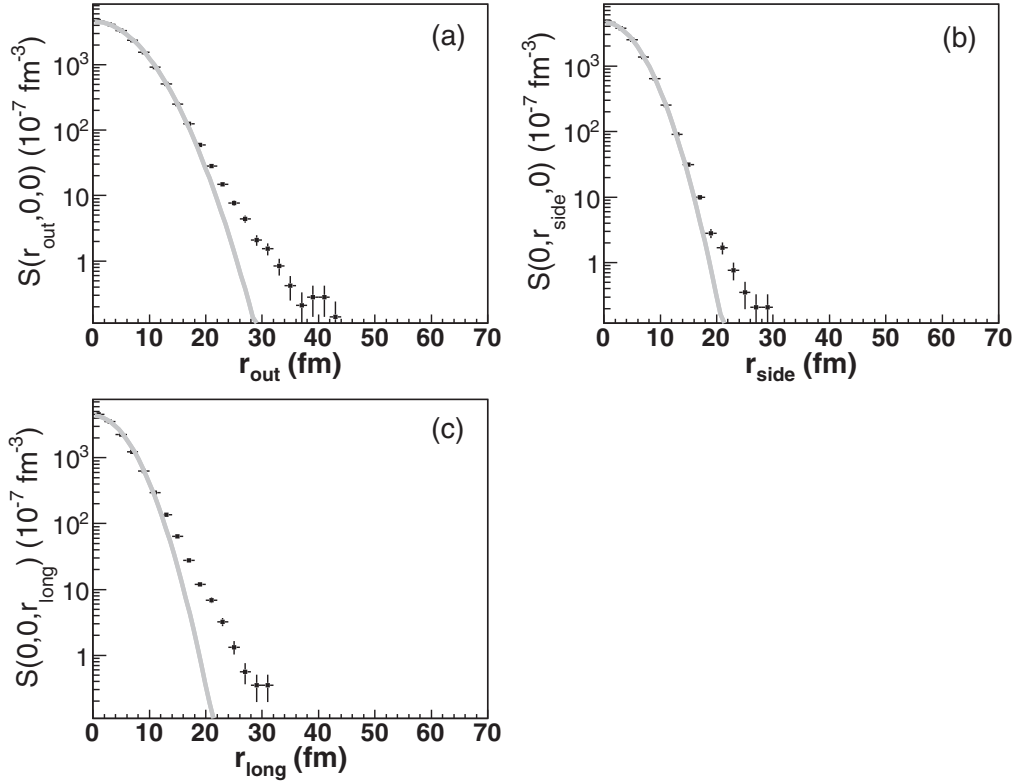


FIG. 1. The $p\Lambda$ source function projections calculated in HKM (markers) and the Gaussian fits to them (lines). The simulations correspond to 10% most central collisions at $\sqrt{s_{NN}} = 200$ GeV in conditions of the STAR experiment at RHIC [4]. Pair transverse momentum and rapidity cuts correspond to those in the experiment. The cut $k^* < 50$ MeV/c is also applied to select the pairs from correlation effect domain, where the $r^* - k^*$ correlations are small.

frame) can be described by the formula

$$S(\mathbf{r}^{*(k)}) = \frac{\sum_{n=1}^{N_{ev}} \sum_{i_1^n, i_2^n} \prod_{\alpha=1}^3 [\delta_{\Delta_\alpha}(r_\alpha^{*(k)} - r_{i_1^n \alpha}^* + r_{i_2^n \alpha}^*) / \Delta_\alpha]}{\sum_{n=1}^{N_{ev}} \sum_{i_1^n, i_2^n} 1}. \quad (8)$$

Here $\mathbf{r}_{i_1^n}^*$ and $\mathbf{r}_{i_2^n}^*$ are the pair rest frame three-coordinates of particles 1 and 2 produced in the n th event, $\mathbf{r}^{*(k)}$ is the three-coordinate of the center of the k th histogram bin, the function $\delta_\Delta(x) = 1$ if $|x| < \Delta/2$ and 0 otherwise, and Δ_α is the size of the α projection of the histogram bin.

III. RESULTS AND DISCUSSION

The $p\Lambda$ source function projections calculated in HKM together with the corresponding Gaussian fits are presented in Fig. 1. Here the out-side-long coordinate system is used, where the *out* axis is directed along the pair total momentum in a longitudinally comoving system, the *long* direction coincides with the beam axis, and the *side* axis is perpendicular to the latter two ones.

One can notice that HKM predicts a $p\Lambda$ source function having non-Gaussian tails. Similar behavior was observed experimentally [10] and reproduced in HKM simulations [9] for the pion source function. In $p\Lambda$ case these tails appear partially because of the averaging over a wide p_T interval. We also see that in different directions the corresponding Gaussian fits have different widths (especially the *out* projection is much

wider than the other ones). These peculiarities of the HKM separation distribution require, in principle, the generalization of the existing Lednický-Lyuboshitz formula (6) to the cases of anisotropic non-Gaussian source functions and heavy tails.¹ However, for simplicity and since in the described STAR experiment only a one-dimensional correlation function is analyzed with no respect to spatial directions, in the current study we are going to stay within the isotropic Gaussian approximation (6) and for this aim utilize the angle-averaged r_0 value. It can be extracted from the Gaussian fit to the angle-averaged source function (see Fig. 2) $S(r) = 1/(4\pi) \int_0^{2\pi} \int_0^\pi S(r, \theta, \phi) \sin \theta d\theta d\phi$.

In Figs. 3–6 we present experimental $p - \Lambda \oplus \bar{p} - \bar{\Lambda}$ and $\bar{p} - \Lambda \oplus p - \bar{\Lambda}$ correlation functions, measured by the STAR collaboration in 10% most central Au+Au collisions at the top RHIC energy $\sqrt{s_{NN}} = 200$ GeV together with the fits performed within the Lednický and Lyuboshitz analytical model.

For the baryon-baryon case (Fig. 3) the scattering lengths f_0^S and effective radii d_0^S values are taken from [13] ($f_0^S = 2.88$ fm, $f_0^t = 1.66$ fm, $d_0^S = 2.92$ fm, $d_0^t = 3.78$ fm). Thus,

¹Note, however, that at such a model extension the number of its free parameters will increase significantly, and utilization of the generalized formula for the reliable description of experimental data will require putting additional constraints on these parameters.

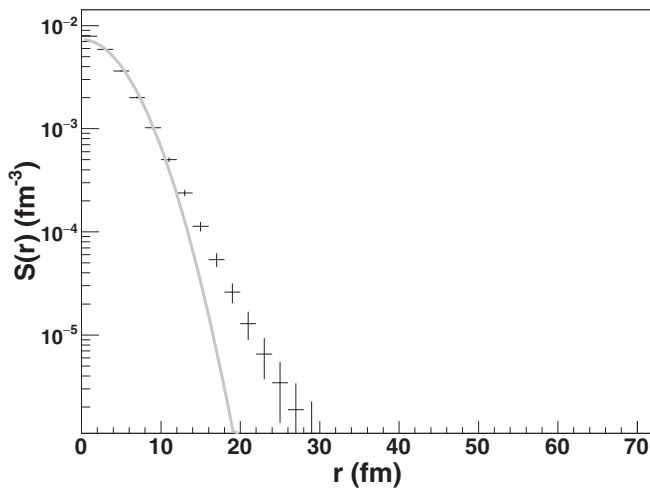


FIG. 2. The $p\Lambda$ angle-averaged source function calculated in HKM (markers) and the Gaussian fit to it (line). The simulation conditions are the same as given in the caption of Fig. 1

the source radius r_0 is the only free parameter in the STAR fit [4] (light curve). Fitting gives $r_0^{\text{exp}} = 3.09 \pm 0.30^{+0.17}_{-0.25} \pm 0.2$ fm. In our own fit (dark curve) all the parameters are fixed, and $r_0^{\text{HKM}} = 3.23$ fm is determined from a Gaussian fit (5) to the HKM $\mathbf{r}^*$ distribution calculated in the pair rest frame for $r^* < 70$ fm. One can see that the HKM model radius is consistent with that extracted from the STAR data in [4].

In the baryon-antibaryon case (Fig. 4), to reduce the number of free parameters both singlet and triplet scattering amplitudes are assumed to be equal, $f^s = f^t = f$ (approximately corresponding to spin-averaged scattering length f_0), and both effective radii are set to zero, $d_0^s = d_0^t = 0$, in the STAR fit. The scattering length should have a positive imaginary part $\text{Im } f_0 > 0$ describing the contribution of annihilation channels and leading to a wide dip in the correlation function at

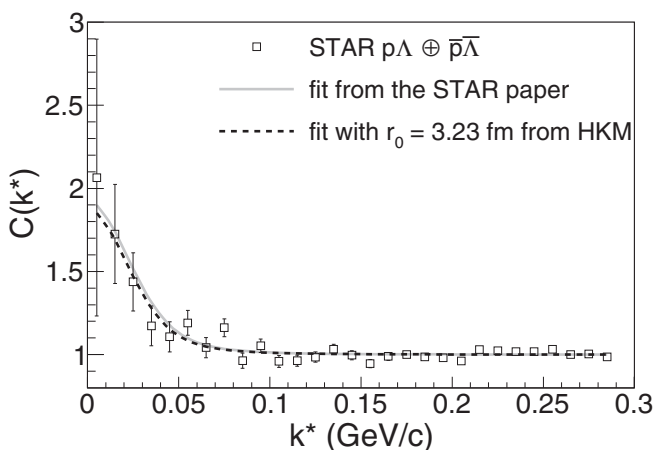


FIG. 3. The $p - \Lambda \oplus \bar{p} - \bar{\Lambda}$ correlation function measured by STAR (open markers), the corresponding fit according to (6) with parameters fixed as in the STAR paper [4] within the Lednický and Lyuboshitz analytical model [1] (light solid line) and our fit within the same model with the source radius r_0 extracted from the HKM calculations (dark dashed line).

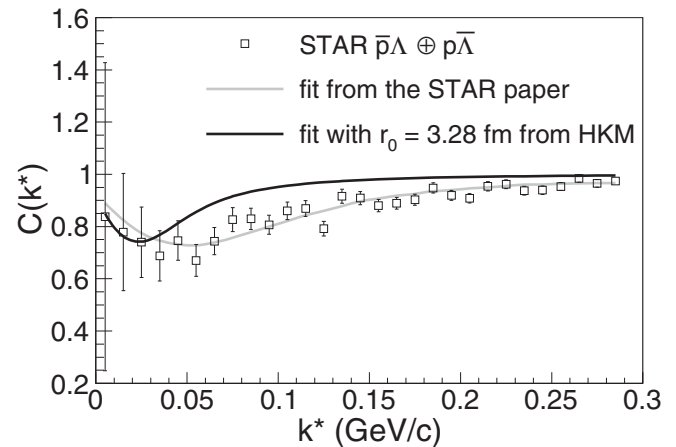


FIG. 4. The same as in Fig. 3 for the $\bar{p} - \Lambda \oplus p - \bar{\Lambda}$ correlation function.

intermediate k^* values due to the last term in Eq. (6). Thus, the model has three free parameters $\text{Re } f_0$, $\text{Im } f_0$, and r_0 in [4] and two free parameters $\text{Re } f_0$, $\text{Im } f_0$ in our fit. STAR obtained a $p\Lambda$ source radius value $r_0 = 1.50 \pm 0.05^{+0.10}_{-0.12} \pm 0.3$ fm (light curve), which is ~ 2 times smaller than the $p\Lambda$ one, although there is no apparent physical reason for such a difference. Both radii can be expected to have similar values, and the HKM source radius for the baryon-antibaryon case $r_0^{\text{HKM}} = 3.28$ fm is expectedly close to the corresponding baryon-baryon one. But at this source radius the fitting curve (dark) is too narrow to describe the data points.

However, residual correlations were not taken into account in the STAR analysis [4,14]. Constructing the experimental correlation function, one usually supposes that only the pairs composed of two primary particles are correlated, and the rest of the pairs, which include secondary or misidentified particles, are supposed to be uncorrelated. However, among such pairs so-called residual correlations can exist. They occur if the parent of a particle from such a pair was correlated with another particle forming the pair. For example, if Σ^+ , correlated with some Λ , decays into proton and π^0 , then this proton will be residually correlated with the Λ . The interactions in most of such pairs are unknown, so at the moment there is no possibility to reliably refine the constructed experimental correlation function from the effect of residual correlations. However, one can try to account for the residual correlations at least phenomenologically using a simple analytical approximation to the residual correlation function.

Note that the effect of the residual correlations has presumably only a minor influence on the baryon-baryon $p\Lambda$ correlation function, since in this case there are not so many inelastic channels open for parent FSI near threshold and, if open, they are usually suppressed, also being near threshold. So, $\text{Im } f_0$ for parent pairs (giving negative contribution to residual correlation function) is expected to be small and the sign of $\text{Re } f_0$ may vary from one parent pair to another, thus likely reducing the net effect of $\text{Re } f_0$ on residual correlation function.

In contrast to this, in the case of $\bar{p}\Lambda$ correlation function there is a number of annihilation channels significantly

contributing to $\text{Im } f_0$ and leading to a substantial negative contribution to residual correlation function.

In the case when the measured baryon-antibaryon correlation function is not corrected for purity, the fitted uncorrected correlation function is expressed through the true one in (6) similar to (1):

$$C_{\text{uncorr}}(k^*) = \lambda(k^*)C(k^*) + [1 - \lambda(k^*)], \quad (9)$$

The pair purity $\lambda(k^*)$ in our calculations is extracted according to (9) from the plots of $C_{\text{uncorr}}(k^*)$ and $C(k^*)$ provided in [15].

The first term in formula (9) corresponds to the pairs of correlated (primary only) particles, and the second one represents the contribution of the uncorrelated pairs, where one or both particles are misidentified or secondary ones. Assuming that among the latter there can be residually correlated pairs, one should modify this expression to account for the residual correlations as well.

In the recent paper [14], the accounting for residual correlations is performed by summarizing all the contributions from different parent pairs to the full correlation function, making, however, a number of simplifying assumptions. Particularly, the purity k^* dependence is neglected in their analysis. As for the scattering parameters f_{0i} and d_{0i} of the parent systems and the corresponding parent source radii r_{0i} , the effective range parameters d_{0i} are neglected and f_{0i} and r_{0i} are assumed to be equal to universal baryon-antibaryon values.

In contrast with [14], we propose here the alternative approach avoiding such a detailed calculation (containing, however, a number of assumptions) of the residual correlations. Instead, we are aiming to describe them by introducing an effective residual correlation function $C_{\text{res}}(k^*)$ for a fraction $\alpha(k^*)$ of the pairs of particles supposed earlier in Eq. (9) to be uncorrelated. Then,

$$C_{\text{uncorr}}(k^*) = 1 + \lambda(k^*)[C(k^*) - 1] + \alpha(k^*)[C_{\text{res}}(k^*) - 1]. \quad (10)$$

Obviously, Eq. (10) reduces to Eq. (9) if either $C_{\text{res}}(k^*) = 1$ or $\alpha(k^*) = 0$. One may approximate $\alpha(k^*)$ with a constant (e.g., put $\alpha = 0.49$, which is the total fraction of the pairs containing daughter particles as given in Table III of [4]) or assume it be proportional to the fraction of nonprimary particles: $\alpha(k^*) = \tilde{\alpha}[1 - \lambda(k^*)]$, where $\tilde{\alpha}$ is a fit parameter.

Choosing the concrete form of $C_{\text{res}}(k^*)$ requires some additional analysis. First of all, one should take into account that, due to the phase space suppression of small k^* values, the baryon-antibaryon correlations are dominated by the effect of wide annihilation dips in parent correlation functions related to $\text{Im } f_{0i}$ through the negative last term in Eq. (6). The parent decays widen these dips and wash out possible structures at small k^* related to $\text{Re } f_{0i}$. Further, the contribution to $C_{\text{res}}(k^*)$ from a given parent correlation function recovers the latter for k^* larger than the parent decay momenta, i.e., for sufficiently large $z = 2k^*r_0$ [16]. Moreover, following from large z behavior of the functions $F_1(z) \rightarrow (2z)^{-2}$ and $F_2(z) \rightarrow z^{-1}$, the parent correlation functions approach unity from below according to a universal inverse power law $\propto z^{-n}$, where the power $n > 2$ increases with the number of terms essentially contributing in the effective range expansion in a given k^* interval. Particularly, $n = 3$ if one may neglect

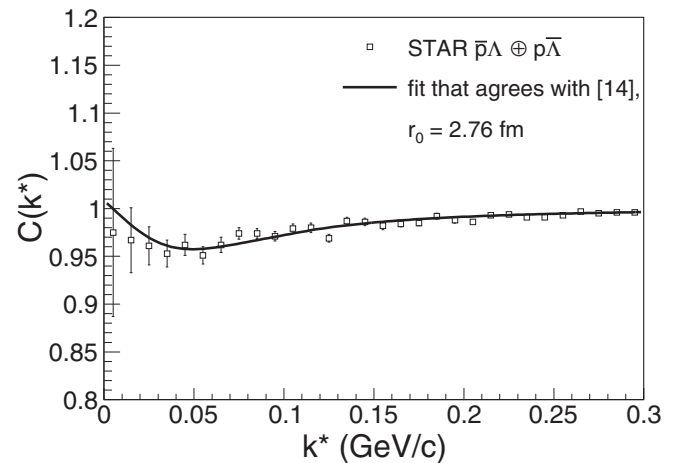


FIG. 5. Our fit (black line) to the STAR purity uncorrected $\bar{p} - \Lambda \oplus p - \bar{\Lambda}$ correlation function as presented in [14] (open markers) according to (10), (6), with $C_{\text{res}}(k^*)$ term in the form (12). In this particular fitting the source radius r_0 is a free parameter. The extracted fit parameter values are $r_0 = 2.76 \pm 0.13$ fm, $\text{Re } f_0 = 0.59 \pm 0.19$ fm, $\text{Im } f_0 = 0.85 \pm 0.14$ fm, with $\chi^2/\text{ndf} = 1.37$.

already the effective range parameters d_{0i} (i.e., neglect the k^* dependence of the effective range function):

$$C_i(z) - 1 \rightarrow \frac{-u + v/(\sqrt{\pi}z)}{z[(1 + uz/2)^2 + (vz/2)^2]} \rightarrow z^{-3}, \quad (11)$$

where $u = \text{Im } f_{0i}/r_{0i}$, $v = \text{Re } f_{0i}/r_{0i}$. For practical calculations, one may follow [14] and assume approximately the same source radii and scattering parameters for all baryon-antibaryon systems. Then, one can approximate $C_{\text{res}}(z)$ at large enough z by the $\bar{p}\Lambda$ correlation function $C(z)$ and approximately account for the washing out effect of parent decays by smoothly tailing the latter at some value $z = z_t$ to a slowly varying function $1 - A + Bz^c$:

$$\begin{aligned} C_{\text{res}}(z, z \leq z_t) &= 1 - A + Bz^c, \\ C_{\text{res}}(z, z > z_t) &= C(z). \end{aligned} \quad (12)$$

Using the effective expression (12) for C_{res} and data points from [14], one can reproduce the results obtained in [14] but in a much simpler way. So, assuming $\alpha(k^*) = 0.49$ and $\lambda(k^*) = 0.15$ as in [14] and fixing $z_t = 3$, $c = 1.5$ based on the analysis of various contributions to the residual correlation function including those in Fig. 4 in [14], the fit results are $r_0 = 2.76 \pm 0.13$ fm, $\text{Re } f_0 = 0.59 \pm 0.19$ fm, and $\text{Im } f_0 = 0.85 \pm 0.14$ fm, with $\chi^2/\text{ndf} = 1.37$ (see Fig. 5). They agree within the errors with the result from [14]: $r_0 = 2.83 \pm 0.12$ fm, $\text{Re } f_0 = 0.49 \pm 0.21$ fm, and $\text{Im } f_0 = 1.00 \pm 0.21$ fm. The fitted r_0 value agrees with the fit result for $p\Lambda$ system; however, it is substantially smaller than the HKM prediction of 3.28 fm. As a result, the fit with the radius fixed at the HKM value leads to an unsatisfactory description of the correlation function.

Therefore, sticking with the HKM result, one is enforced to apply a more flexible parametrization of C_{res} , avoiding the assumption of the universal form of baryon-antibaryon correlation functions at $z > z_t$ with a constant effective range function (i.e., with neglected d_0 and higher order expansion

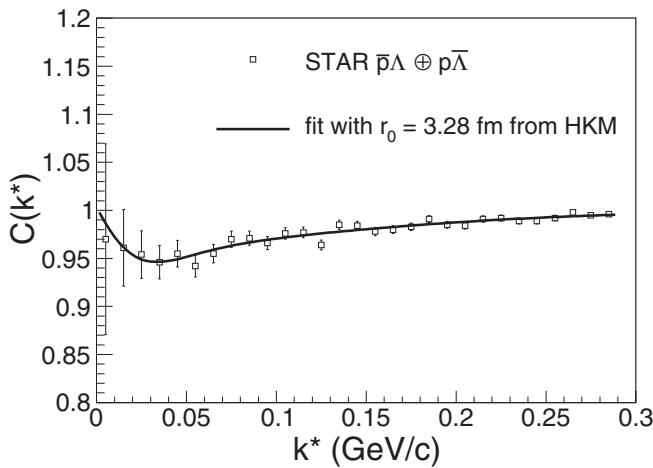


FIG. 6. The purity uncorrected $\bar{p} - \Lambda \oplus p - \bar{\Lambda}$ correlation function measured by STAR [15] (open markers) and our fit to it according to (10) and (6) (black line), with the Gaussian parametrization (13) for the residual correlation term $C_{\text{res}}(k^*)$. The source radius r_0 was fixed at a value extracted from the HKM calculations. The extracted fit parameters are $\text{Re } f_0 = 0.14 \pm 0.66$ fm, $\text{Im } f_0 = 1.53 \pm 1.31$ fm, $\beta = 0.034 \pm 0.005$, and $R = 0.48 \pm 0.05$ fm, with $\chi^2/\text{ndf} = 0.87$.

parameters). It is worth noting that, even if keeping the universality assumption, the account of additional complex expansion parameters would make the fit quite unstable and unpractical at given statistical errors. Instead, one can use the effective Gaussian parametrization with reasonable behavior at small and large k^* values [17,18]:

$$C_{\text{res}}(k^*) = 1 - \tilde{\beta} e^{-4k^{*2}R^2}, \quad (13)$$

where $\tilde{\beta} = A > 0$ is the annihilation (wide) dip amplitude and $R \ll r_0$ is the dip inverse width.

Choosing further the fraction of residually correlated particles as $\alpha(k^*) = \tilde{\alpha}[1 - \lambda(k^*)]$, one may notice that the parameters $\tilde{\alpha}$ and $\tilde{\beta}$ enter in (10) only through a product $\tilde{\alpha}\tilde{\beta}$; the latter can be substituted by a single fit parameter β .

In Fig. 6 we present the result of such a fit of the experimental $\bar{p}\Lambda \oplus p\bar{\Lambda}$ correlation function. The fit quality is quite good ($\chi^2/\text{ndf} = 0.87$) and the fitted parameters are $\text{Re } f_0 = 0.14 \pm 0.66$ fm, $\text{Im } f_0 = 1.53 \pm 1.31$ fm, $\beta = 0.034 \pm 0.005$, and $R = 0.48 \pm 0.05$ fm. Unfortunately, due to the decoupling of the form of the residual correlation function from the scattering parameters, the statistics now appears to be insufficient for their reliable determination.

IV. CONCLUSIONS

Study of baryon and antibaryon correlations provides a powerful tool for probing space-time evolution of heavy ion collisions and for extracting the parameters of strong interaction between emitted particles.

We reproduced the $p - \Lambda \oplus \bar{p} - \bar{\Lambda}$ and $\bar{p} - \Lambda \oplus p - \bar{\Lambda}$ correlation functions, measured in 10% most central Au+Au collisions by STAR at $\sqrt{s_{NN}} = 200$ GeV, using the Lednický and Lyuboshitz analytical formalism with the average source radii extracted from the hydrokinetic model (HKM). To take into account the residual correlations influencing baryon-antibaryon femtoscopic effects, a modified analytical approximation has been applied. The values of the $p\Lambda$ and $p\bar{\Lambda}$ source radii calculated in HKM are similar, in agreement with theoretical expectations, and consistent with experimental result for $p - \Lambda \oplus \bar{p} - \bar{\Lambda}$. The significantly smaller source size obtained by the STAR Collaboration for $\bar{p} - \Lambda \oplus p - \bar{\Lambda}$ pairs can be explained by neglecting residual correlations in the data analysis.

The real and imaginary parts of the spin averaged scattering length have been extracted for baryon-antibaryon pairs when residual correlations are taken into account. We analyze the different forms of effective corrections for the residual baryon-antibaryon correlations, and find that the simple Gaussian form results in the best fit quality.

The hydrokinetic model including a detailed description of particle correlations allows for a precise study of heavy ion collisions. New high statistics data from RHIC and LHC will provide measurements of various particle pairs, including baryon-antibaryon ones, allowing us to investigate the particle interactions in these pairs. A consistent approach for a wide class of observables will help us to understand complex and unknown features of the evolution of heavy ion collisions.

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3.2. Баріон-баріонні кореляції на LHC

Proton- Λ correlation functions at energies available at the CERN Large Hadron Collider taking into account residual correlations

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The theoretical analysis of the $\bar{p}\text{-}\Lambda \oplus p\text{-}\bar{\Lambda}$ correlation function in 10% most central Au+Au collisions at Relativistic Heavy Ion Collider (RHIC) energy $\sqrt{s_{NN}} = 200$ GeV shows that the contribution of residual correlations is a necessary factor for obtaining a satisfactory description of the experimental data. Neglecting the residual correlation effect leads to an unrealistically low source radius, about 2 times smaller than the corresponding value for $p\text{-}\Lambda \oplus \bar{p}\text{-}\bar{\Lambda}$ case, when one fits the experimental correlation function within Lednický-Lyuboshitz analytical model. Recently an approach that accounts effectively for residual correlations for the baryon-antibaryon correlation function was proposed, and a good RHIC data description was reached with the source radius extracted from the hydrokinetic model (HKM). The $\bar{p}\text{-}\Lambda$ scattering length, as well as the parameters characterizing the residual correlation effect—annihilation dip amplitude and its inverse width—were extracted from the corresponding fit. In this paper we use these extracted values and simulated in HKM source functions for Pb+Pb collisions at the LHC energy $\sqrt{s_{NN}} = 2.76$ TeV to predict the corresponding $p\Lambda$ and $p\bar{\Lambda}$ correlation functions.

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I. INTRODUCTION

The study of $p\Lambda$ correlation functions (CF) (along with the study of other two-particle correlations such as $\pi\pi$, KK) allows one to obtain the information about the evolution of the matter formed in relativistic nuclear collisions, in particular about the collective flow.

Another great possibility which is open for a researcher in baryon-(anti)baryon correlation analysis is the study of strong interaction between the particles of different sorts with the final-state interaction (FSI) correlation technique [1–3]. Since the Large Hadron Collider (LHC) copiously produces hadrons of different species, including multistrange, charmed, and beauty ones, the advantage of this approach is the ability to analyze the interactions even in exotic particle pairs, which is hardly achieved by other means.

Fitting the experimental correlation function with some analytical formula, e.g., the Lednický-Lyuboshitz model [1], allows one to extract the quantities describing both the interaction in corresponding pairs and the size of the particle emission region. Of course, if the source function $S(\mathbf{r}^*)$, describing the spatial structure of the pair emission, is known, an extraction of unknown interaction parameters becomes more reliable. The corresponding spatial structure can be obtained from a realistic collision model that simulates the evolution of the system formed in high-energy heavy-ion collisions. In the recent paper [4] the hybrid variant of the hydrokinetic model (HKM) [5–7] was used for this purpose. The choice of HKM is highly reliable since it is known to provide successful simultaneous descriptions of a wide class of bulk observables in nuclear collision experiments at RHIC and LHC [8]. The model also reproduces well [9] the source functions for pion and kaon pairs in Au+Au collisions at the top RHIC energy [10], including non-Gaussian tails observed in certain experimental source function projections.

In Ref. [4], in order to extract unknown $\bar{p}\Lambda$ scattering length, the corresponding experimental correlation function for 10% most central Au+Au collisions at top RHIC energy, measured by the STAR Collaboration [11], was fitted with the Lednický-Lyuboshitz formula using the effective source radius r_0 extracted from the HKM source function. It was also found that r_0 values obtained in HKM for baryon-baryon and baryon-antibaryon correlations are expectedly close, while in the STAR experimental analysis [11], where the source radii were considered as free fit parameters, the extracted $\bar{p}\Lambda$ radius was ~ 2 times smaller than the $p\Lambda$ one. One can assume that this apparent difference is due to neglect of residual correlations in Ref. [11]. Such correlations can exist between secondary protons and Λ s if their parents were correlated (or the parent of one particle was correlated with another particle in the pair). To take into account the residual correlation contribution to the baryon-antibaryon CF, a modified analytical formula was introduced in Ref. [4]. As a result, a good description of the experimental $\bar{p}\Lambda$ correlation function is obtained and the $\bar{p}\Lambda$ spin-averaged scattering length is extracted from the corresponding fit.

In the present paper we are going to apply the method developed in Ref. [4] for description of the RHIC data to predict the $p\Lambda$ and $\bar{p}\Lambda$ correlation functions in the 5% most central Pb+Pb collisions at the LHC energy $\sqrt{s_{NN}} = 2.76$ TeV, utilizing the results of Ref. [4] as a starting point.

II. FORMALISM

The experimental correlation functions presented in Ref. [11] are purity corrected. They are obtained from the measured ones as

$$C_{\text{corr}}(k^*) = \frac{C_{\text{meas}}(k^*) - 1}{\lambda(k^*)} + 1, \quad (1)$$

where $C_{\text{meas}}(k^*)$ and $C_{\text{corr}}(k^*)$ are the measured and the corrected CF respectively, and $\lambda(k^*)$ is pair purity. The latter is defined as the fraction of pairs consisting of primary, correctly identified particles.

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As in Ref. [4] we model the purity-corrected LHC baryon-baryon correlation function with Lednický-Lyuboshitz analytical formula [1]:

$$C(k^*) = 1 + \sum_S \rho_S \left[\frac{1}{2} \left| \frac{f^S(k^*)}{r_0} \right|^2 \left(1 - \frac{d_0^S}{2\sqrt{\pi}r_0} \right) + \frac{2 \operatorname{Re} f^S(k^*)}{\sqrt{\pi}r_0} F_1(2k^*r_0) - \frac{\operatorname{Im} f^S(k^*)}{r_0} F_2(2k^*r_0) \right], \quad (2)$$

where $F_1(z) = \int_0^z dx e^{x^2 - z^2}/z$ and $F_2(z) = (1 - e^{-z^2})/z$. This formula is derived starting from the basic equation $C(k^*) = \langle |\Psi_{-\mathbf{k}^*}^S(\mathbf{r}^*)|^2 \rangle$, where the wave function $\Psi_{-\mathbf{k}^*}^S$ represents the stationary solution of the scattering problem with the opposite sign of the vector $\mathbf{k}^*$. The angle brackets mean averaging over the total spin S and the distribution of the relative distances $S(\mathbf{r}^*)$. Since typically the source radius can be considered much larger than the range of the strong interaction potential, $\Psi_{-\mathbf{k}^*}^S$ can be approximated at small k^* by the s -wave solution in the outer region:

$$\Psi_{-\mathbf{k}^*}^S(\mathbf{r}^*) = e^{-i\mathbf{k}^* \cdot \mathbf{r}^*} + \frac{f^S(k^*)}{r^*} e^{ik^* \cdot \mathbf{r}^*}. \quad (3)$$

The scattering amplitude $f^S(k^*)$ here is taken in the effective range approximation

$$f^S(k^*) = \left(\frac{1}{f_0^S} + \frac{1}{2} d_0^S k^{*2} - ik^* \right)^{-1}, \quad (4)$$

where f_0^S is the scattering length and d_0^S is the effective radius for a given total spin $S = 1$ or $S = 0$. The singlet and triplet weights ρ_i for unpolarized particles (supposing the polarization $P = 0$) are $\rho_0 = 1/4(1 - P^2) = 1/4$ and $\rho_1 = 1/4(3 + P^2) = 3/4$ correspondingly.

As for the baryon-antibaryon case, following Ref. [4], we fit the experimentally measured correlation function $C_{\text{uncorr}}(k^*)$ with the following analytical expression:

$$C_{\text{uncorr}}(k^*) = 1 + \lambda(k^*)(C(k^*) - 1) + \alpha(k^*)(C_{\text{res}}(k^*) - 1), \quad (5)$$

where $\lambda(k^*)$ is purity or the fraction of correctly identified pairs consisting of primary particles, $C(k^*)$ is “true” correlation function approximated by Eq. (2), $\alpha(k^*)$ is the fraction of secondary particles which are residually correlated, $\alpha(k^*) = \tilde{\alpha}[1 - \lambda(k^*)]$, and $C_{\text{res}}(k^*)$ is the residual correlation contribution. The latter is taken in the Gaussian form [4,12],

$$C_{\text{res}}(k^*) = 1 - \tilde{\beta} e^{-4k^{*2}R^2}, \quad (6)$$

where $\tilde{\beta} = A > 0$ is the annihilation (wide) dip amplitude and $R \ll r_0$ is the dip inverse width. Since $\tilde{\alpha}$ and $\tilde{\beta}$ enter Eq. (5) only as a product $\tilde{\alpha}\tilde{\beta}$, the latter is treated as a single parameter β at fitting.

The source radii r_0 in both $p\Lambda$ and $\bar{p}\Lambda$ cases are extracted from the Gaussian fit $S_{\text{fit}}(r^*) = (2\sqrt{\pi}r_0)^{-3} e^{-\frac{r^{*2}}{4r_0^2}}$ to the angle-averaged source function,

$$S(r^*) = 1/(4\pi) \int_0^{2\pi} \int_0^\pi S(r^*, \theta, \phi) \sin \theta d\theta d\phi, \quad (7)$$

calculated in the hydrokinetic model [5–7]. The latter simulates the evolution of the matter in relativistic nuclear collision as consisting of two stages—(1) the hydrodynamic expansion of the matter in local thermal and chemical equilibrium and (2) gradual system decoupling, beginning when the equilibrium is lost. The first stage is described within ideal hydrodynamics and for the second one the hydrokinetic approach is utilized, based on the Boltzmann equations in the integral form, with switching to Ultrarelativistic Quantum Molecular Dynamics (UrQMD) cascade at a spacelike hypersurface. In the current study HKM is taken in its simplified hybrid form [8] with sudden switch from hydrodynamic evolution to the cascade at the hadronization hypersurface defined by the isotherm $T = 165$ MeV.

The model output consists of generated particle momenta and coordinates that are further used to build different observables. The considered angle-averaged source function histograms are filled using the following procedure (here r^* is the particle spatial separation in the pair rest frame):

$$S(r^{*(k)}) = \frac{\sum_{n=1}^{N_{\text{ev}}} \sum_{i_1^n, i_2^n} [\delta_\Delta(r^{*(k)} - r_{i_1^n}^* + r_{i_2^n}^*) / (4\pi(r_{i_1^n}^* - r_{i_2^n}^*)^2 \Delta)]}{\sum_{n=1}^{N_{\text{ev}}} \sum_{i_1^n, i_2^n} 1}. \quad (8)$$

Here $r_{i_1^n}^*$ and $r_{i_2^n}^*$ are the pair rest frame r coordinates of particles 1 and 2 produced in the n th event, $r^{*(k)}$ is the r coordinate of the k th histogram bin center, the function $\delta_\Delta(x) = 1$ if $|x| < \Delta/2$ and 0 otherwise, and Δ is the size of the histogram bin.

III. RESULTS AND DISCUSSION

The initial conditions (IC) for HKM calculations simulating the considered case of 5% most central Pb+Pb collisions at the LHC energy $\sqrt{s_{NN}} = 2.76$ TeV are described in detail in Refs. [8,13]. We assume longitudinal boost invariance, so that the IC are specified in the plane transverse to the beam axis only. The transverse energy density profile $\epsilon_i(\mathbf{r}_T)$ at the starting time $\tau_i = 0.1$ fm/c corresponds to Monte Carlo Glauber model and is calculated in GLISSANDO code [14], where the overall scale factor ϵ_0 , being the maximal initial energy density, is fixed basing on the experimental mean charged particle multiplicity. The initial transverse flow is absent in the present calculations.

In Fig. 1 one can see the angle averaged $p\Lambda$ source function $S(r^*)$ obtained in the hydrokinetic model for the considered LHC collisions together with the Gaussian fit to it. The source radius value extracted from this fit is $r_0 = 3.76$ fm, which is about 1.15 times larger than for the RHIC case. The $\bar{p}\Lambda$ source function fitting results in the same source radius value.

Having obtained r_0 from HKM and fixing f_0^S and d_0^S according to the paper [15] for the baryon-baryon case, we can model the corresponding correlation function (see Fig. 2). As compared to RHIC, the LHC $p\Lambda$ CF is slightly more narrow (the two corresponding Gaussian widths differ by a factor of ~ 1.13) and has a lower intercept of about 1.7.

To build the corresponding baryon-antibaryon correlation function one should determine the values of parameters entering Eqs. (5) and (6) for the LHC case. As well as for RHIC we assume $f^s = f^t = f$ and $d_0^s = d_0^t = 0$. The

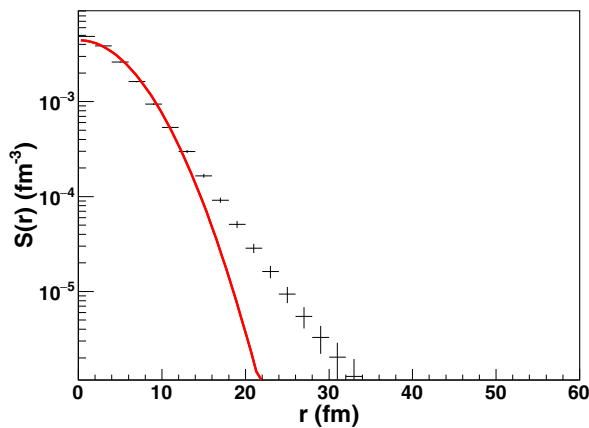


FIG. 1. (Color online) $p\Lambda$ angle averaged source function calculated in the HKM (markers) and the Gaussian fit to it (line). The calculations correspond to 5% most central Pb+Pb collisions at LHC energy $\sqrt{s_{NN}} = 2.76$ TeV, in pseudorapidity range $|\eta| < 0.8$. Proton p_T range is $0.7 < p_T < 4$ GeV/c, and for Λ s it is $0.7 < p_T < 5$ GeV/c. The extracted $r_0 = 3.76$ fm.

source radius is again fixed from HKM calculation, $r_0 = 3.76$ fm. As for the real and imaginary parts of the scattering length, $\text{Re } f_0$ and $\text{Im } f_0$, they characterize the antiproton- Λ strong interaction and hence are not changed when switching from RHIC to LHC. We can use the values extracted from the fit to RHIC $\bar{p}\Lambda$ correlation function shown in Fig. 6 of Ref. [4], where the Gaussian parametrization (6) for the residual correlation contribution $C_{\text{res}}(k^*)$ is applied, $\text{Re } f_0 = 0.14 \pm 0.66$ fm and $\text{Im } f_0 = 1.53 \pm 1.31$ fm. The parameter $\beta = \tilde{\alpha}\tilde{\beta}$ describes the strength of residual correlations and the fraction of residually correlated nonprimary particles. So, this parameter depends on particle interaction kinematics. Thus, it also can be taken the same as

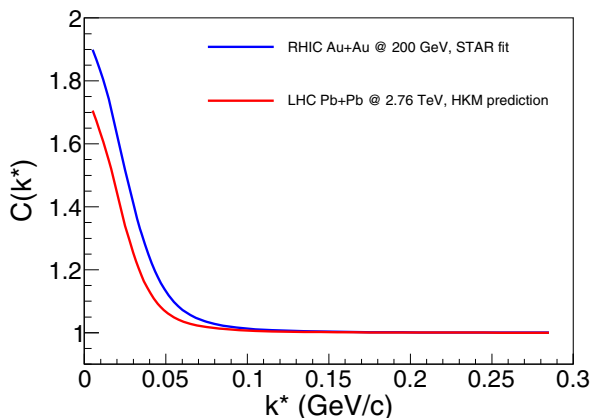


FIG. 2. (Color online) HKM prediction for purity-corrected $p\text{-}\Lambda \oplus p\text{-}\bar{\Lambda}$ correlation function in the LHC Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV, $c = 0 - 5\%$, $|\eta| < 0.8$, with $0.7 < p_T < 4$ GeV/c for protons and $0.7 < p_T < 5$ GeV/c for Λ s (red [light gray] line). The LHC source radius value calculated in HKM is $r_0 = 3.76$ fm. The Lednický-Lyuboshitz fit to the top RHIC energy correlation function, corresponding to the STAR experiment [11], with $r_0 = 3.23$ fm extracted from the HKM source function is presented for comparison (blue [dark gray] line).

TABLE I. The fractions of $p\Lambda$ pairs, primary and coming from different decays, calculated in HKM. These fractions are quite similar for RHIC and LHC cases.

Pair	Fraction (%)
$p_{\text{prim}}\text{-}\Lambda_{\text{prim}}$	38
$p_{\Lambda}\text{-}\Lambda_{\text{prim}}$	16
$p_{\Sigma^+}\text{-}\Lambda_{\text{prim}}$	3
$p_{\text{prim}}\text{-}\Lambda_{\Sigma^0}$	12
$p_{\Lambda}\text{-}\Lambda_{\Sigma^0}$	5
$p_{\Sigma^+}\text{-}\Lambda_{\Sigma^0}$	1
$p_{\text{prim}}\text{-}\Lambda_{\Xi}$	17
$p_{\Lambda}\text{-}\Lambda_{\Xi}$	7
$p_{\Sigma^+}\text{-}\Lambda_{\Xi}$	1

for RHIC, $\beta = 0.034 \pm 0.005$. As for the R parameter, which is some effective size associated with residual correlations ($R_{\text{RHIC}} = 0.48 \pm 0.05$ fm), one can suppose that for LHC it will be larger than for RHIC, approximately by a factor equal to the source radii ratio $r_0^{\text{LHC}}/r_0^{\text{RHIC}} \approx 1.15$, i.e., $R_{\text{LHC}} = R_{\text{RHIC}}(r_0^{\text{LHC}}/r_0^{\text{RHIC}})$. Such an assumption gives $R_{\text{LHC}} = 0.55 \pm 0.06$ fm.¹ Note that since the mentioned parameter values have errors, the LHC correlation function cannot be predicted exactly, but only up to uncertainties caused by these errors in the fit parameters. The uncertainty in the predicted correlation function is calculated as $\Delta C_{\text{uncorr}}(k^*) = \sqrt{\sum_{i=1}^4 \left(\frac{\partial C_{\text{uncorr}}(k^*)}{\partial x_i}\right)^2 \sigma_i^2 + 2 \sum_{1 \leq i < j \leq 4} \frac{\partial C_{\text{uncorr}}(k^*)}{\partial x_i} \frac{\partial C_{\text{uncorr}}(k^*)}{\partial x_j} \sigma_{ij}}$, where x_i are the four fit parameters, $\text{Re } f_0$, $\text{Im } f_0$, β , and R ; $\sigma_i = \Delta x_i$ are the corresponding parameter standard errors; and σ_{ij} are the covariances of parameters x_i and x_j .

The determination of purity $\lambda(k^*)$ at the LHC is not so explicit, since it depends not only on the fraction of secondary pairs, coming from resonance decays, but also on the experimental setup, or more precisely, on the fraction of misidentified pairs. To clarify this issue we have calculated the fractions of pairs made by particles having different origins (primary or coming from certain decays) in the hydrokinetic model for RHIC and LHC collisions. The results obtained in both cases are quite close and are presented in Table I. Comparing these fractions with those listed for the RHIC case in Table III of Ref. [11], one can see that experimental fraction of $p_{\text{prim}}\text{-}\Lambda_{\text{prim}}$ pairs $\lambda = 0.15$ is about 2.5 times lower than in HKM, likely due to misidentification, which takes place in the experiment. In HKM simulations, on the contrary, all the produced particles are correctly identified, which leads to such a difference between corresponding fraction values. However, since HKM is a realistic model that describes both RHIC and LHC bulk observables well, based on its results one can conclude that true purities, understood as the fractions of primary pairs, at RHIC and LHC should be quite similar.

In such a situation we demonstrate three different plots for the LHC $\bar{p}\text{-}\Lambda \oplus p\text{-}\bar{\Lambda}$ correlation function (see Figs. 3–5).

¹The calculations show that the model correlation function depends weakly on R changing by a close to unity factor.

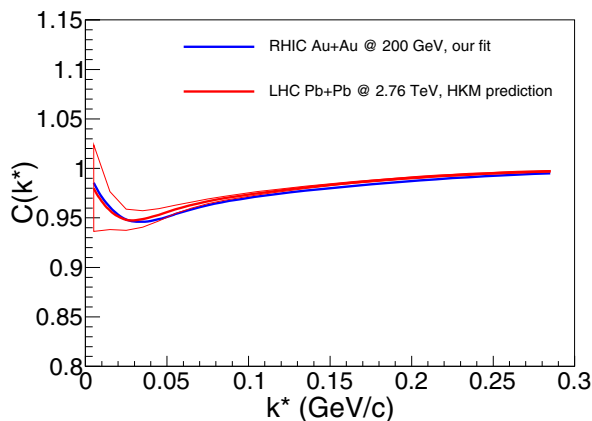


FIG. 3. (Color online) Same as in Fig. 2 for purity uncorrected $\bar{p}-\Lambda \oplus p-\bar{\Lambda}$ correlation function. The HKM source radius for LHC is $r_0 = 3.76$ fm. The purity $\lambda(k^*)$ is the same as for the RHIC case [11]. The scattering length real and imaginary parts, $\text{Re } f_0$ and $\text{Im } f_0$, are taken from the fit to RHIC CF that corresponds to Fig. 6 from Ref. [4], where the HKM source radius $r_0 = 3.28$ fm and the Gaussian parametrization (6) for the residual correlation contribution $C_{\text{res}}(k^*)$ are utilized. For the LHC fit the $C_{\text{res}}(k^*)$ parameter β coincides with that for RHIC, while parameter R is scaled by the factor $r_0^{\text{LHC}}/r_0^{\text{RHIC}}$. The LHC fit is determined up to uncertainty, caused by the errors in parameters $\text{Re } f_0$, $\text{Im } f_0$, β , and R (this is illustrated by the contour around the LHC curve).

The first one demonstrates the model CF with $\lambda_{\text{LHC}}(k^*)$ the same as for RHIC [11], $\lambda_{\text{LHC}}(k^*) = \lambda_{\text{RHIC}}(k^*)$. This function is again more narrow than for RHIC. In Fig. 4 one can see our prediction for the LHC correlation function with $\lambda_{\text{LHC}}(k^*) = 2.5\lambda_{\text{RHIC}}(k^*)$, where the factor 2.5 corresponds to the ratio of the corresponding primary pair fractions in HKM and in the STAR experiment. Figure 5 shows the purity and residual correlation-corrected CFs for LHC and RHIC. They are expressed [as follows from Eq. (5)] through the uncorrected ones as $C(k^*) = 1 + [C_{\text{uncorr}}(k^*) - 1]/\lambda(k^*) - \alpha(k^*)[C_{\text{res}}(k^*) - 1]/\lambda(k^*)$. The latter so-called true function

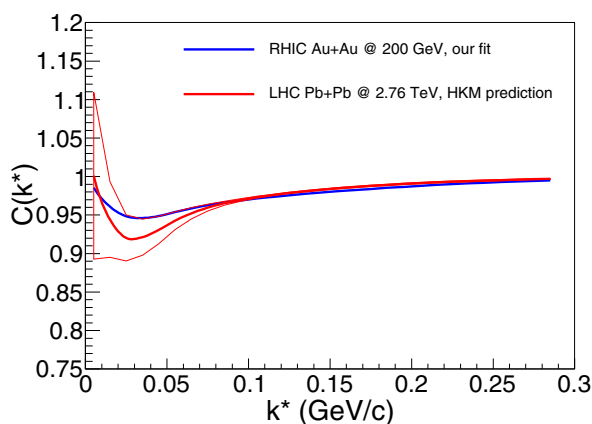


FIG. 4. (Color online) Same as in Fig. 3, but LHC purity $\lambda(k^*)$ is scaled as compared to RHIC according to the primary $p\Lambda$ pair fraction in HKM simulations, where there is no misidentification problem.

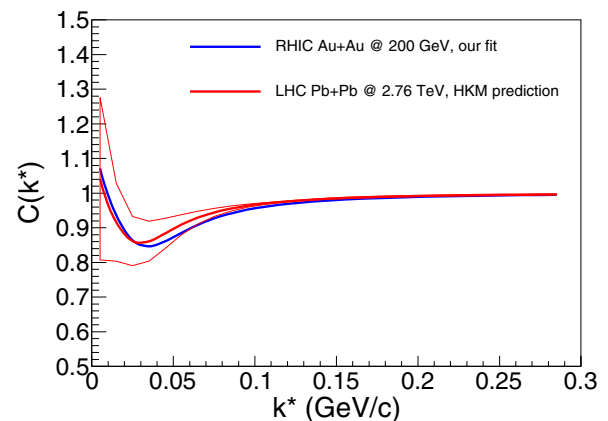


FIG. 5. (Color online) Same as in the previous two figures, but the correlation functions are corrected for purity and residual correlations, i.e., $C(k^*) = 1 + [C_{\text{uncorr}}(k^*) - 1]/\lambda(k^*) - \alpha(k^*)[C_{\text{res}}(k^*) - 1]/\lambda(k^*)$.

can be easily compared with the experimental result (corrected in the same way for purity and residual correlations), since it does not depend on the fraction of misidentified particles in the concrete experiment. As compared to RHIC, the LHC curve apparently has smaller amplitude and width.

IV. CONCLUSIONS

The first predictions for $p\Lambda$ and $\bar{p}\Lambda$ correlation functions, calculated within Lednický-Lyuboshitz and hydrokinetic (HKM) models and accounting for the residual correlation effect, are presented for 5% most central Pb+Pb LHC collisions at the energy $\sqrt{s_{NN}} = 2.76$ TeV. The functions' behavior is predicted based on the results previously obtained for Au+Au collisions at the top RHIC energy.

Both $p\Lambda$ and $\bar{p}\Lambda$ correlation function curves in the LHC case are slightly more narrow than those at RHIC.

The LHC source radii r_0 calculated in HKM for baryon-baryon and baryon-antibaryon cases are similar, $r_0 = 3.76$ fm. They are about 1.15 times larger than the corresponding RHIC radii.

The pair purities (the fractions of pairs consisting of primary particles), calculated in HKM for RHIC and LHC, are very close and are about 2.5 times larger than the experimental primary pair fraction in the STAR Collaboration experiment at the RHIC. This difference is most probably because of particle misidentification in the experiment. That is why we present results for purity uncorrected $\bar{p}\Lambda$ correlation functions in two variants: with purity $\lambda(k^*)$ similar to that in the experiment at RHIC and with RHIC purity scaled by a factor of 2.5, which corresponds to HKM purity. The measured LHC correlation function should lie somewhere between these two limiting possibilities. Depending on the concrete experimental setup and the related fraction of misidentified particles, the real curve can be closer to one or another variant.

We also demonstrate the purity and residual correlation corrected baryon-antibaryon CF which should not depend on the experiment details and thus can be easily compared with experimental result. This function for LHC has smaller amplitude than for RHIC.

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3.3. Кореляції у парах $p\Xi$ з урахуванням кулонівської взаємодії

PRODUCTION AND CORRELATIONS OF STRANGE MESONS AND BARYONS AT RHIC AND LHC IN HYDROKINETIC MODEL

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Dedicated to Andrzej Bialas in honour of his 80th birthday

The recent results on the theoretical analysis of particle production and correlation in relativistic heavy-ion collisions at the LHC and RHIC within the hydrokinetic model (HKM) and its extended version — integrated hydrokinetic model (iHKM) are addressed. The study of strange K meson spectrum and femtoscopy scales is discussed along with the pion ones for the case of LHC Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV. The m_T -dependence of spectra and longitudinal femtoscopy scales at the LHC, obtained in HKM simulations, is compared with the results given by simple analytical formulas including the effective temperature on the hypersurface of maximal particle emission, emission proper time, and transverse flow intensity. The influence of $K^*(892)$ resonance decays and hadron re-scatterings at the afterburner stage of the collision on the interferometry radii is analyzed. The related problem of $K^*(892)$ effective identification and reliable yield measurement in view of hadron re-scatterings is also investigated for RHIC and LHC energy cases. The application of the FSI formalism with account for residual correlation effect to modeling of the $p\Lambda$ and $p\Sigma$ correlation functions using the source functions calculated in HKM is also considered.

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1. Introduction

The study of spatiotemporal extents of the emission sources for different particle species, produced in relativistic heavy-ion collisions, serves as a good instrument for the study of the properties of hot and dense matter created in these processes, since the structure of particle emission is essentially defined by the collision dynamics [1–4]. The most common direct method for such an analysis is the correlation femtoscopy or intensity interferometry [5], which allows one to measure the interferometry radii, associated with the homogeneity lengths of a particle emission region. They are usually extracted from a Gaussian fit to the two-particle momentum correlation function. Determining the dependence of the femtoscopy scales on pair transverse average momentum $k_T = |\mathbf{p}_{T1} + \mathbf{p}_{T2}|/2$ or transverse mass m_T ($m_T^2 = m^2 + (|\mathbf{p}_{T1} + \mathbf{p}_{T2}|/2)^2 = m^2 + k_T^2$) for different particle species allows to test the applicability of different matter evolution scenarios and particle emission pictures in the collision process. For instance, if one supposes the hydrodynamic approximation with negligible transverse flow to be justified for $A + A$ collision, then the longitudinal radii $R_L(m_T)$ should have similar $m_T^{-1/2}$ behavior for pions and kaons, even up to complete m_T -scaling in the common freeze-out case [2, 3]. However, in reality, the particle emission picture is much more complicated and the results are a product of interplay of a number of various factors, and it requires a thorough analysis to be performed in a very detailed $A + A$ collision model.

The hydrokinetic model (HKM) [6], which simulates the evolution process of the system formed in relativistic heavy-ion collisions, was designed to describe and predict different bulk observables measured in experiments at RHIC and LHC. In particular, a good description was obtained in HKM for pion, kaon, proton and all charged particle spectra at different centralities together with the corresponding elliptic flows [6]. A prediction as for reduction of the $R_{\text{out}}/R_{\text{side}}$ ratio at the LHC as compared to the RHIC was made in HKM [7] and then experimentally confirmed. The pion and kaon femtoscopic scales at RHIC [6, 8] and LHC [6, 9] were also calculated in the model. Moreover, not only the interferometry radii, but also the non-Gaussian source functions for pions and kaons were successfully described for the top RHIC energy and predicted for the LHC one [10]. In addition to the results related to heavy-ion collision, a good description of pion interferometry radii in the LHC $p + p$ collisions at $\sqrt{s} = 7$ TeV was reached in hydrokinetic model with the quantum uncertainty principle corrections applied to a quasi-classical event generator results [11].

As for the m_T -scaling between pion and kaon femtoscopy radii at the LHC, HKM predicts [9] that it is violated, and the k_T scaling (for $k_T > k_0 \approx 0.4$ GeV/ c) takes place instead. The reasons for such a behavior are

analyzed within HKM in [12] and the results of that study are discussed in the current paper below. It appears that the breaking of m_T -scaling is caused by the interplay of the following factors: strong transverse collective flows, non-Gaussian shape of the correlation function, particle re-scatterings and resonance decays at the afterburner stage of the system's evolution, where produced particles continue to re-scatter and annihilate, gradually escaping from the fireball to finally travel towards the detectors without interaction.

The intensity of particle collisions at the afterburner stage can be probed by analyzing the effectiveness of strange $K(892)^*$ resonances registration by their decays into $K\pi$ pairs. The point is that the interactions between daughter hadrons do not allow to identify all the produced K^* s, so the resulting fraction of restored resonances strongly depends on the hadron interaction intensity at 3–5 fm/c after the afterburner stage beginning. In addition, the experimental particle number ratios, regarded as a source of information about the collision dynamics and the properties of matter at the chemical freeze-out (at the temperature $T \approx 160$ MeV), also diverge from their true values. In this paper, we consider how the hadronic re-scatterings at the afterburner stage affect the observability of strange $K(892)^*$ resonance utilizing the integrated hydrokinetic model (iHKM) for the case of Pb+Pb collisions at the LHC. We consider different centrality classes at the collision energy $\sqrt{s_{NN}} = 2.76$ TeV. The calculation results for the LHC case are then compared with experimental data from RHIC presented by the STAR Collaboration.

At last, we deal with the problem of the final state interaction (FSI) analysis of baryon–(anti)baryon correlations. Namely, taking the results obtained earlier within the Lednicky–Lyuboshitz analytical model for the $p\Lambda$ case as a starting point, we present theoretical predictions as for the $p\Xi$ correlation functions (CF), obtained within the so-called Koonin–Pratt (KP) formalism. In this approach, the pair CF is expressed through the squared modulus of the pair relative wave-function averaged over the Gaussian time-integrated pair separation distribution (the source function). The latter is defined by the source radius, extracted from the fit to the source function calculated in iHKM. The effect of residual correlations is also taken into account using an effective analytical approximation introduced as an additional term into the model expression for the correlation function.

2. π and K meson spectra and femtoscopy scales

2.1. Analytical formula for femtoscopy scales

The system's evolution simulation in hydrokinetic model consists of two stages — the hydrodynamic expansion of the (locally) thermally and chemically equilibrated matter and gradual system decoupling, which starts when

the equilibrium gets lost. For the description of the first stage, the ideal hydrodynamics approximation is utilized and the second one is described within the hydrokinetic approach, based on the integral Boltzmann equations. Then, on some spacelike hypersurface, one performs switching to UrQMD hadronic cascade [13]. For the calculations presented here, the simplified HKM variant is utilized [14], where a sudden switch is performed from collective matter evolution to a particle cascade at the hadronization hypersurface, chosen to be the isotherm $T = 165$ MeV. As a result of the model, one gets a set of produced particle momenta and coordinates that serve for building necessary observables.

The initial conditions for the hydro evolution in HKM are set by specifying the initial transverse energy density profile $\epsilon_0(\mathbf{r}_T)$ and the initial transverse flow $y_0(\mathbf{r}_T)$ at the initial proper time τ_0 . For the case of $A + A$ collisions, we take $\tau_0 = 0.1$ fm/c, and $\epsilon_0(\mathbf{r}_T)$ is generated in GLISSANDO MC Glauber code [15]. The normalization constant ϵ_0 , being the maximal initial energy density, is fixed from the mean charged particle multiplicity in the considered experiment.

In order to analyze the role of different factors in defining the particle emission picture in HKM and find the way of determination of the approximate particle maximal emission time in the experiment, we consider here the logic of the spectra formation and some analytical estimates for the spectra and femtoscopy scales. In the hydrodynamic approach based on Boltzmann equations [16, 17], it can be shown that, although the freeze-out in general case has a finite time width, the momentum spectra are still well-described within the Cooper–Frye prescription (CFp) [18]. In such a case, the freeze-out duration is defined by the inverse of the particle collision rate at the maximal emission points for particles with momentum p , $(t_\sigma(\mathbf{r}, p), \mathbf{r})$. In most cases, the hypersurface consisting of all such points does not completely enclose the originally dense matter. Since for each concrete p the related hypersurface piece σ_p is always spacelike, the utilization of such generalized Cooper–Frye prescription allows to avoid automatically the general problem of standard CFp, namely the negative contributions to the particle spectrum from non-spacelike parts of the common freeze-out hypersurface.

Following the results of [17, 19, 20], let us regard the hypersurface $\tau = \tau_{\text{m.e.}} = \text{const}$ of constant proper time (limited in transverse dimension $\mathbf{r}_T$) as the hypersurface of maximal emission for particles with comparably soft momenta p (for the hard momentum particles with $p_T > 0.8$ GeV/c, such hypersurface is not suitable due to space-time correlation presence [19, 20]). Then, the bosonic Wigner function in most central events will be given by

$$f_{\text{l.eq.}}(x, p) = \frac{1}{(2\pi)^3} [\exp(\beta p \cdot u(\tau_{\text{m.e.}}, \mathbf{r}_T) - \beta\mu) - 1]^{-1} \rho(\mathbf{r}_T) \quad (1)$$

with the Gaussian cutoff factor [19, 20], suppressing the contribution from high-momentum particles, emitted from rapidly moving fluid elements with $\cosh \eta_T(r_T) \gg 1$,

$$\rho(\mathbf{r}_T) = \exp[-\alpha(\cosh \eta_T(r_T) - 1)]. \quad (2)$$

Here, $\beta = 1/T$ is the temperature reciprocal at $\tau_{\text{m.e.}}$, $x^\mu = (\tau \cosh \eta_L, \mathbf{r}_T, \tau \sinh \eta_L)$ is the 4-coordinate with longitudinal rapidity $\eta_L = \text{arctanh } v_L = \frac{1}{2} \ln \frac{t+x_L}{t-x_L}$, transverse rapidity $\eta_T = \text{arctanh } v_T(r_T)$, and $u^\mu(x) = (\cosh \eta_L \cosh \eta_T, \frac{\mathbf{r}_T}{r_T} \sinh \eta_T, \sinh \eta_L \cosh \eta_T)$ — the hydrodynamic velocity.

According to [19, 20] and in the view of improved CFp [17], the parameter $\alpha = R_v^2/R_T^2$, where R_T is the transverse homogeneity length in r_T (near $r_T = 0$ for small k_T) along the hypersurface $\tau = \tau_{\text{m.e.}} = \text{const}$, and R_v is the hydrodynamic length, $R_v = (v'(r_T))^{-1}$, near the same r_T . The small α means a very intensive flow, such that the hydrodynamic length R_v appears to be much smaller than the homogeneity length R_T . The case of very large α corresponds to $R_v = \infty$, *i.e.* absence of the transverse flow.

Within introduced formalism, the boson single particle spectrum and the correlation function will be written as follows:

$$p_0 \frac{d^3N}{d^3p} = \int_{\sigma_{\text{m.e.}}(p)} d\sigma_\mu p^\mu f_{\text{l.eq.}}(x, p), \quad (3)$$

$$C(p, q) \approx 1 + \frac{\left| \int_{\sigma_{\text{m.e.}}(k)} d\sigma_\mu k^\mu f_{\text{l.eq.}}(x, k) \exp(iqx) \right|^2}{\left(\int_{\sigma_{\text{m.e.}}(k)} d\sigma_\mu k^\mu f_{\text{l.eq.}}(x, k) \right)^2}, \quad (4)$$

where $q = p_1 - p_2$, $k^\mu = \left(\sqrt{m^2 + \left(\frac{\mathbf{p}_1 + \mathbf{p}_2}{2} \right)^2}, \frac{\mathbf{p}_1 + \mathbf{p}_2}{2} \right)$. Here the smoothness and mass shell approximations are applied, so that $k \approx p = (p_1 + p_2)/2$ and the 4-vector q has only three independent components. The latter can be selected along the beam axis, $q_{\text{long}} \equiv q_l$, along the pair transverse momentum vector $\mathbf{k}_T$, $q_{\text{out}} \equiv q_o$, and along *side* direction, $q_{\text{side}} \equiv q_s$, orthogonal to both *long* and *out* directions.

Applying then the saddle point method (in complex plane) to the calculation of the spectra and correlation function (4) in Boltzmann approximation for longitudinally boost-invariant expansion, one will obtain in LCMS [21]

$$C(k, q_l, q_s = q_o = 0) = 1 + \frac{\exp \left[\frac{2}{\lambda^2} \left(1 - \sqrt{1 + \tau^2 \lambda^4 q_l^2} \right) \right]}{[1 + \tau^2 \lambda^4 q_l^2]^{3/2}} \xrightarrow{k_T \rightarrow \infty} 1 + \exp(-\lambda_L^2 q_l^2), \quad (5)$$

where according to [19, 20], λ is defined as

$$\lambda^2 = \frac{\lambda_L^2}{\tau^2} = \frac{T}{m_T} (1 - \bar{v}_T^2)^{1/2}. \quad (6)$$

Here, $\lambda_L = \tau \sqrt{\frac{T}{m_T} (1 - \bar{v}_T^2)^{1/2}}$ is the longitudinal homogeneity length in presence of the transverse flow, $\bar{v}_T$ is the transverse velocity in the saddle point, $\bar{v}_T = k_T / (m_T + \alpha T)$, $T = T_{\text{m.e.}}$ is the temperature at the hypersurface of maximal emission, $\tau = \tau_{\text{m.e.}}$. The dependence on q_o and q_s is neglected since the correlation function behavior in *long* direction does not depend on the transverse velocity profile at the hypersurface of maximal emission, depending only on the parameter α . The *out* and *side* projections are, on the contrary, quite sensitive to the details of the velocity profile.

At large $m_T/T \gg 1$, when the correlation function shape is Gaussian, the interferometry radii coincide with homogeneity lengths. In the case of pure Bjorken expansion (no transverse flow, $\bar{v}_T = 0$), one has $R_L = \lambda_L = \tau \sqrt{\frac{T}{m_T}}$ (first obtained in [2]). In [22], the authors proposed the correction to the radii for small $m_T/T \approx 1$, probably applicable for pions at zero transverse flow

$$R_L^2 = \lambda_L^2 \rightarrow \lambda_L^2 \frac{K_2\left(\frac{m_T}{T}\right)}{K_1\left(\frac{m_T}{T}\right)}, \quad (7)$$

where K_n are modified Bessel functions. As one can see from (5), at $m_T/T \approx 1$, the correlation function is essentially non-Gaussian, so the radius R_L describes only the peak of the correlation function, that corresponds to the Gaussian approximation at small q_l , $C(q_l) = 1 + \exp(-R_L^2 q_l^2)$ ¹.

Supposing boost-invariant longitudinal expansion, from (5) at small q_l one can obtain for the radii at transverse flow with arbitrary velocity profile

$$R_L^2(k_T) = \tau^2 \lambda^2 \left(1 + \frac{3}{2} \lambda^2\right). \quad (8)$$

Comparing this result with (7) for pure Bjorken picture ($\alpha \rightarrow \infty$ in our notation), we find that the difference between two results in all actual k_T intervals for pion pairs is only 1–3%. However, our expression (8) is justified for the case of collective flows of arbitrary intensity, which is important when describing the LHC collisions.

2.2. Fitting the HKM results

Analyzing the m_T -dependence of R_L in the approximation of 1D hydrodynamics, one finds that m_T -scaling should take place — such radius

¹ For the detailed analysis of different analytical approximations for the femtoscopy scales, see [21].

behavior was obtained already in [2], namely $R_L \propto \sqrt{T/m_T}$. Application of (7) and (8) in this case (it corresponds to $\alpha = \infty \Rightarrow \bar{v}_T = 0$) also results in scaling, $R_L = R_L(\tau, m_T/T)$.

However, the results of HKM simulations for LHC energies show the absence of m_T scaling [9]. To investigate the physical reasons leading to this, we calculated the pion and kaon radii in HKM with UrQMD cascade switched off (but with resonance decays still implemented) and tried to fit the obtained points with (8) at $\alpha = \infty$. As one can see, in Fig. 1 the fits are quite bad ($\chi^2/\text{n.d.f.} = 2562.31$ for pions and $\chi^2/\text{n.d.f.} = 585.24$ for kaons), and there is no m_T -scaling as well as in the case of full HKM simulations.

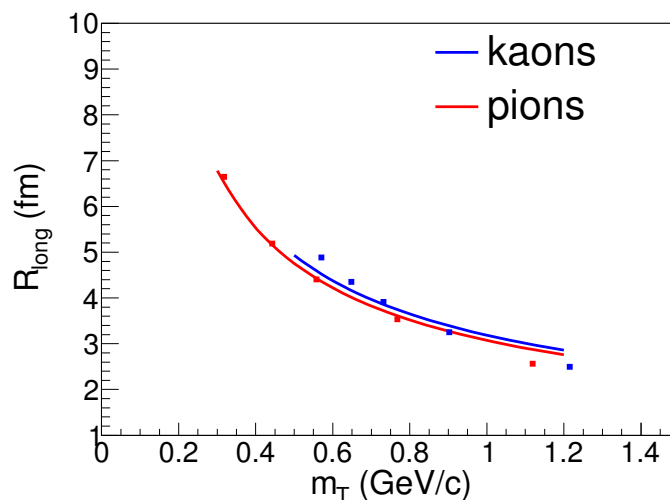


Fig. 1. The m_T dependencies of longitudinal femtoscopy radius R_L calculated in HKM model without re-scattering stage for $K^{\text{ch}}K^{\text{ch}}$ and $\pi^-\pi^-$ pairs (markers) together with the corresponding fits (lines) according to formula (8) where transverse flow is absent ($\alpha = \infty$). The temperature is $T = 165$ MeV. The results are related to $\sqrt{s_{NN}} = 2.76$ GeV Pb+Pb collisions at the LHC, $c = 0\text{--}5\%$, $|\eta| < 0.8$, $0.14 < p_T < 1.5$ GeV/c.

The quality of the fits grows dramatically if one sets α in (8) to have a finite value in order to account for non-zero transverse flow (see Fig. 2). Now, R_L depends on both m_T/T and k_T/T variables, so the scaling between analytical curves disappears. Thus, one of the reasons of m_T -scaling violation in HKM is the existence of strong transverse flow in simulations related to LHC heavy-ion collision experiments. The satisfactory pion and kaon radii description is obtained with the same temperature parameter value, $T = 165$ MeV, and maximal emission times $\tau_\pi = 7.41$ fm/c for pions and $\tau_K = 7.56$ fm/c for kaons. For both mesons, $\alpha = 2.8$.

Apart from the transverse flow, scaling can be violated because of non-Gaussian form of the corresponding correlation functions and the re-scattering interactions at the afterburner stage of system's evolution. For analyzing

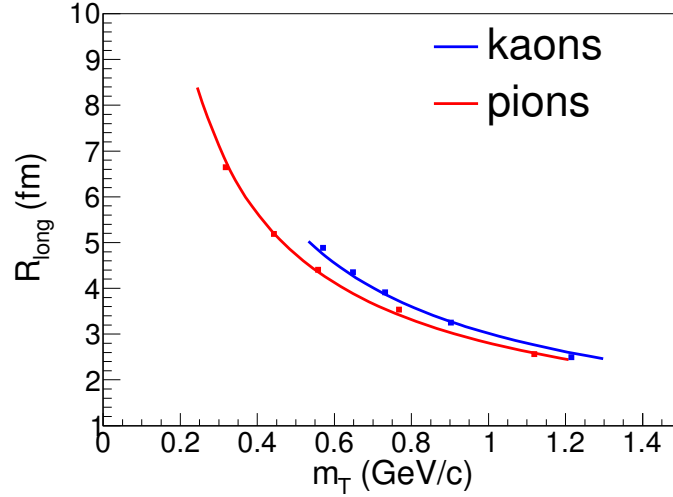


Fig.2. The same as in Fig. 1 but fits account now for the intensive transverse flow, $\alpha = 2.8$. The time parameters for pions and kaons are $\tau_\pi = 7.41$ fm/ c and $\tau_K = 7.56$ fm/ c correspondingly, $T_\pi = T_K = 165$ MeV.

the influence of these factors, the results of calculations in full HKM (with UrQMD cascade) should be used. The parameters T and α appearing in the fitting formula for radii can be fixed from the fit to respective particle momentum spectra, *e.g.* from the combined fit to pion and kaon spectra calculated in HKM with account for the transverse flow. Fitting the spectra can be carried out based on the following formula, obtained in the same approximation as (5) [20]

$$p_0 \frac{d^3 N}{d^3 p} \propto \exp \left[- (m_T/T + \alpha) (1 - \bar{v}_T^2)^{1/2} \right]. \quad (9)$$

Figure 3 shows the result of the combined pion–kaon spectrum fit (9) to HKM points together with the experimental data [23] in p_T -range of 0.5–1.0 GeV/ c . The temperature parameter was put to be the same for both particle species, and the extracted parameter values are the following: $T = 144 \pm 3$ MeV, $\alpha_\pi = 5.0 \pm 3.5$ and $\alpha_K = 2.2 \pm 0.7$. The obtained values with the respective errors are substituted then to (8) in order to fit the femtoscopy scales m_T -dependence. Since the formulas (7) and (8) are related to the Gaussian radii, describing only the small q region when the correlation function is non-Gaussian, we use the limited fitting range, $q = 0$ –0.04 GeV/ c , at extraction the interferometry radii from the full HKM. The corresponding radii m_T dependence is shown in Fig. 4 together with the analytical fit according to Eq. (8), with T and α constrained based on the combined spectra fit. The pion and kaon times of maximal emission extracted from the fit are $\tau_\pi = 9.44 \pm 0.02$ fm/ c and $\tau_K = 12.40 \pm 0.04$ fm/ c , respectively, while the rest of parameters take values $T_\pi = 147$ MeV, $T_K = 141$ MeV, $\alpha_\pi = 8.5$ and $\alpha_K = 1.5$.

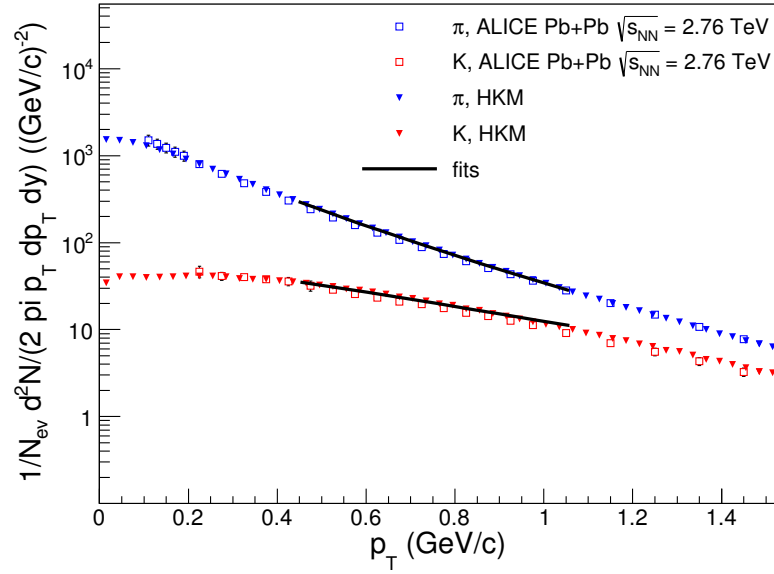


Fig. 3. Pion (upper/blue) and kaon (lower/red) momentum spectra in Pb+Pb collisions at the LHC, $\sqrt{s_{NN}} = 2.76$ GeV. Open squares show experimental values [23], triangles show HKM results, black lines correspond to combined (with the same temperature T) pion and kaon spectra fit according to (9).

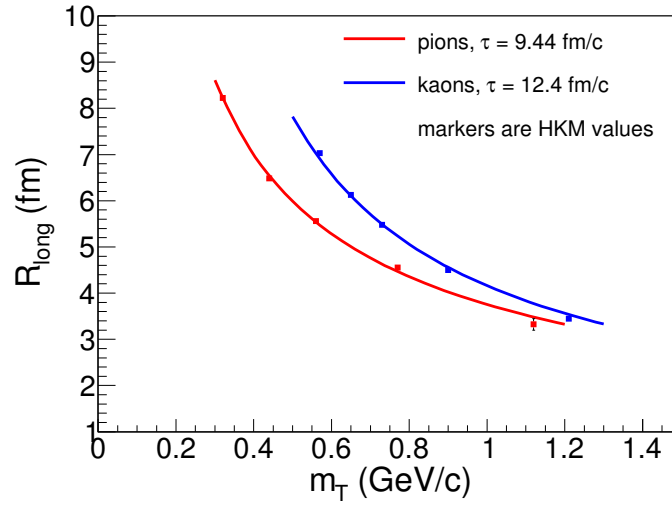


Fig. 4. The same as in Figs. 1 and 2 but radii are calculated in the full HKM model including re-scattering stage. To reduce the effect of the non-Gaussian correlation functions, we take more narrow fitting range for them, $q = 0\text{--}0.04$ GeV/ c . The fit parameters T and α correspond to combined pion and kaon spectra fitting. At $T_\pi = 147$ MeV, $T_K = 141$ MeV, $\alpha_\pi = 8.5$ and $\alpha_K = 1.5$, extracted maximal emission times are $\tau_\pi = 9.44 \pm 0.02$ fm/ c and $\tau_K = 12.40 \pm 0.04$ fm/ c .

Another way of comparison of the pion and kaon emission pictures in HKM is examining the plots based on the respective invariant emission functions $G(x, p) = p^0 \frac{d^7N}{d^4x d^3p}$. In Fig. 5, one can see the reduced emission function

averaged over all momentum angles $g(r_T, \tau; p_T) = \frac{p^0 d^6 N}{dr_o dr_s d\eta d\tau dp_T dy} \Big|_{r_s=0}$ for pions and kaons at $0.2 < p_T < 0.3$ GeV/ c in the central rapidity bins for both space-time η and momentum y rapidities². The times of the maximal emission extracted from the fits to interferometry radii are in good agreement with the presented HKM emission picture. As one can notice, the kaon radiation has two maxima: the first one at the proper time $\tau = 10$ fm/ c and the second, broader and less pronounced, is at $\tau \approx 14$ –15 fm/ c . The origination of this second local maximum must be connected with decays of strange $K^*(892)$ resonance into $K\pi$ pairs containing additional $K^\pm(493.7)$ particles. Then, how the single obtained fit parameter $\tau_K \approx 12$ fm/ c should be interpreted? Apparently, it can be understood as some effective (or mean) maximal emission time for kaons, corresponding to the actual two emission time maxima.

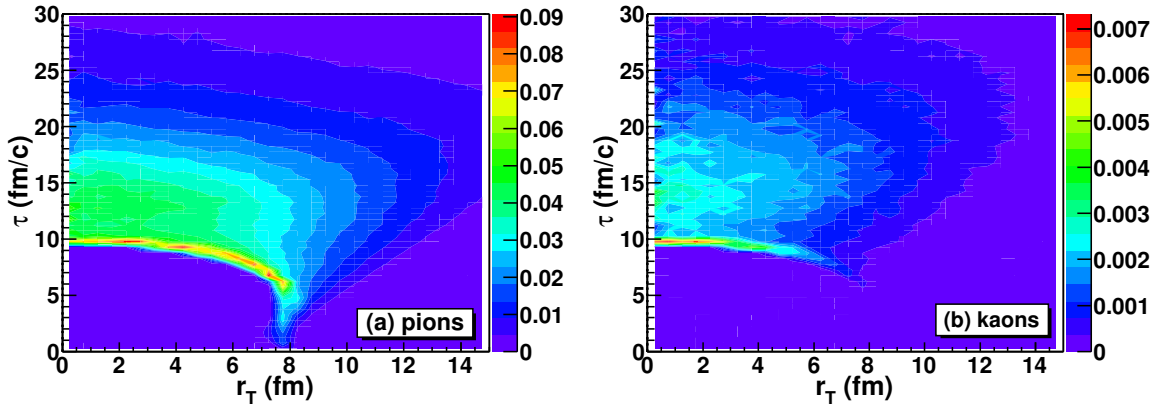


Fig. 5. The momentum angle averaged emission functions per units of space-time and momentum rapidities $g(\tau, r_T, p_T)$ [fm^{-3}] (see the body text) for pions (a) and kaons (b) obtained from the HKM simulations of Pb+Pb collisions at the LHC $\sqrt{s_{NN}} = 2.76$ GeV, $0.2 < p_T < 0.3$ GeV/ c , $|y| < 0.5$, $c = 0$ –5%.

This “mean” time is larger than the time of pion maximal emission, and the inequality between two values is much bigger in the case when hadron re-scatterings are switched on in the model (“full” mode). Such peculiarity can be explained by the consideration that in the regime when only resonance decays are implemented, the additional kaon emission source leads only to deviation of the CF shape from the Gaussian one, since fast K^* free streaming with subsequent decays into $K^\pm$ does not remind the hydrodynamic expansion of fluid elements, forming the source of primary kaons. In contrast, the re-scattering stage supposes that K^* s are involved in some kind of collective motion, thus affecting the longitudinal radius connected with the maximal emission time.

² Such a form of the emission function, in *out* and *side* r_T components, was used in [19–21] when deriving the analytical approximation for the correlation function.

As a result of performed fitting for the full HKM R_L points and the HKM momentum spectra, we arrive at different T and α parameter values for pions and kaons. It means that re-scatterings and resonance decays also lead to the m_T -scaling destruction along with already mentioned strong transverse flow. The influence of the re-scattering on violation of m_T -scaling was also noted in [24].

Now, let us return again to the HKM *long* radii behavior fitting. Previously considered points were extracted from the Gaussian fits to HKM correlation functions in a reduced q -range, $q = 0\text{--}0.04$ GeV/ c . However, in the experimental analysis, one typically utilizes a broader interval, *e.g.* $q = 0\text{--}0.2$ GeV/ c [25]. Applying (8) for fitting the HKM radii obtained using such extended range for q , one still gets a good description for pions (Fig. 6, gray/red line) with parameters constrained by combined spectra fit ($T_\pi = 141$ MeV, $\alpha_\pi = 1.82$, $\tau_\pi = 10.34 \pm 0.06$ fm/ c), but for kaons ($T_K = 141$ MeV, $\alpha_K = 1.5$, and $\tau_K = 11.09 \pm 0.02$ fm/ c), the description is unsatisfactory (see Fig. 6, solid black/blue line). In order to get an adequate fit for the kaon femtoscopy radii related to a wide q -region, one should remove restrictions on the α_K parameter. In Fig. 6, such a fit is demonstrated with the temperature $T = 144 \pm 3$ MeV still constrained, but with free α_K and τ_K (dashed black/blue line). The extracted maximal emission time $\tau_K = 12.65 \pm 1.58$ fm/ c at $T = 146$ MeV and $\alpha = 0.02$ is in good agreement with the previous results for narrow- q radii and the particle emission structure represented by Fig. 5.

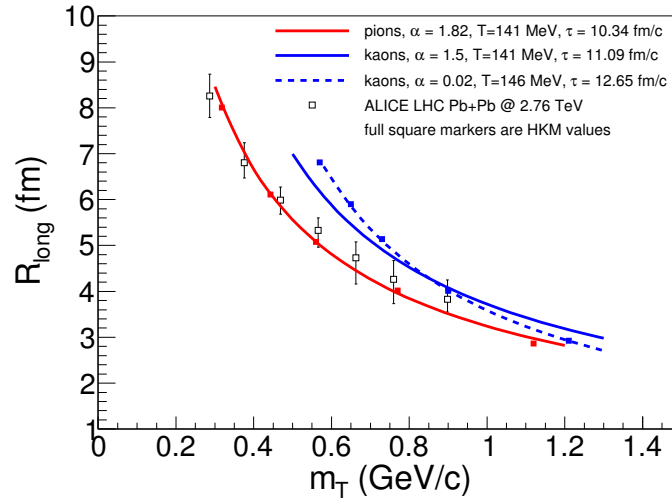


Fig. 6. The same as in Fig. 4, but the radii are extracted from the fits to full HKM correlation functions in a wide q range, $q = 0\text{--}0.2$ GeV/ c . As one can see, the significant deviation of the CF shape from the Gaussian one leads to a distorted radii m_T behavior. At fitting, it can be compensated by decreasing α_K as compared to that obtained from the spectra fit. Experimental data [25] for pions are demonstrated for comparison.

As a conclusion, the longitudinal femtoscopy scales corresponding to the peak of the non-Gaussian correlation function can be well-described by the T and α parameters extracted from the combined pion and kaon spectra fit, giving a reliable estimate for the times of maximal particle emission. Still good estimates can be reached fitting the CF in a wide q range, however, the α_K parameter for kaons should be left unconstrained. As for the very small resulting α_K in this case, it can indicate that due to strong influence of the re-scattering processes at the afterburner stage of the collision, the interferometry radius m_T behavior cannot be described in a pure hydrodynamic approach. The deviation of the correlation function shape from the Gaussian one becomes more pronounced due to re-scatterings. Thus, the re-scattering stage plays a quite important role in m_T scaling violation.

In addition, we present the results for the source radii r_0 dependence on m_T for different meson and baryon particle pairs (see Fig. 7). These radii were extracted from the Gaussian fits to the angle-averaged HKM source functions $S(r) = 1/(4\pi) \int_0^{2\pi} \int_0^\pi S(r, \theta, \phi) \sin \theta d\theta d\phi$. The points in the figure present HKM points, and the lines show the fits (8) to these points. The parameter T at the fitting was constrained in accordance with the combined pion-kaon spectra fit result $T = 144 \pm 3$ MeV, while α and τ were left to vary freely. With the parameters $T = 141$ MeV for all the particle pairs, $\tau_{\pi\pi} = 11.47 \pm 0.03$ fm/c, $\tau_{KK} = 11.26 \pm 0.04$ fm/c, $\tau_{pp} = 11.30 \pm 0.13$ fm/c, $\tau_{p\Lambda} = 12.44 \pm 0.29$ fm/c, the fits describe the HKM results unexpectedly good. The parameter α for all the pairs is about 10^4 and more, *i.e.* actually $\alpha \rightarrow \infty$. This simply means that there are no transverse flows in the pair

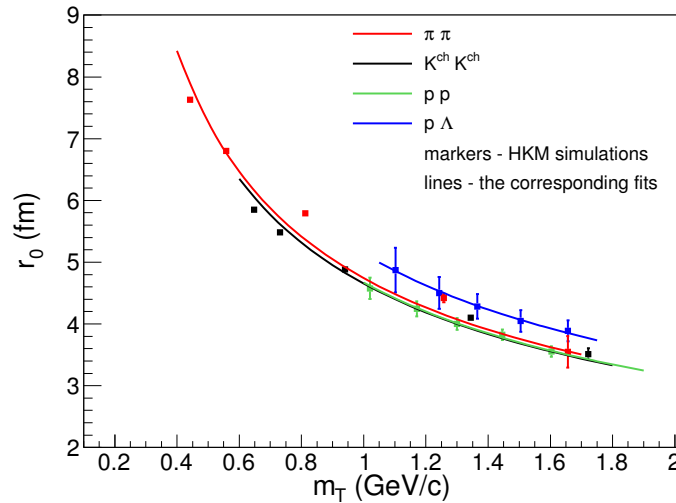


Fig.7. The m_T dependencies of $\pi\pi$, $K^{\text{ch}}K^{\text{ch}}$, pp and $p\Lambda$ source radii r_0 extracted from corresponding angle-averaged source functions calculated in HKM for $\sqrt{s_{NN}} = 2.76$ GeV Pb+Pb collisions at the LHC, $c = 0-5\%$, $|\eta| < 0.8$. Transverse momentum ranges are $0.14 < p_T < 1.5$ GeV/c for pions and kaons, $0.7 < p_T < 4.0$ GeV/c for protons and $0.7 < p_T < 5.0$ GeV/c for Lambdas.

rest frame. The transverse velocity saddle point for fluid elements in rest is naturally zero. To our surprise, we see that for demonstrated radii, the m_T -scaling takes place.

3. Strange K^* resonance identification in iHKM

The clarification of the re-scattering stage role in the kaon emission picture is closely connected with the problem of $K(892)^*$ resonance experimental identification. Being itself the source of secondary kaons, the K^* can also serve as a probe for properties of the system, formed in heavy-ion collision at the late stages of its evolution.

In current section, we present the results of simulations in the integrated hydrokinetic model (iHKM) [26] concerning the $K(892)^*$ yields in Pb+Pb collisions at the LHC energy $\sqrt{s_{NN}} = 2.76$ TeV for the case of 8 centrality classes: $c = 0$ –5%, $c = 5$ –10%, $c = 10$ –20%, $c = 20$ –30%, $c = 30$ –40%, $c = 40$ –50%, $c = 50$ –60%, and $c = 60$ –70%. The iHKM model is an improved version of HKM, which includes, in addition to the latter, an energy-momentum transport model of the prethermal stage of the matter evolution in the collision and the viscous (not ideal as in HKM) hydrodynamics. The results below are obtained in the full publication being prepared [27].

Experimental identification of $K(892)^{*0}$ resonances is performed through the track forks of their decays into $K^+\pi^-$ pairs. Such restoration is complicated by several factors. First, because of comparably (with long-lived resonances) short $K(892)^*$'s lifetime (about 4 fm/c), the intensive re-scattering processes occurring at the “afterburner” stage of the matter evolution results in the two opposing effects:

- (a) the reduction of the $K\pi$ pairs number identified as K^* due to momentum diffusion of kaons and pions in the daughter pair, and
- (b) the enhancement of K^* yield caused by pseudo-elastic interactions between hadrons, producing extra resonances.

Also the kaon–pion correlations of various origins, *e.g.* event-by-event elliptic flow and residual ones, are subject to misinterpretation and may be taken for the K – π bound state. The experimental misidentification issue also plays some role here. And at last, the imperfection of the invariant mass criterion, applied to the relevant $K\pi$ pairs selection may result in rejection of the pairs of interest or accept of the wrong pairs.

The joint impact of the first two competing effects on the K^* identification effectiveness can be studied within iHKM framework. For this purpose, one calculates the two K^* numbers: the number of actual K^{*0} particles, generated at the hadronization hypersurface and coming from the resonance

decays, and the number of $K^+\pi^-$ pairs, identified as coming from K^{*0} decay after the UrQMD cascade stage³. The rapidity and transverse momentum cuts for $K^+\pi^-$ pairs are chosen to be $|y| < 0.5$ and $0.2 < k_T < 10$ GeV/ c . We use the following criterion for the pair $K^+\pi^-$ relevance to K^{*0} decay: all the spatial coordinates of particle last collision points have to differ by less than 0.01 fm, $|x_i^K - x_i^\pi| < 0.01$ fm, while the pair invariant mass should differ from the invariant mass of $K(892)^{*0}$, which is $M_{K^*} = 895.94$ MeV, by less than 100 MeV.

The dependence of the fraction of “identified” K^* s obtained in iHKM on the collision centrality is demonstrated in Fig. 8 for Pb+Pb processes at the LHC energy $\sqrt{s_{NN}} = 2.76$ TeV. As one can see, this fraction increases from 0.78 for 5% most central collisions to 0.92 for the peripheral collisions with $c = 60$ –70%. Actually, this is an expected behavior — in the central collisions rather extensive, hot and dense long-living system is formed, transforming subsequently into strongly re-scattering particles, that leads eventually to the problems with K^* identification. Such situation does not take place in the periphery collisions, where most of the nucleons in colliding nuclei pass near each other without intensive interaction, and effective interaction volume is relatively small.

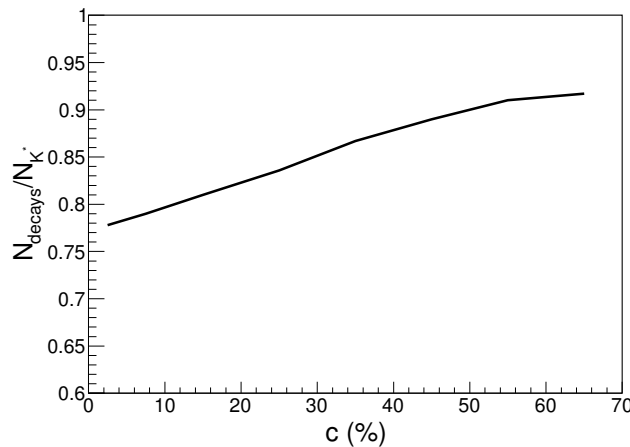


Fig. 8. The fraction of $K^+\pi^-$ pairs coming from $K(892)^*$ decay, which can be identified as K^* daughters in iHKM simulations after the particle re-scattering stage modeled within UrQMD hadron cascade. The simulations correspond to LHC Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV with different centralities.

The reason for the fraction of identifiable K^* not to reach unity even for essentially non-central collisions may lie in the imperfection of invariant mass criterion. Integrating the Breit–Wigner invariant mass distribution for $K(892)^{*0}$ in the range of $(M_{K^*} - 100 \text{ MeV}, M_{K^*} + 100 \text{ MeV})$, one will

³ Usually in the experiment, the $\bar{K}^{*0}$ and $K^{*\pm}$ yields are analyzed as well, however our simulations show that the identification efficiency for these resonances is very close to the results for K^{*0} , so in what follows, we will mention only K^{*0} s, implying that the same concerns also $\bar{K}^{*0}$ and $K^{*\pm}$ resonances.

obtain the result close to 0.85. It means that in theory, only 85% of K^* s are restorable using the described criterion. A possible explanation of a larger value 0.92, obtained in iHKM, can be related to misidentification of $K\pi$ pairs, coming from other resonance decays with close invariant mass values and sufficiently large widths, as coming from $K(892)^{*0}$ decays.

The analogous model simulations for the long-living resonances, such as $\phi(1020)$ with lifetime about 50 fm/ c which decays into K^+K^- and $K_L^0K_S^0$ pairs, demonstrate a rather good observability of this particles for all the considered centrality classes. The fraction of identified $\phi(1020)$ is even about 10% larger than unity, that can be the manifestation of KK correlations. Such a result indicate that lifetime of $\phi(1020)$ is much larger than the duration of the afterburner stage, so the re-scatterings do not reduce the yield of these resonances, rather opposite effect takes place: the recombination due to the KK interaction at active phase of the post-hydrodynamic stage is additional source of $\phi(1020)$.

The particle number ratios, such as K^*/K^+ ratio, can be the source of information about the dynamics of particle production in the heavy-ion collisions. Here, we demonstrate the K^*/K^+ number ratios calculated in the iHKM model for the LHC and RHIC energies. The calculations are performed at the two different stages: on the hadronization hypersurface (which is isotherm $T = 165$ MeV in our simulations), and in the end of hadron cascade stage. The primary K^*/K ratio values for all the considered centralities are about 0.5. The final-stage results are presented in Tables I and II and Fig. 9 for the Au+Au collisions at top RHIC energy and for Pb+Pb collisions at the LHC energy $\sqrt{s_{NN}} = 2.76$ TeV. The calculation results for RHIC are compared with the corresponding experimental data [28]. Here, kaon pseudorapidity cut is applied, $|\eta| < 0.8$, and selected kaon transverse momentum is limited to $0.2 < p_T < 10$ GeV/ c . As for the K^* resonances, they were again “reconstructed” from the decays into $K^+\pi^-$ pairs with $|y| < 0.5$ and $0.2 < k_T < 10$ GeV/ c . The iHKM calculations give the results that agree with the RHIC experimental analysis data within the errors, as it can be seen from the tables and figure. In both RHIC and LHC cases, the considered particle number ratio decreases approximately twice in the end of the collision as compared to the “primordial” result at the hypersurface of hadronization. The “afterburner” K^*/K ratio slightly increases when collision centrality is decreasing.

TABLE I

The comparison of K^*/K^+ ratio calculated in iHKM for the case of RHIC Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV and the experimental data [28] for different centrality classes.

c	K^*/K STAR	K^*/K iHKM
0–10%	$0.23 \pm 0.01 \pm 0.05$	0.21
10–30%	$0.24 \pm 0.02 \pm 0.05$	0.21
30–50%	$0.26 \pm 0.02 \pm 0.06$	0.22
50–80%	$0.26 \pm 0.02 \pm 0.05$	0.23

TABLE II

The K^*/K^+ ratio as calculated in iHKM for the case of LHC Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV for the events from different centrality classes.

c	K^*/K iHKM
0–5%	0.19
5–10%	0.19
10–20%	0.20
20–30%	0.20
30–40%	0.21
40–50%	0.22
50–60%	0.23
60–70%	0.24

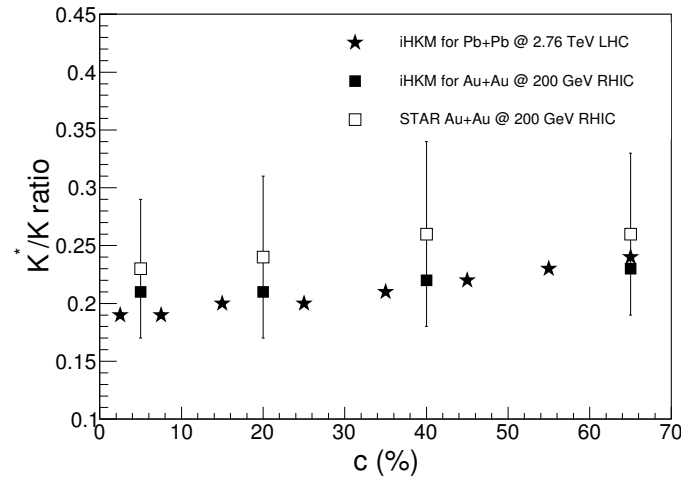


Fig. 9. The comparison of K^*/K^+ ratio calculated in iHKM for the case of RHIC Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV and the experimental data [28] for different centrality classes. The predictions for K^*/K^+ ratio in the LHC Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV for the events from different centrality classes are presented too.

4. Baryon–baryon correlations

This section is devoted to the theoretical description and predictions of baryon–(anti)baryon correlations induced by the strong and Coulomb final state interactions in such baryon pairs in relativistic heavy-ion collisions. The common approach to this issue is based on expressing the correlation function through the source function $S(\mathbf{r}^*)$ and the relative pair wave-function $\psi(\mathbf{r}^*, \mathbf{q}^*)$ using the so-called Koonin–Pratt equation [29–35]

$$C(\mathbf{q}^*) = \int d^3r^* S(\mathbf{r}^*) |\psi(\mathbf{r}^*, \mathbf{q}^*)|^2. \quad (10)$$

At typical sizes of particle emission sources, formed in the high-energy collisions, if one studies the correlation between hadrons interacting solely strongly, the Lednicky–Lyuboshitz analytical model, utilizing the asymptotic expression for $\psi(\mathbf{r}^*, \mathbf{q}^*)$ at large distances, can be successfully used for analytical correlation function approximation [29]:

$$\Psi_{-\mathbf{k}^*}^S(\mathbf{r}^*) = e^{-i\mathbf{k}^* \cdot \mathbf{r}^*} + \frac{f^S(k^*)}{r^*} e^{i\mathbf{k}^* \cdot \mathbf{r}^*}, \quad (11)$$

$$C(k^*) = 1 + \sum_S \rho_S \left[\frac{1}{2} \left| \frac{f^S(k^*)}{r_0} \right|^2 \left(1 - \frac{d_0^S}{2\sqrt{\pi}r_0} \right) + \frac{2 \operatorname{Re} f^S(k^*)}{\sqrt{\pi}r_0} F_1(2k^*r_0) - \frac{\operatorname{Im} f^S(k^*)}{r_0} F_2(2k^*r_0) \right], \quad (12)$$

where $F_1(z) = \int_0^z dx e^{x^2 - z^2}/z$ and $F_2(z) = (1 - e^{-z^2})/z$.

The particle strong interaction is characterized by the scattering amplitude $f^S(k^*)$, which in the effective range approximation is given by

$$f^S(k^*) = \left(\frac{1}{f_0^S} + \frac{1}{2} d_0^S k^{*2} - i k^* \right)^{-1}, \quad (13)$$

where f_0^S is the scattering length and d_0^S is the effective radius for a given total spin $S = 1$ or $S = 0$.

Fitting the experimental CF with (12) allows one to infer the unknown f_0^S and d_0^S parameters, defining the interaction within the pair. This procedure becomes improved if the source radius r_0 , entering (12), is extracted from the Gaussian fit to the source function, calculated in realistic collision model. In the recent papers [36, 37], such an approach, based on the source functions calculated in HKM and supplemented with the method of effective accounting for residual correlations, was applied to the description of pA and $\bar{p}A$ correlations in 10% most central Au+Au collisions at the top

RHIC energy [36] (see Figs. 10 and 11), and was used to make a prediction as for $p\Lambda$ CF in the 5% most central Pb+Pb collisions at the LHC energy $\sqrt{s_{NN}} = 2.76$ TeV [37] (see Figs. 12 and 13). The spin-averaged $p\Lambda$ scattering length value was extracted [36] from the fit to the RHIC experimental CF, measured by the STAR Collaboration [38].

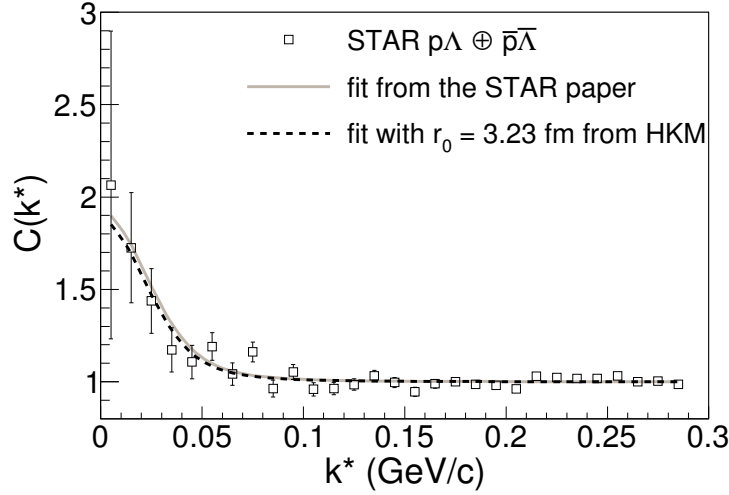


Fig. 10. The $p\Lambda \oplus \bar{p}\bar{\Lambda}$ correlation function measured by STAR (open markers), the corresponding fit according to (12) with parameters fixed as in the STAR paper [38] within the Lednický and Lyuboshitz analytical model [29] (gray solid line) and our fit within the same model with the source radius r_0 extracted from the HKM calculations (black dashed line).

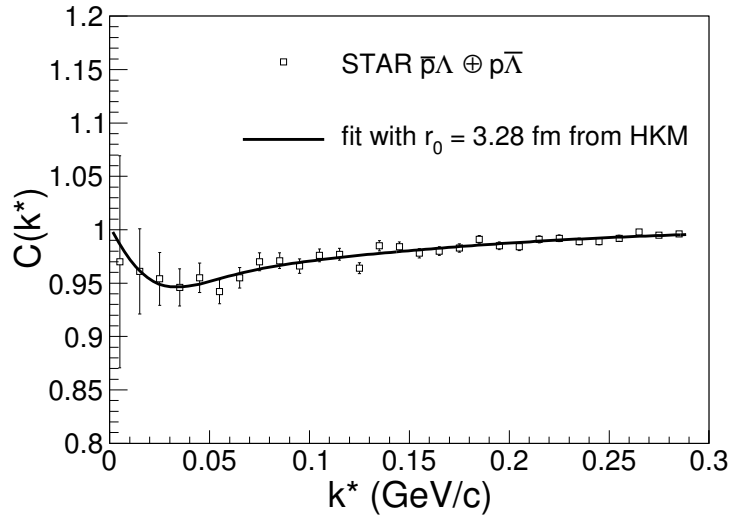


Fig. 11. The purity uncorrected $\bar{p}\Lambda \oplus p\bar{\Lambda}$ correlation function measured by STAR [39] (open markers) and our fit to it (black line), with the Gaussian parametrization (22) for the residual correlation term $C_{\text{res}}(k^*)$. The source radius r_0 was fixed at a value extracted from the HKM calculations. The extracted fit parameters are $\text{Re } f_0 = 0.14 \pm 0.66$ fm, $\text{Im } f_0 = 1.53 \pm 1.31$ fm, $\beta = 0.034 \pm 0.005$ and $R = 0.48 \pm 0.05$ fm, with $\chi^2/\text{n.d.f.} = 0.87$.

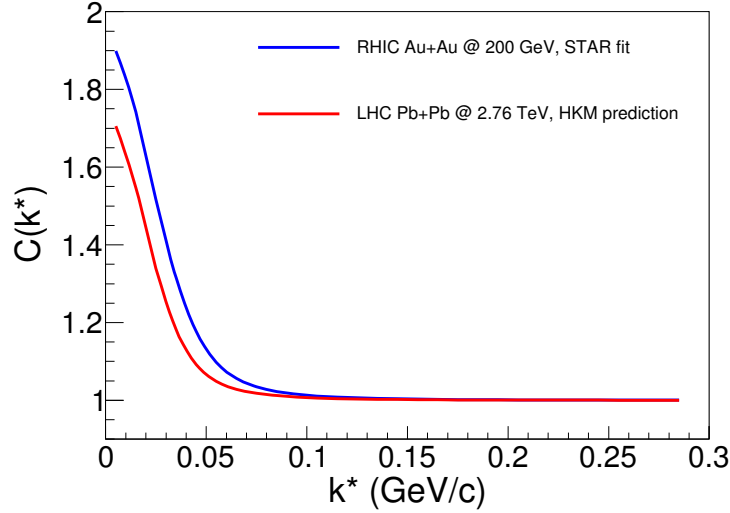


Fig. 12. The HKM prediction for purity corrected $p\text{-}\Lambda \oplus \bar{p}\text{-}\bar{\Lambda}$ correlation function in the LHC Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV, $c = 0\text{--}5\%$, $|\eta| < 0.8$, with $0.7 < p_T < 4$ GeV/c for protons and $0.7 < p_T < 5$ GeV/c for Lambdas (gray/red line). The LHC source radius value calculated in HKM is $r_0 = 3.76$ fm. The Lednický–Lyuboshitz fit to the top RHIC energy correlation function, corresponding to the STAR experiment [38], with $r_0 = 3.23$ fm extracted from the HKM source function is presented for comparison (black/blue line).

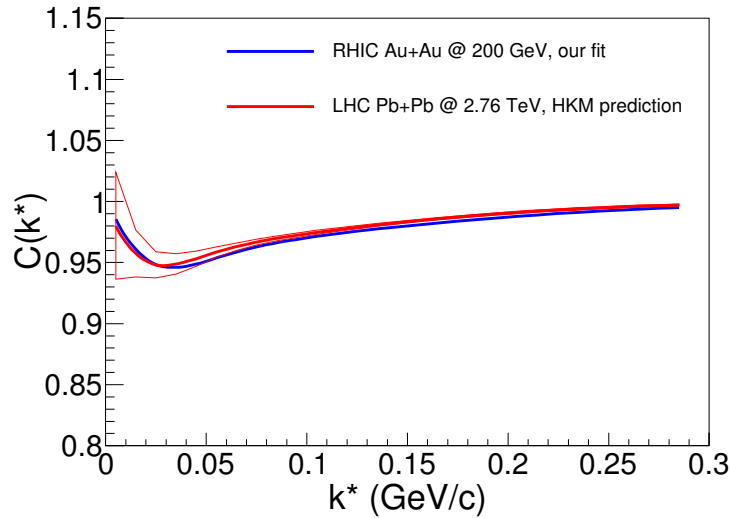


Fig. 13. The same as in Fig. 12 for purity uncorrected $\bar{p}\text{-}\Lambda \oplus p\text{-}\bar{\Lambda}$ correlation function. The HKM source radius for LHC is $r_0 = 3.76$ fm. The purity $\lambda(k^*)$ is the same as for the RHIC case [38]. The scattering length real and imaginary parts, $\text{Re } f_0$ and $\text{Im } f_0$, are taken from the fit to RHIC CF that corresponds to Fig. 6 from [36], where HKM source radius $r_0 = 3.28$ fm and the Gaussian parametrization (22) for the residual correlation contribution $C_{\text{res}}(k^*)$ are utilized. For the LHC fit, the $C_{\text{res}}(k^*)$ parameter β coincides with that for RHIC, while parameter R is scaled by the factor $r_0^{\text{LHC}}/r_0^{\text{RHIC}}$. The LHC fit is determined up to errors in parameters $\text{Re } f_0$, $\text{Im } f_0$, β and R , that is illustrated by the gray band around the LHC curve.

However, when we wish to apply the developed method to the description of the correlations in pairs, where the Coulomb interaction is also present, we should accordingly modify the theoretical formula for the correlation function. Following [40], we describe long-range Coulomb interaction with the amplitude

$$\psi_{-\mathbf{k}^*}(\mathbf{r}^*) = e^{i\delta_C} \sqrt{A_C(\eta)} \left[e^{-i\mathbf{k}^* \mathbf{r}^*} F(-i\eta, 1, i\xi) + f_C(k^*) \frac{\tilde{G}(\rho, \eta)}{r^*} \right], \quad (14)$$

where $\xi = \mathbf{k}^* \mathbf{r}^* + k^* r^* \equiv \rho(1 + \cos \theta^*)$, $\rho = k^* r^*$, $\eta = (k^* a)^{-1}$, $a = (\mu z_1 z_2 e^2)^{-1}$ is the two-particle Bohr radius including the sign of the interaction, $\delta_C = \arg \Gamma(1 + i\eta)$ is the Coulomb s-wave phase shift, $A_C(\eta)$ is the Coulomb penetration factor,

$$F(\alpha, 1, z) = 1 + \alpha z/1!^2 + \alpha(\alpha + 1)z^2/2!^2 + \dots \quad (15)$$

is the confluent hypergeometric function and $\tilde{G} = \sqrt{A_C}(G_0 + iF_0)$ is a combination of the regular (F_0) and singular (G_0) s-wave Coulomb functions

$$\tilde{G}(\rho, \eta) = P(\rho, \eta) + 2\eta\rho [\ln |2\eta\rho| + 2C - 1 + \chi(\eta)] B(\rho, \eta). \quad (16)$$

Here $C \doteq 0.5772$ is the Euler constant, the functions

$$\begin{aligned} B(\rho, \eta) &= \sum_{s=0}^{\infty} B_s, & B_0 &= 1, & B_1 &= \eta\rho, & \dots \\ P(\rho, \eta) &= \sum_{s=0}^{\infty} P_s, & P_0 &= 1, & P_1 &= 0, & \dots \end{aligned} \quad (17)$$

are defined by the following recurrence relations:

$$\begin{aligned} (n+1)(n+2)B_{n+1} &= 2\eta\rho B_n - \rho^2 B_{n-1}, \\ n(n+1)P_{n+1} &= 2\eta\rho P_n - \rho^2 P_{n-1} - (2n+1)2\eta\rho B_n, \end{aligned} \quad (18)$$

$B(\rho, \eta) \equiv F_0/(\rho\sqrt{A_C}) \rightarrow \sin(\rho)/\rho$ and $P(\rho, \eta) \rightarrow \cos(\rho)$ in the limit $\eta\rho \equiv r^*/a \rightarrow 0$. The function

$$\chi(\eta) = h(\eta) + iA_C(\eta)/(2\eta), \quad (19)$$

where the function $h(\eta)$ is expressed through the digamma function $\psi(z) = \Gamma'(z)/\Gamma(z)$ as follows

$$h(\eta) = [\psi(i\eta) + \psi(-i\eta) - \ln(\eta^2)]/2. \quad (20)$$

Composing the full relative wave function as a superposition of the Coulomb (14) and strong interaction (11) amplitudes and substituting the result into (10), where the source function is a Gaussian fit $S_{\text{fit}}(r^*) = (2\sqrt{\pi}r_0)^{-3}e^{-\frac{r^{*2}}{4r_0^2}}$ to the one-dimensional HKM source function $S(r^*) = 1/(4\pi) \int_0^{2\pi} \int_0^\pi S(r^*, \theta, \phi) \sin\theta d\theta d\phi$, one gets the analytical equation for the correlation function. This equation is rather complicated, so we do not list it here, limiting our presentation by the results of numerical calculations based on it.

In what follows, we consider some of results obtained in preparing full publication [41], the case of $p\Xi^-$ correlation function in the 5% most central Pb+Pb LHC collisions at the energy $\sqrt{s_{NN}} = 2.76$ TeV. To account for the residual correlation effect, we follow the procedure developed in [36, 37]. Having a correlation function not corrected for purity, we fit it with the formula

$$C_{\text{uncorr}}(k^*) = 1 + \lambda(k^*)(C(k^*) - 1) + \alpha(k^*)(C_{\text{res}}(k^*) - 1), \quad (21)$$

where $\lambda(k^*)$ is the pair purity or the fraction of correctly identified pairs consisting of primary particles, $C(k^*)$ is “true” correlation function obtained according to (10), $\alpha(k^*)$ is the fraction of secondary particles which are residually correlated, $\alpha(k^*) = \tilde{\alpha}(1 - \lambda(k^*))$ and $C_{\text{res}}(k^*)$ is the residual correlation contribution. We take the latter in the Gaussian form [36, 37, 42]

$$C_{\text{res}}(k^*) = 1 - \tilde{\beta}e^{-4k^{*2}R^2}, \quad (22)$$

where $\tilde{\beta} = A > 0$ is the annihilation (wide) dip amplitude and $R \ll r_0$ is the dip inverse width. One can notice that $\tilde{\alpha}$ and $\tilde{\beta}$ enter (21) only as a product $\tilde{\alpha}\tilde{\beta}$, so that it can be treated at fitting as a single parameter β .

In Figs. 14–16, one can see our predictions for the $p\Xi^-$ and $p\Xi^+$ correlation function at the LHC. In the calculations, we use the source radius extracted from iHKM source functions for $p\Xi$ pairs, $r_0 = 3.1$ fm. Strong interaction scattering lengths and effective radii are assumed to be the same as for $p\Lambda$ pairs in [43] ($f_0^s = 2.88$ fm, $f_0^t = 1.66$ fm, $d_0^s = 2.92$ fm, $d_0^t = 3.78$ fm) and as obtained for $\bar{p}\Lambda$ in [36] ($\text{Re } f_0 = 0.14 \pm 0.66$ fm and $\text{Im } f_0 = 1.53 \pm 1.31$ fm, $d_0 = 0$). The parameters R and β describing residual correlations are taken from [37], $\beta = 0.034 \pm 0.005$ and $R_{\text{LHC}} = 0.55 \pm 0.06$ fm. In the first figure, we present “true” CF, corresponding to the case of pair purity $\lambda = 1$ for two cases: application of the described FSI formalism at all the distances r between p and Ξ^- in the pair rest frame, and both Coulomb and strong interactions switched off at small distances $r < 1$ fm. The second figure shows the correlation functions $p\Xi^-$ with λ different from unity. Here, similarly

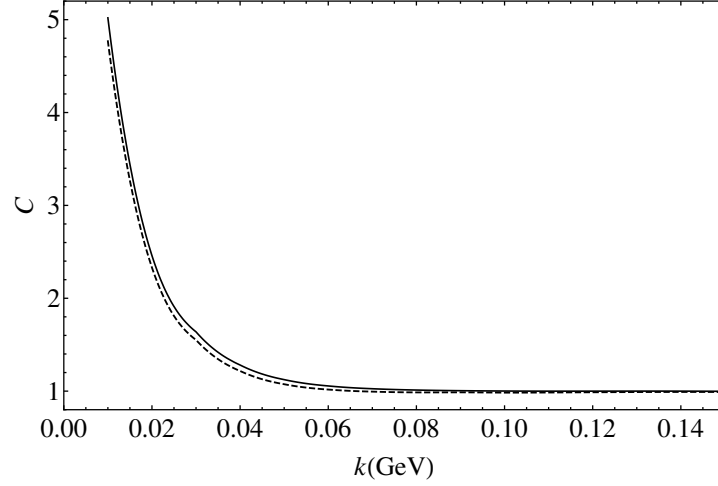


Fig. 14. The pure (purity $\lambda = 1$) baryon–baryon correlation function between primary proton and cascade, $p\Xi^-$ obtained in iHKM simulations. The Gaussian radius in the source function distribution in iHKM is 3.1 fm. The scattering lengths for strong interactions are supposed to be the same as in $p\Lambda$ systems. The solid line is related to assuming the Coulomb plus strong FSI exist at all distances r between the two baryons in the rest system of the pair. The dashed line corresponds to switching off all interactions at $r < 1$ fm. Therefore, the realistic results, accounting for the baryon kernel in short-range interaction and electromagnetic form-factors of baryons is supposed to be between these two curves.

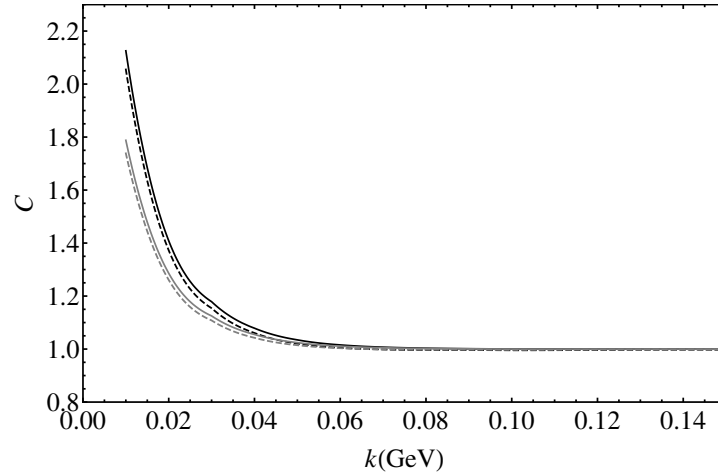


Fig. 15. The prediction for observed baryon–baryon correlation function proton and cascade, $p\Xi^-$ obtained in iHKM simulations. The purity that is result of long-lived resonance decays is found in iHKM as $\lambda_{\text{res}} = 0.28$. The gray lines account in addition for the purity connected with particle misidentification and some other detector aspects. We put this additional factor as 0.7, so that the gray lines are corresponding to $\lambda = 0.7 \lambda_{\text{res}} = 0.196$. The other notations and parameters as in Fig. 14.

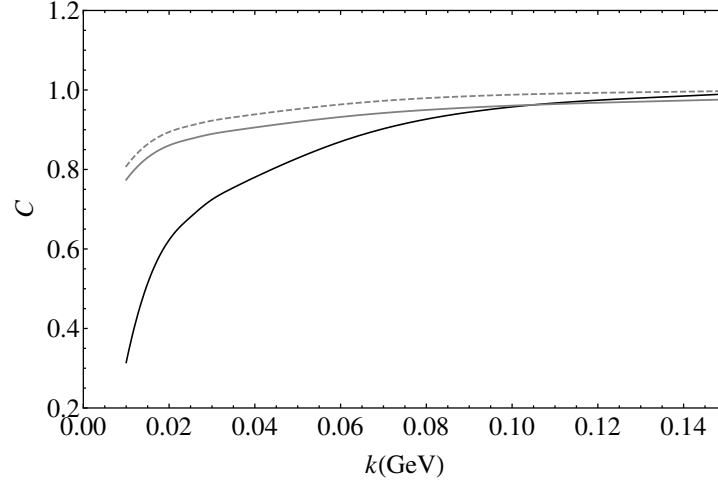


Fig. 16. The baryon–antibaryon correlation function for primary proton and anti-cascade, $p\Xi^+$, obtained in iHKM simulations. The Gaussian radius in the source function distribution in iHKM is 3.1 fm. The scattering lengths for strong interactions are supposed to be the same as for $\bar{\Lambda}$ extracted from STAR data in Ref. [36]. The black solid line is related to purity $\lambda = 1$ in primary baryon–antibaryon system. The gray dashed line correspond to the purity that is result of long-lived resonance decays as it is found in iHKM, $\lambda_{\text{res}} = 0.28$. Solid gray line corresponds to account for the residual correlations among primary parents of p or/and Ξ^+ . The parameters of such a correction are taken from top RHIC energies [36] and adjusted for LHC space scale by using iHKM similarly to the way it was done for $p\Lambda$ in [37] .

to [37], we suppose purity to be close to the fraction of primary proton-cascade pairs in iHKM simulations, $\lambda = 0.28$ (see Table III), perhaps with some deviations from this value caused by different experimental issues. We

TABLE III

The fractions of $p\Xi^-$ pairs, primary and coming from different decays calculated in iHKM.

Pairs	Fractions [%]
$p_{\text{prim}}-\Xi_{\text{prim}}^-$	28
$p\Lambda-\Xi_{\text{prim}}^-$	12
$p\Sigma^+-\Xi_{\text{prim}}^-$	2
$p_{\text{prim}}-\Xi_{\Xi(1530)}^-$	38
$p\Lambda-\Xi_{\Xi(1530)}^-$	16
$p\Sigma^+-\Xi_{\Xi(1530)}^-$	3
$p_{\text{prim}}-\Xi_{\Omega^-}^-$	< 0.7
$p\Lambda-\Xi_{\Omega^-}^-$	< 0.3
$p\Sigma^+-\Xi_{\Omega^-}^-$	< 0.1

try to effectively account for the latter using the scaling factor 0.7 for the gray curves in Fig. 15. In Fig. 16, the result for baryon–antibaryon case (*i.e.* for $p\Xi^+$ pairs) is demonstrated. The black solid line corresponds to pure CF, $\lambda = 1$, while dashed gray line is related to $\lambda = 0.28$. The solid gray line shows the model CF where the residual correlations effect was taken into account in the way described above.

5. Summary

We have overviewed the recent results on the analysis of meson and baryon emission structure in relativistic heavy-ion collisions, obtained within the hydrokinetic and integrated hydrokinetic models. The detailed spatiotemporal picture of particle emission that is calculated in HKM for the cases of top RHIC and LHC energy collisions allows one to describe the behavior of a wide class of bulk observables and investigate the physical nature of observed effects, revealing the peculiar properties of the matter formed in the collision, and the process of its evolution.

Using simple analytical approximations, one can, based on the results of numerical calculations, determine various characteristics of the particle emission process, in particular, the time of maximal emission for particles of different species, effective temperature, collective flow parameters. Varying the model parameters allows one to investigate the role of different factors on the eventually obtained results. Thus, the violation of m_T scaling in the longitudinal interferometry radii dependence between pions and kaons can be explained by the influence of strong transverse flow and hadron re-scatterings at the late stages of the radiating system's evolution. The resonance decays, being the source of secondary particles production, also make a contribution to the final emission picture. In such a way, the K^* resonance decays produce the additional amount of kaons which are then subject of intensive re-scatterings at the afterburner stage, that significantly affects the observed behavior of interferometry radii and the extracted maximal emission time.

The dynamics of such re-scattering processes can be probed with the study of K^* observability, the possibility to restore these resonances through the products of their decays. The iHKM simulations show that in central collisions at the LHC, up to 25% of produced $K(892)^*$ cannot be identified due to dissipation of their decay daughters. This implies a quite strong effect of the re-scattering stage on the experimental measurement results, in particular on the femtoscopy scales.

The model simulations also help in FSI correlation analysis, allowing to calculate the emission source functions and facilitate the related fitting procedure. Another useful model application is finding the realistic pair purity values and investigating the contribution of the different factors on

it. Taking into account also the residual correlation effect, one can describe or predict the correlation functions of interest within different analytical models. In this paper, we presented such predictions as for $p\Xi$ correlation functions at the LHC energy.

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ВИСНОВКИ

У дисертаційній роботі в рамках реалістичних моделей (передусім, інтегрованої гідрокінетичної моделі — іНКМ) детально досліджено властивості систем, що утворюються в релятивістських зіткненнях важких ядер, та механізми формування м'яких адронних спостережуваних на різних стадіях еволюції даних систем. Ціла низка одержаних результатів свідчить про те, що більш адекватний опис народження адронів у зіткненні досягається в рамках підходу “поступового фріз-ауту”, який передбачає поступове формування імпульсних спектрів та виходів частинок в ході еволюції утвореної системи, що включає в себе довгу адронну стадію. Перелічимо головні отримані результати:

- Інтегровану гідрокінетичну модель ядро-ядерних зіткнень (іНКМ) вперше відкалібровано для опису широкого класу спостережуваних м'якої фізики (виходи частинок різних сортів, їх відношення, спектри поперечного імпульсу, коефіцієнти імпульсної анізотропії, кореляційні функції та відповідні фемтоскопічні масштаби) у зіткненнях ядер свинцю на Великому адронному колайдері (ЛНС) при енергії $\sqrt{s_{NN}} = 5.02$ TeV.
- Однаково гарний опис наявних експериментальних даних та передбачення для фемтоскопічних масштабів піонів і каонів вперше отримано у моделі іНКМ при двох різних рівняннях стану кварк-глюонної матерії на гідродинамічній стадії еволюції системи. Збереження узгодженості теоретичного опису з експериментом при переході до іншого рівняння стану досягається перенормуванням початкової густини енергії у центрі системи ϵ_0 . Таким чином, встановлено, що наявні результати експериментальних вимірювань м'яких адронних спостережуваних не дозволяють надати чітку перевагу одному з двох розглянутих рівнянь стану та визначити точні величини температури партиклізації і густини енергії на ранній стадії зіткнення.
- Встановлено, що адронна стадія зіткнення, яка у іНКМ описується в рам-

ках адронного каскаду UrQMD, компенсує різницю у значеннях м'яких спостережуваних, яка може мати місце в кінці гідродинамічної стадії внаслідок використання різних рівнянь стану КГП. Цей висновок говорить на користь концепції “поступового фріз-ауту”, тобто поступового формування імпульсних спектрів та виходів частинок в процесі еволюції системи, що включає в себе довгу адронну стадію. Зокрема, адронна стадія має ключові значення для успішного опису виходів резонансів $K^*(892)$ та відношень числа народжених протонів до числа пі-мезонів.

- В рамках моделі іНКМ з'ясовано, що фемтоскопічні масштаби π - та K -мезонів у випадку енергії зіткнень $\sqrt{s_{NN}} = 5.02$ TeV близькі до відповідних значень при енергії $\sqrt{s_{NN}} = 2.76$ TeV. Ефект певного зростання множинності народжених частинок внаслідок збільшення енергії зіткнення частково компенсується більшими градієнтами колективної швидкості при енергії 5.02 TeV. Підтверджено порушення скейлінгу радіусів піонів і каонів за поперечною масою пари m_T і наявність скейлінгу за середнім поперечним імпульсом k_T (при не надто малих k_T), що раніше спостерігалось в рамках розрахунків у гідрокінетичній моделі.
- В області надм'яких імпульсів ($m_T < 200$ MeV/c) у моделях іНКМ та LQTH виявлено відхилення поведінки фемтоскопічних піонних радіусів від степеневого закону виду am_T^{-b} . Натомість, вдається єдиною формулою описати залежність радіусів від поперечної швидкості пари β_T , що дає можливість екстраполяції відомих залежностей $R(m_T)$ при $m_T > 200$ MeV/c на область більш низьких імпульсів. Незвичний характер залежності $R(m_T)$ для піонів з наднизькими імпульсами може свідчити про своєрідне відокремлення даних частинок від решти системи, їх триваліше утримання в її центрі та утворення своєрідної “пастки” для надм'яких піонів.
- У моделі НКМ з'ясовано, що на адронній стадії зіткнення відбувається інтенсивна взаємодія короткоживучих резонансів $K^*(892)$ (час життя яких становить близько 5 фм/с) та продуктів їх розпаду з густим адронним середовищем. Для центральних зіткнень така взаємодія призводить спо-

чатку до “втрати” близько 70% первинно утворених K^* , а потім до утворення шляхом рекомбінації додаткових 50% від початкової кількості K^* . В результаті спостерігається недостача числа ідентифікованих K^* у 20%. Ефект зменшується при переході від центральних до периферичних зіткнень. Виходи ж довгоживучих резонансів $\phi(1020)$ (з часом життя порядку 50 фм/с), навпаки, не залежать від центральності зіткнення.

- У інтегрований гідрокінетичній моделі отримано оцінки на часи випромінювання піонів і K -мезонів у зіткненнях важких ядер на колайдерах LHC та RHIC. Несподіваним результатом з точки зору гідродинамічних міркувань є одержане в роботі більше значення ефективного часу емісії каонів у порівнянні з піонами. Даний результат можна пояснити сильним впливом взаємодій між адронами на фінальній стадії зіткнення, зокрема перерозсіянь і рекомбінації продуктів розпаду резонансів $K^*(892) \leftrightarrow K\pi$, на результуючі каонні спостережувані. Для більш периферичних зіткнень та при зниженні їх енергії часи емісії частинок обох сортів зменшуються, разом з інтенсивністю поперечного колективного потоку та ефективною температурою спектрів.
- В моделі iHKM встановлено, що тривалість гідродинамічної стадії для зіткнень ядер золота при енергії $\sqrt{s_{NN}} = 200$ GeV і ядер свинцю при енергіях $\sqrt{s_{NN}} = 2.76$ TeV та $\sqrt{s_{NN}} = 5.02$ TeV майже однакова і становить 8 – 10 фм/с. Головними причинами близькості отриманих значень, попри різницю в енергії зіткнення, є інтенсивне, суттєво тривимірне гідродинамічне розширення системи, яке починається з певного моменту власного часу, та схожі співвідношення між розмірами системи та середньою величиною швидкості звуку на гідродинамічній стадії в усіх вказаних випадках.
- Запропоновано метод оцінки загального часу життя системи, що формується у релятивістському зіткненні ядер, на основі фітування спектрів та фемтоскопічних радіусів піонів і каонів в області малих поперечних імпульсів. Для зіткнень ядер свинцю на Великому адронному колайдері отримано значення часу життя системи порядку 17 фм/с, що говорить про наявність

у таких зіткненнях довгої адронної стадії, яка грає важливу роль у формуванні кінцевих спектрів та фемтоскопічних радіусів.

- На основі формалізму Куніна-Пратта та Ледніцкі-Любошица, а також розрахунків у гідрокінетичній моделі запропоновано просту аналітичну модель для опису баріон-баріонних та баріон-антибаріонних кореляційних функцій з урахуванням ефекту залишкових кореляцій. В рамках запропонованого підходу вдалося вирішити проблему видимої двократної відмінності між радіусами джерел емісії пар $p\Lambda$ і $p\bar{\Lambda}$ в експериментальній роботі колаборації STAR [125] щодо зіткнень ядер золота за найвищої енергії прискорювача RHIC. Визначено довжину розсіяння сильної взаємодії у парах $p\bar{\Lambda}$ та $\bar{p}\Lambda$. На основі отриманих результатів зроблено передбачення кореляційних функцій для зіткнень при енергії колайдера LHC з урахуванням сильної та кулонівської взаємодії між баріонами.

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ДОДАТОК А. СПИСОК ПУБЛІКАЦІЙ ЗДОБУВАЧА ЗА ТЕМОЮ ДИСЕРТАЦІЇ ТА ВІДОМОСТІ ПРО АПРОБАЦІЮ РЕЗУЛЬТАТІВ ДИСЕРТАЦІЇ

СПИСОК НАУКОВИХ ПРАЦЬ, В ЯКИХ ОПУБЛІКОВАНО ОСНОВНІ НАУКОВІ РЕЗУЛЬТАТИ ДИСЕРТАЦІЇ

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