

# Bogolyubov Institute for Theoretical Physics

## XXIV Davydov Lectures

Kiev, December, 26, 2017

# Alex Abrikosov: life and scientific biography

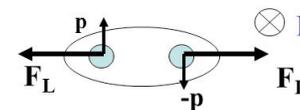


Andrey Varlamov

SPIN-CNR, Italy

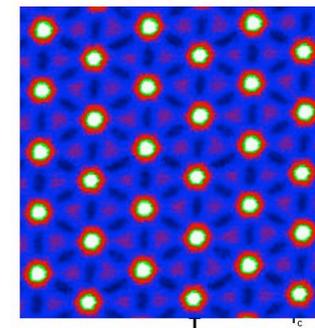
Doctor H.C. of Bogolyubov Institute

Orbital effect

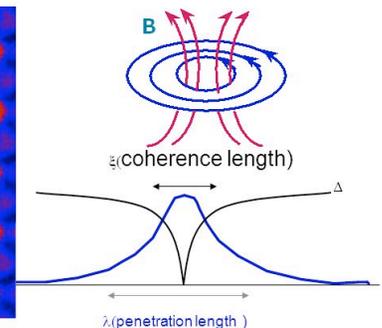


Vortex

Flux quantum



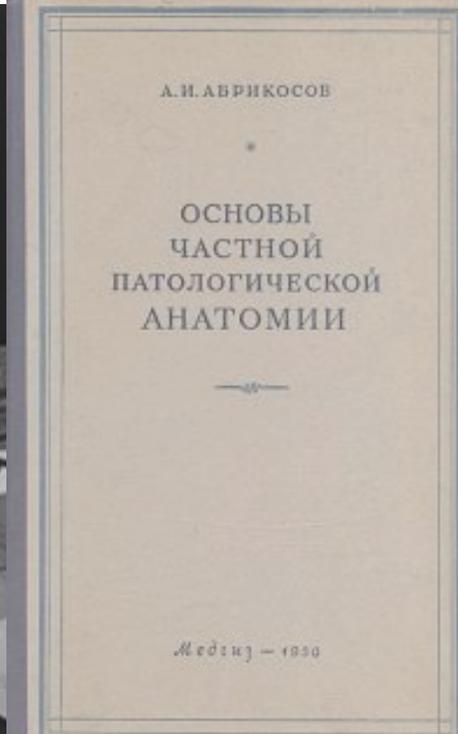
Vortex lattice in NbSe<sub>2</sub>  
(STM)

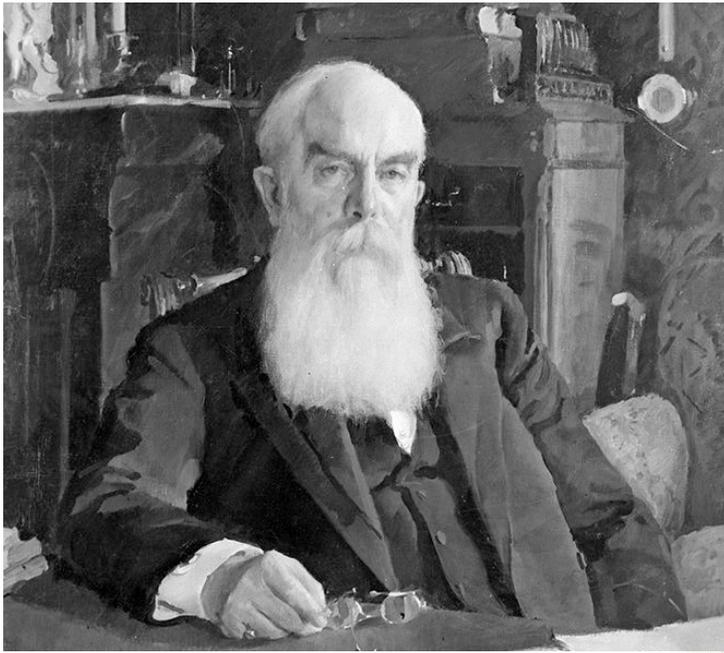


$$\Phi_0 = hc/2e = 2.07 \times 10^{-7} \text{ Oe} \cdot \text{cm}^2$$

# I. The family and childhood

Alexei Abrikosov was born on 25 June 1928 in Moscow in the family of well-known physicians: Alexei Ivanovich Abrikosov and Fani Wulf





The Abrikosov's family was known in the Russian Empire from the early 19th century as the founders and owners of a confectionery shop which gradually became a large concern and was awarded the title of «Supplier to the Court of His Imperial Majesty».

Alexei Ivanovich Abrikosov-the older



Dmitri Ivanovich Abrikosov (1876 - 1951 ), uncle of Alexei Abrikosov, was the last (1917 - 1925 ) ambassador (Charge d'Affaires) of the Russian Empire in Japan



Uncle - Dmitri Ivanovich Abrikosov

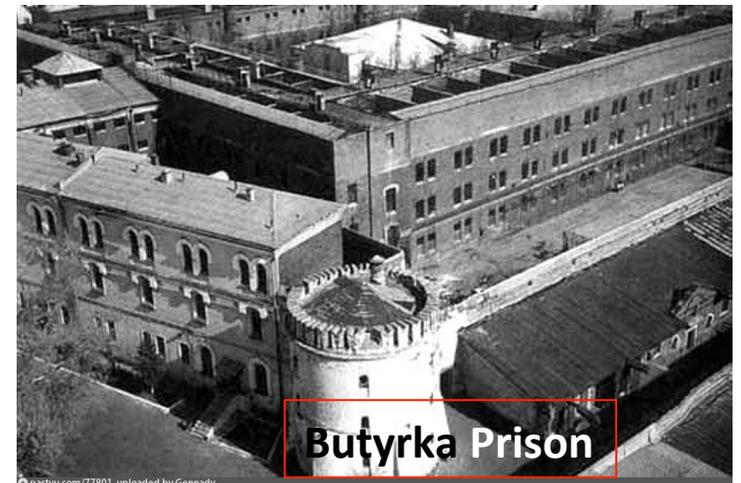


## Anna Ivanovna Abrikosova (Mother Catherine of Siena) - aunt of Alexei Abrikosov

23 January 1882, Moscow, Russian Empire – 23 June 1936, **Butyrka Prison**, Moscow, Soviet Union

Anna Ivanovna Abrikosova was a Russian Catholic Religious Sister, literary translator, and victim of Joseph Stalin's concentration camps. She was also the foundress of a Byzantine Catholic community of the Third Order of St. Dominic which has gained wide attention, even among secular historians of Soviet repression. In an anthology of women's memoirs from the GULAG, Anna Abrikosova is described as, "a woman of remarkable erudition and strength of will". She is also mentioned by name in the first volume of *The Gulag Archipelago* by Alexander Solzhenitsyn.

Since 2002, Mother Catherine's life has been under scrutiny for possible beatification by the Holy See. Her current title is **Servant of God**.





Honorable mention for Alexei Abrikosov

## Years of studying

In 1943, 15 years old, Alexei Abrikosov graduated from high school and entered Moscow Power Engineering Institute. In 1945, he transferred to the Faculty of Physics of Lomonosov Moscow State University.

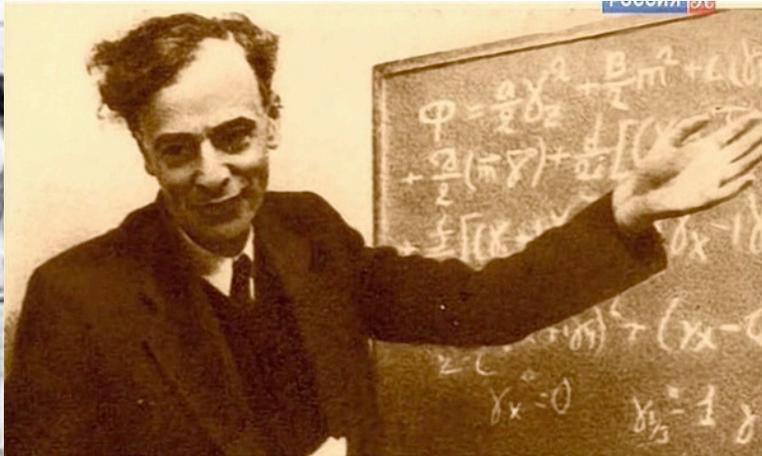


Moscow University old building



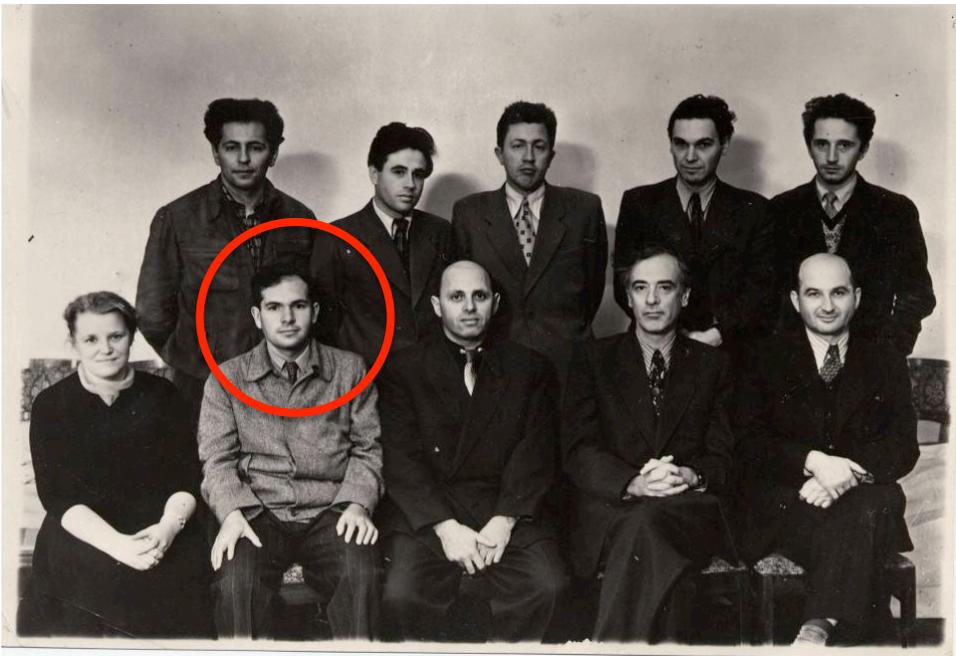
# Landau School of Theoretical Physics

The **Kharkiv Theoretical Physics School** was founded by Lev Landau in Kharkov, Ukraine. It is sometimes referred to as the **Landau school** — more precisely, one might say that Landau's group at Kharkiv was the beginning of the Landau school that, after Landau moved to Moscow, included new generations of theoretical physicists from the countries of the former Soviet Union. Lev Landau was the head of the Kharkhiv Theoretical Physics School from 1932 to 1937.



## II. Landau's favorite pupil

Alexei Abrikosov's scientific growth was directly influenced by Lev Landau. At the age of 19, he passed the **theoretical minimum**, in a year he graduated **cum laude** from the Faculty of Physics of MSU.



Theoretical Group of IPP, 1956.

Stand: S. Gershtein, L. Pitaevsky, L. Vainshtein, R. Arkhipov, I. Dzyaloshinskiy. Sit: Прозорова, А. Abrikosov, I. Khalatnikov, L. Landau, E. Lifshitz.

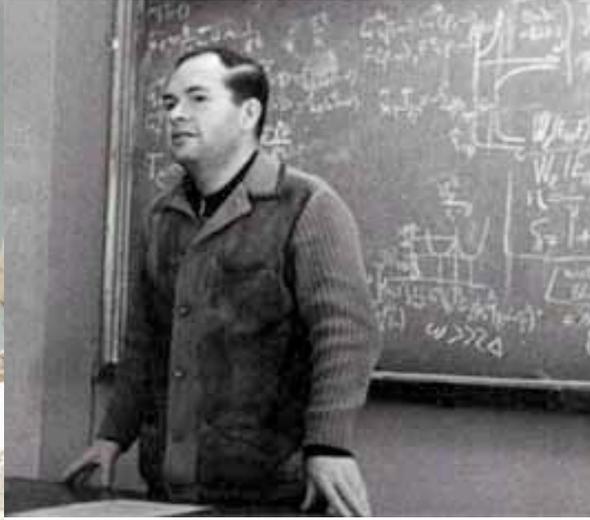
list of students who passed the **theoretical minimum**. Handwriting of Landau

3. Ахисеп	35	к
4. Пауэрман	35	к
5. Мисса	35	
6. Левин	37	к
7. Берестовский	39	д
8. Спорodinский	40	д
9. Халатников	41	д
10. Хуцшвили		д
11. Тер-Мартirosян	47	д
12. Абрикосов	47	д
13. Цорге	49	д
14. Исакян	50	к
15. Липидзе	50	к
16. Суцаков	51	к
17. Клан	51	к
18. Циринский	52	к
19. Зорсков	53	д
20. Дзюльшинский	53	к
21. Архипов	54	к

24. Мехриш	55	
25. Писелский	55	
26. Саидел	55	
27. Бекарев	55	к
28. Шлакун	56	к
29. Бонько	57	к
30. Шаполаз	58	
31. Фальковский	59	
32. Андреев	59	
33. Кокраменко	59	
34. Русиль	59	
35. Маринков	60	
36. Берков	60	



# Assault of the peaks: mountaineering and scientific



In 1950, at the age 22, Alexey Abrikosov defended his PhD thesis devoted to the study of thermal diffusion in fully or partially ionized plasma.

A. Abrikosov and S. Gershtein

A. Abrikosov



**Khorloogiin Choibalsan** was the Communist leader of the Mongolian People's Republic  
Born: February 8, 1895,  
Died: **January 26, 1952,**  
**Moscow, Russia**

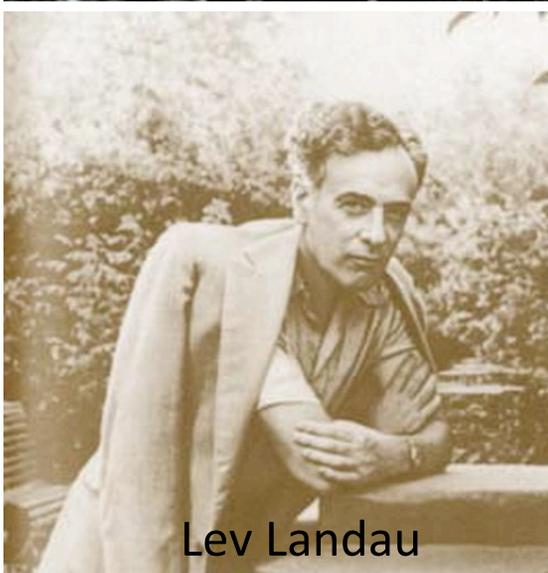
# Almost two of the subsequent decades of Alexei Abrikosov's scientific work were spent at the Institute for Physical Problems (IPP) of RAS



Petr Kapitza



Institute for Physical Problems



Lev Landau



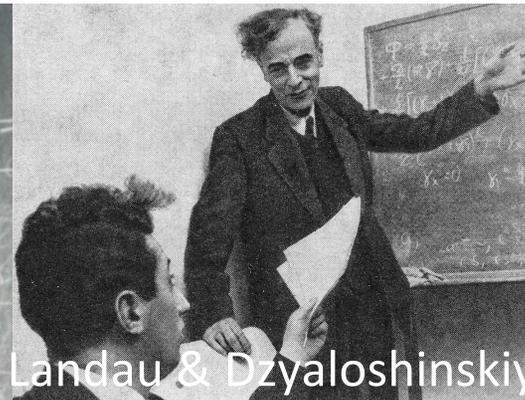
I. Khalatnikov, L. Landau, E. Lifshitz



I. Dzyaloshinskiy



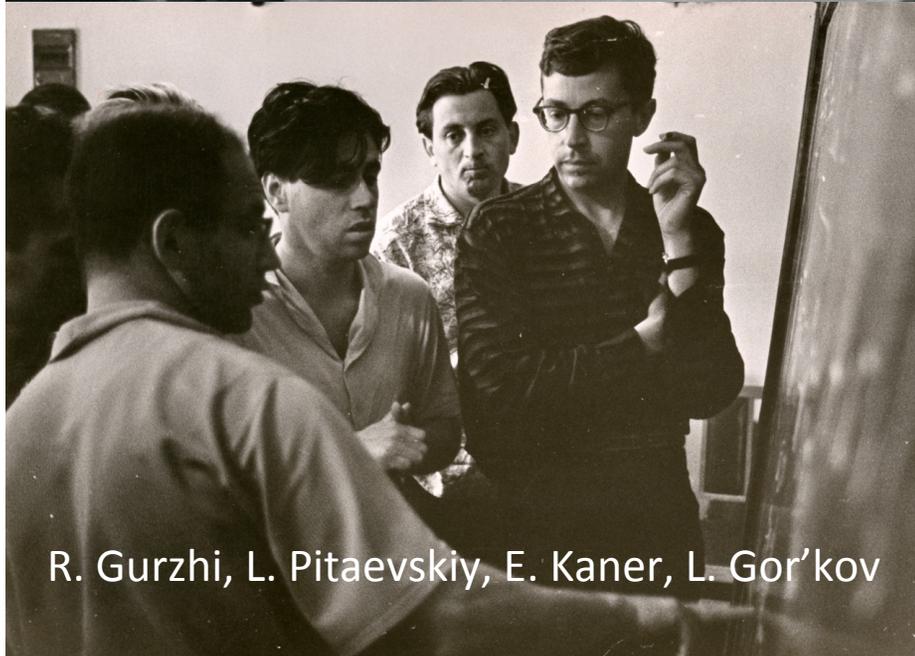
Gor'kov & Abrikosov



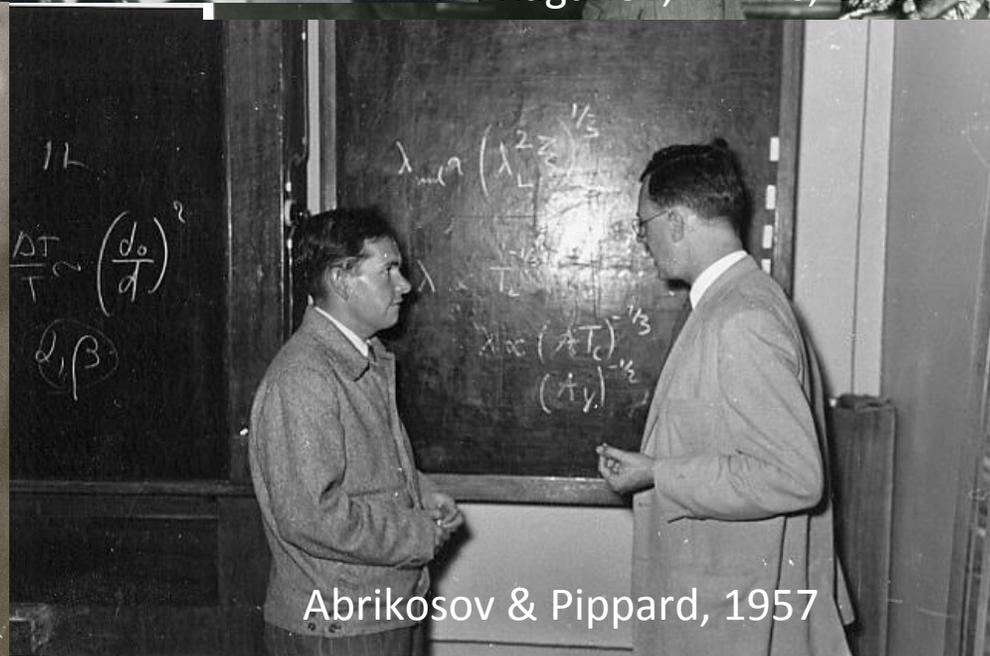
Landau & Dzyaloshinskiy



M. Kaganov, V. Fiks, E. Kaner,



R. Gurzhi, L. Pitaevskiy, E. Kaner, L. Gor'kov



Abrikosov & Pippard, 1957

### III. Discovery of the type II superconductivity



Ginzburg & Landau

1950: Ginzburg-Landau  
Phenomenology  $\Psi$ -Theory of  
Superconductivity

$$\delta\mathcal{G} = \int dV \left\{ \alpha\tau|\Psi|^2 + \frac{b}{2}|\Psi|^4 + \frac{1}{4m} \left| \left( -i\hbar\nabla + \frac{2e}{c}\mathbf{A} \right) \Psi \right|^2 + \frac{H^2}{8\pi} \right\}$$

$$\frac{1}{\kappa^2} \frac{d^2\Psi}{dx^2} + \Psi(1 - A^2) - \Psi^3 = 0,$$

$$\left. \frac{d\Psi}{dx} \right|_b = 0,$$

$$\frac{d^2A}{dx^2} - \Psi^2 A = 0$$

$$\kappa = 3 \left( \frac{2}{7\zeta(3)} \right)^{1/2} \left( \frac{\pi\hbar}{v} \right)^{3/2} \frac{cT_c}{\hbar p_0}$$

Experimentally for superconductors  
of those times  $\kappa \ll 1$

## Great successes of the GL theory

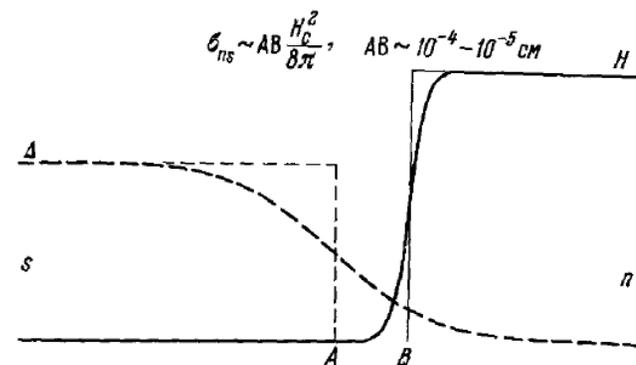
1 *The critical field and magnetization of a thin film. Supercooling and superheating*

2 *The critical current of a thin wire with  $\kappa \ll 1$*

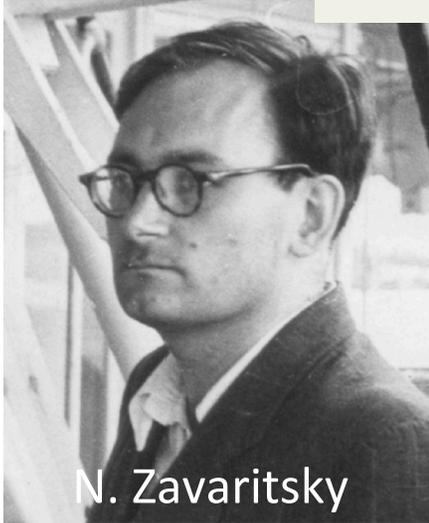
3 *Quantization of the magnetic flux*

4 *Surface energy at the interface between the normal and superconducting phases*

$$\kappa < 1/\sqrt{2}$$



## Experimental Verification of the GL theory



N. Zavaritsky

In 1951-1952, Abrikosov and Zavaritsky, an experimentalist from IPP, were engaged in verifying the predictions of the recently developed Ginzburg-Landau theory of superconductivity concerning critical magnetic fields for thin films.

### From Abrikosov's Nobel lecture:

“Nikolay Zavaritskii, started to measure the critical field of thin films. Theory and experiment fitted perfectly, including the change of the nature of the transition: first order at larger thicknesses and second order at smaller ones. Everything seemed OK but Alexander Shalnikov, the boss of Zavaritskii was not satisfied. He said that the films used by Zavaritskii were bad, since they were prepared at room temperature. The atoms of the metal, evaporated on a glass substrate, could agglomerate, and therefore the film, actually consisted of small droplets.

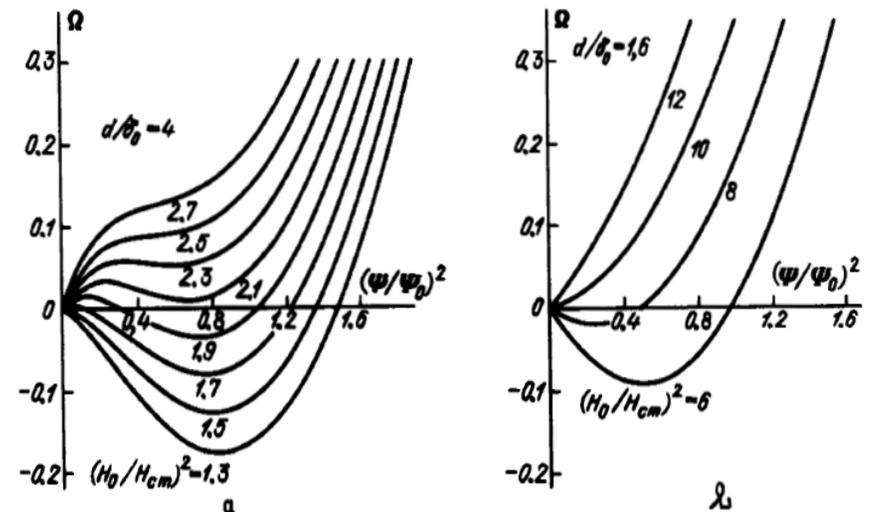


Fig. 109.

## Observed contradictions



A. Shalnikov

In order to avoid that, Shalnikov recommended to maintain the glass substrate at helium temperature during evaporation and until the measurements were finished. Zavaritskii followed this advice, and the result was a surprise: the dependence of the critical field on the thickness, or temperature (the theory contains the ratio of the thickness to the penetration depth, which depends on temperature), did not fit the predictions of the Ginzburg-Landau theory.

Discussing these results with Zavaritskii, we couldn't believe that the theory was wrong: it was so beautiful, and fitted so well the previous data. Therefore we tried to find some solution in the framework of the theory itself, and we found it.

The equations of the theory, where all entering quantities were expressed in corresponding units, depended only on the dimensionless "material" constant, which was later called the Ginzburg-Landau parameter. Its value could be defined from the surface energy between the normal and superconducting phases. The latter, in its turn could be calculated from the period of the structure of the intermediate state. These data for conventional superconductors led to very small values of  $\kappa$  and therefore the calculations in the paper by Ginzburg and Landau were done for this limiting case. It was also established that with increasing value of the surface energy between the superconducting and normal layers would become negative, and since this contradicted the existence of the intermediate state, such a case was not considered.

## Discovery of Abrikosov's vortex state

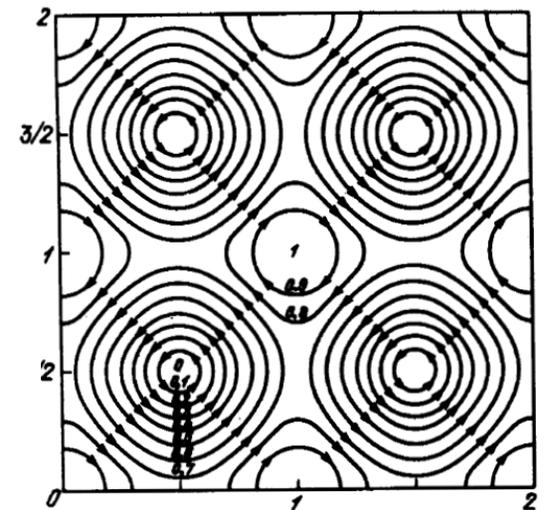
After that, Abrikosov passed to studying the magnetic properties of massive second-order superconductors. He succeeded to find the formal solution of the GL equations with  $\kappa > 1/\sqrt{2}$

$$(-i\kappa^{-1}\nabla + \kappa^{-1}\nabla\chi - \mathbf{A})^2|\Psi| - |\Psi| + |\Psi|^3 = 0.$$

$$\Psi = |\Psi| e^{i\chi}$$

$$\Psi = \sum_{n=-\infty}^{\infty} C_n \exp \left[ ikny - \frac{1}{2} \kappa^2 \left( x - \frac{kn}{\kappa^2} \right)^2 \right].$$

$$\Psi = C \exp \left( -\frac{1}{2} \kappa^2 x^2 \right) \vartheta_3 \left[ 1; (2\pi)^{1/2} \kappa i (x + iy) \right].$$



In result Abrikosov came to the conclusion that the transition from the superconducting to normal state with increasing field proceeds gradually, the field having two critical values

$$\begin{aligned} \varepsilon_0 &= \frac{1}{2} n_s \int_{\xi}^{\delta} \frac{2mv_s^2}{2} 2\pi\rho \, d\rho \\ &= \frac{\pi n_s \hbar^2}{4m} \int_{\xi}^{\delta} \frac{d\rho}{\rho} = \frac{\pi n_s \hbar^2}{4m} \ln\left(\frac{\delta}{\xi}\right) \\ &= \frac{\pi n_s \hbar^2}{4m} \ln \kappa \end{aligned}$$

Lower critical field  $H_{c1}$

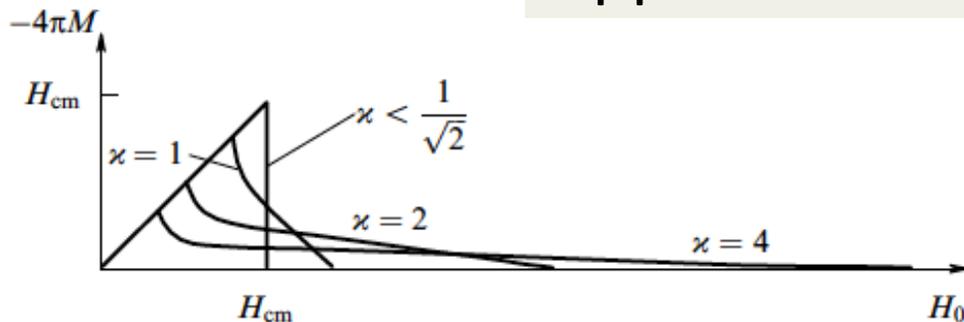
$$\begin{aligned} M &= \frac{1}{2} n_s \frac{2e}{c} \int_{\xi}^{\delta} \rho v_s 2\pi\rho \, d\rho = \pi \frac{e\hbar n_s}{mc} \int_{\xi}^{\delta} \rho \, d\rho \\ &\approx \frac{\pi e\hbar n_s}{2mc} \delta^2 \end{aligned}$$

In the presence of a magnetic field  $H$  the vortex acquires a magnetic energy  $-MH$ . The smallest field at which an elementary vortex can appear corresponds to  $\varepsilon_0 - MH = 0$  or

$$H_{c1} \sim \frac{H_{cm}}{\kappa} \ln \kappa.$$

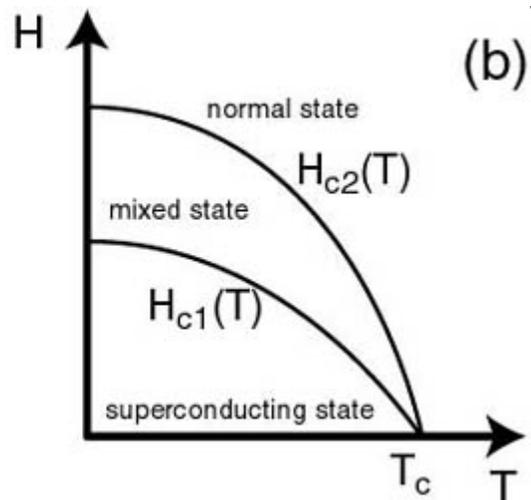
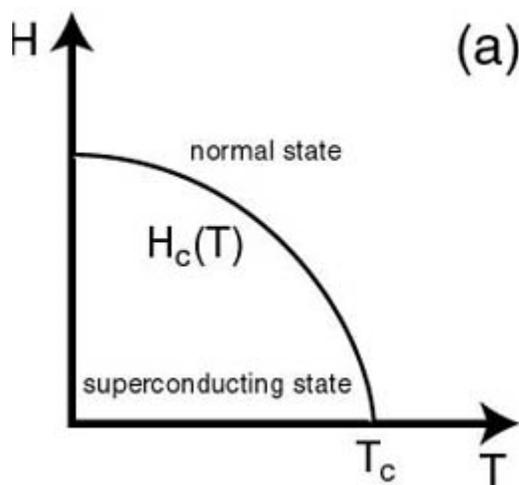
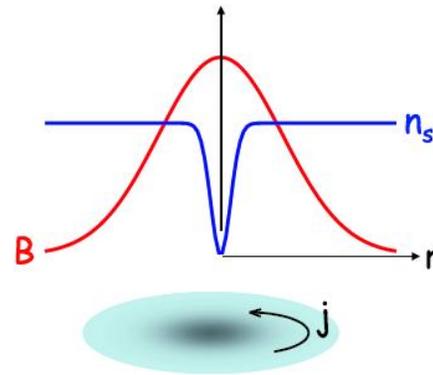
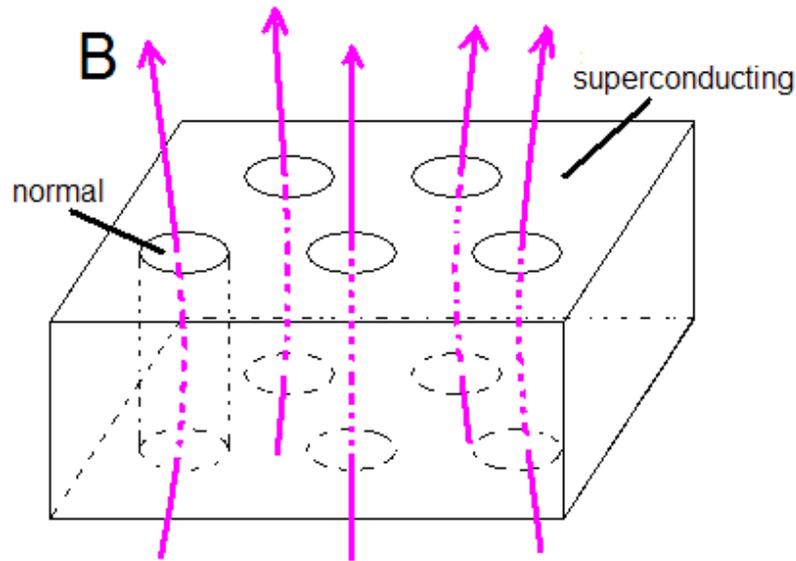
Upper critical field  $H_{c2}$

$$r_L \sim \frac{cp_{\perp}}{eH} > \xi.$$



$$H_{c2} \sim \frac{c\hbar}{e\xi l} \sim \frac{c\hbar}{el} \frac{\Delta}{\hbar v} \sim H_{cm} \frac{\delta_L}{l} \sim H_{cm} \kappa.$$

Between these critical values, the external magnetic field in the form of thin filaments of the magnetic flux, surrounded by vortex currents, gradually penetrates the superconductor. These quantum vortices form a regular structure (now known as the Abrikosov vortex lattice).



Having compared his results with the experimental curves of superconducting alloy magnetization obtained in the 1930s, Abrikosov found a remarkable coincidence. The authors, however, explained their data by the inhomogeneity of the specimens.

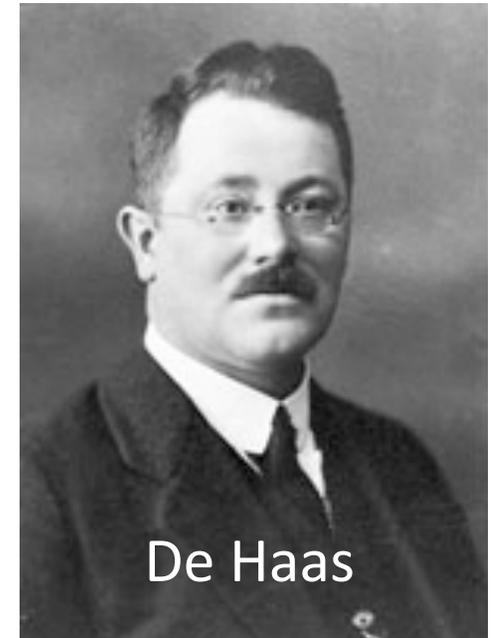
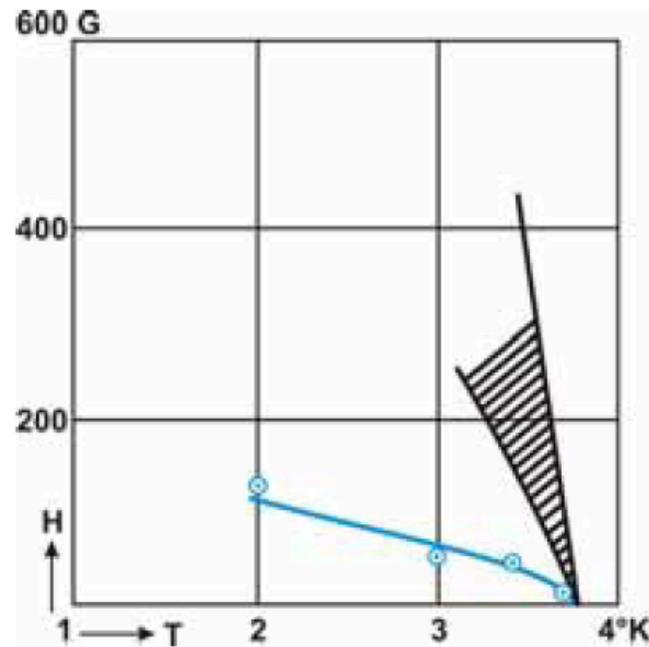
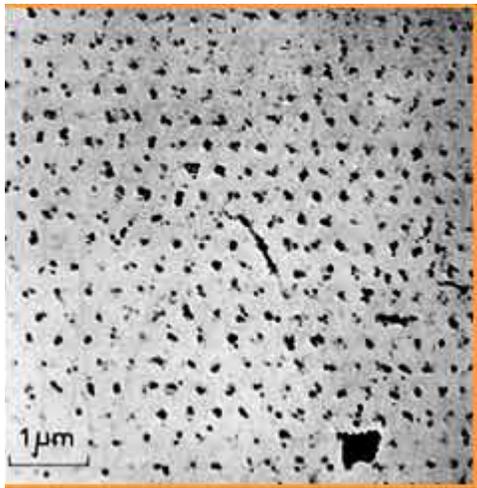
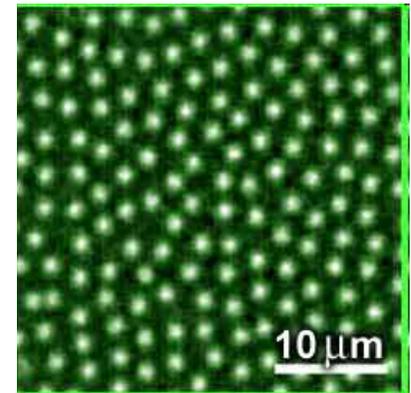
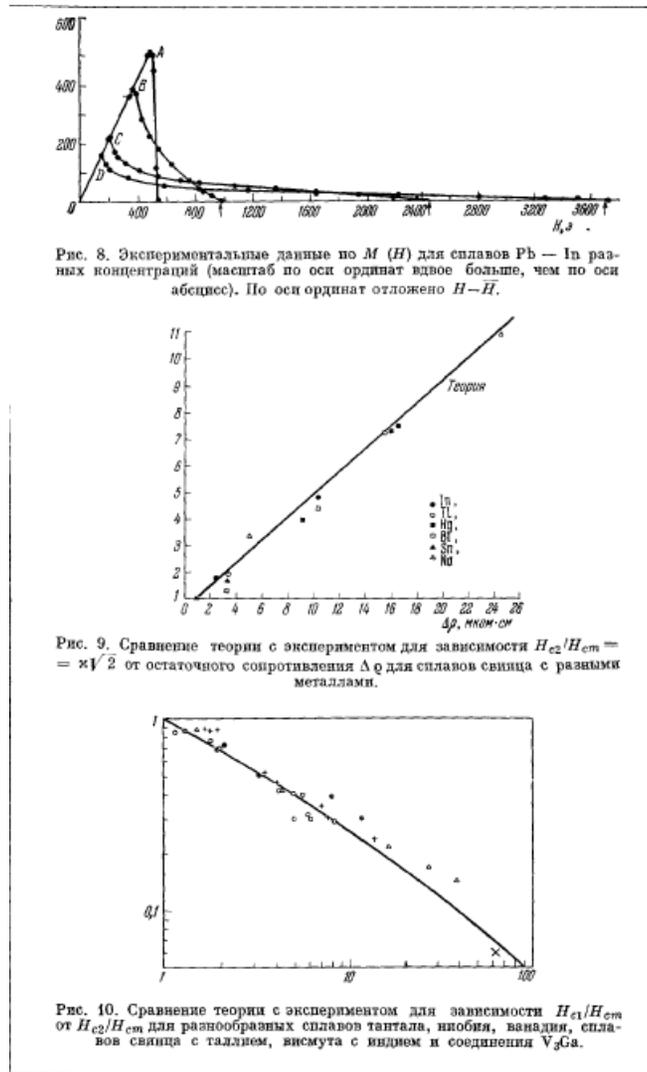


Fig. 11. Temperature dependence of the incipient penetration of magnetic field into the superconducting alloy Pb+64.8wt%Tl. The hatched region denotes the region of gradual flux penetration in magnetic field according to the electrical resistance measurement data (After De Haas & Casimir-Jonker, 1935a).

Abrikosov's paper, without which it is now impossible to imagine the physics and technology of superconductivity, appeared in 1957, but the experimentalists only bought in to the vortex lattice ten years later after direct observations using the magnetic decoration method.



U. Essmann and H. Trauble  
 Max-Planck  
 Institute, Stuttgart  
[Physics Letters](#)  
[24A, 526 \(1967\)](#)



**Magneto-optical  
 image of Vortex  
 lattice, 2001**  
**P.E. Goa et al.**  
**University of Oslo**  
[Supercond. Sci. Technol. 14, 729 \(2001\)](#)



## 1958: Lev Gorkov

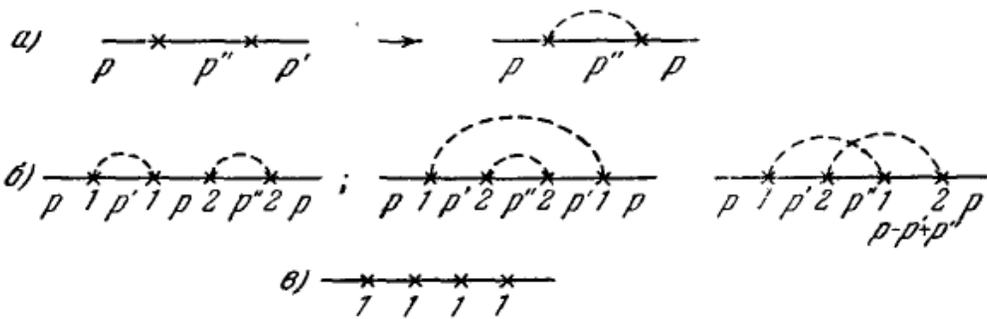
formulates elegant equations of the microscopic theory of superconductivity and **demonstrates the equivalence between the microscopic BCS theory and GL phenomenology at temperatures close to the critical one.**

$$\begin{pmatrix} \left\{ -\frac{\partial}{\partial \tau} + \frac{\nabla^2}{2m} + \mu \right\} & \Delta \\ -\Delta^* & \left\{ \frac{\partial}{\partial \tau} + \frac{\nabla^2}{2m} + \mu \right\} \end{pmatrix} \times \begin{pmatrix} \mathfrak{G}(x-x') & \mathfrak{F}(x-x') \\ \mathfrak{F}^+(x-x') & -\mathfrak{G}(x'-x) \end{pmatrix} = \mathbf{i}. \quad (34.32)$$

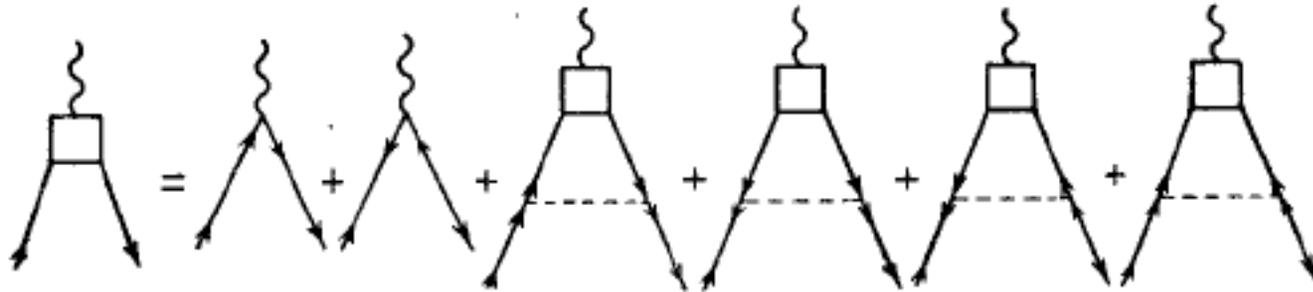
$$\mathfrak{G}_\omega(\mathbf{p}) = -\frac{i\omega + \xi}{\omega^2 + \xi^2 + \Delta^2}, \quad \mathfrak{F}_\omega^+(\mathbf{p}) = \frac{\Delta^*}{\omega^2 + \xi^2 + \Delta^2}.$$

These equations gave the basis for study of nonhomogeneous superconductivity

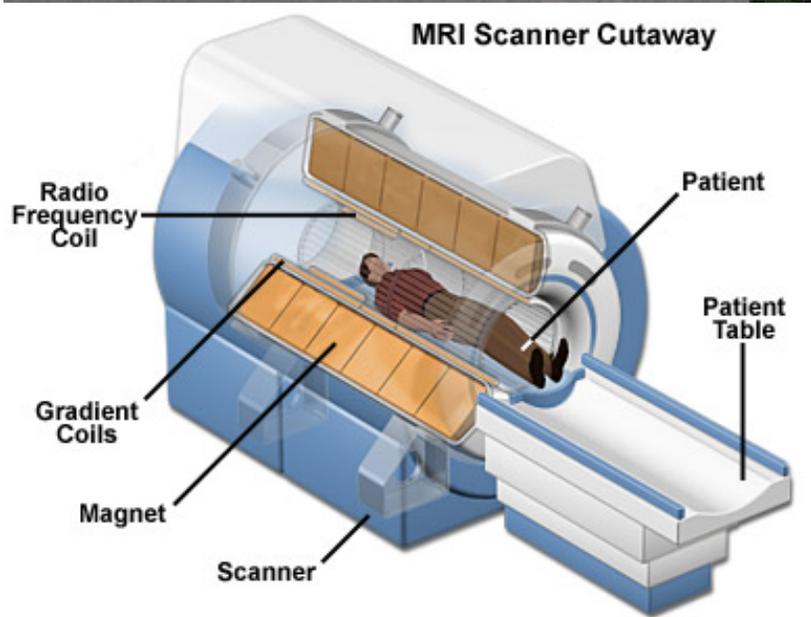
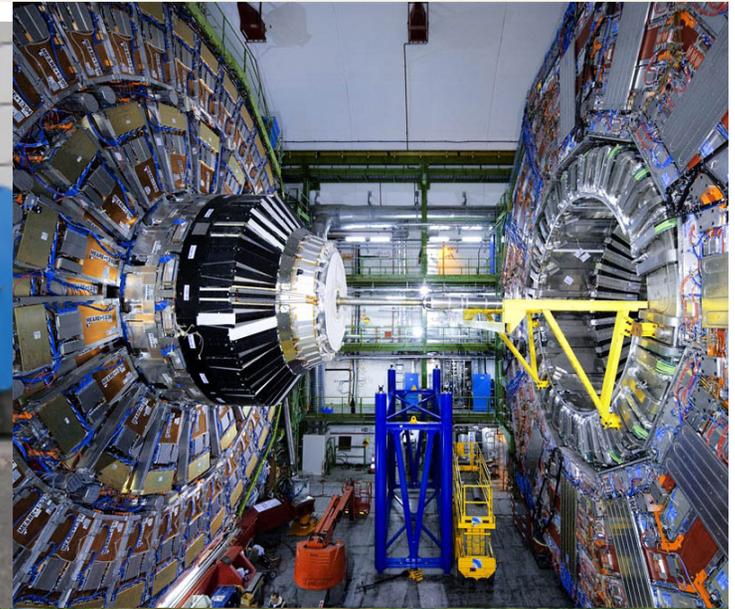
# IV. Abrikosov & Gor'kov, 1959: elaborate the microscopic methods for studies of electron scattering by impurities in normal metals and superconductors



$$Q = \frac{Ne^2}{m} 2\pi T \Delta^2 \sum_{n=1}^{\infty} \frac{1}{(\omega^2 + \Delta^2) \left( \sqrt{\omega^2 + \Delta^2} + \frac{1}{2\tau_{tr}} \right)}$$



# Applications of Superconducting alloys

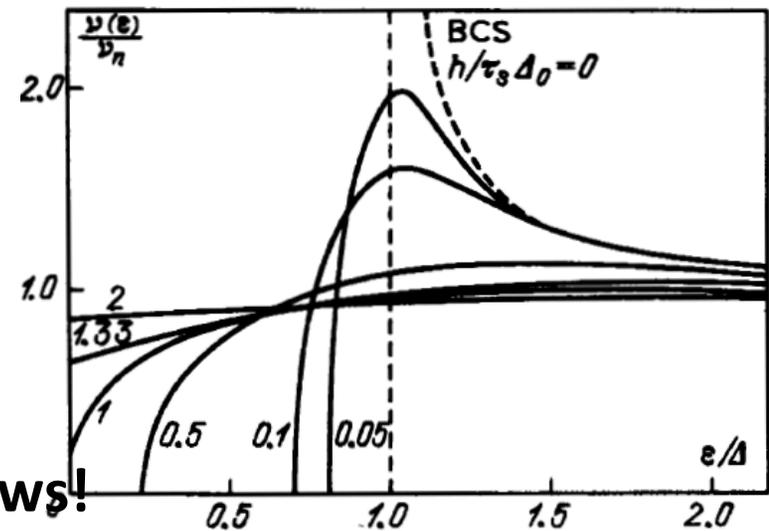


**V. Abrikosov & Gor'kov:** investigate the properties of superconductors with magnetic impurities and discover the phenomenon of gapless superconductivity

$$\ln\left(\frac{T_{c0}}{T_c}\right) = \psi\left(\frac{1}{2} + \frac{1}{2}\rho\right) - \psi\left(\frac{1}{2}\right),$$

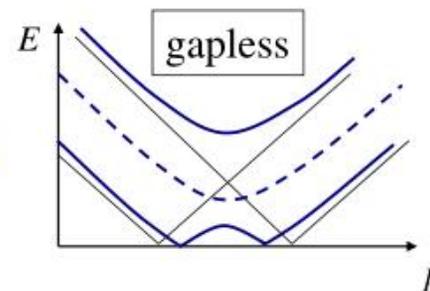
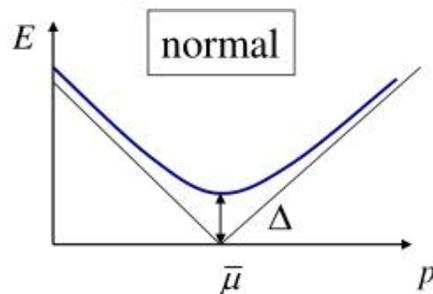
$$\rho = \hbar / (\pi T_c \tau_s)$$

$$0.91 n_{mc} < n_m < n_{mc}.$$



**There is no gap but supercurrent can flow!**

• fermion dispersion:



# VI. Further studies of superconductivity in the newly formulated microscopic theory

## *A SUPERCONDUCTOR IN A HIGH FREQUENCY FIELD*

A. A. ABRIKOSOV, L. P. GOR' KOV, and I. M. KHALATNIKOV

Institute for Physical Problems, Academy of Sciences, U.S.S.R.

Submitted to JETP editor March 4, 1958

J. Exptl. Theoret. Phys. (U.S.S.R.) 35, 265-275 (July, 1958)



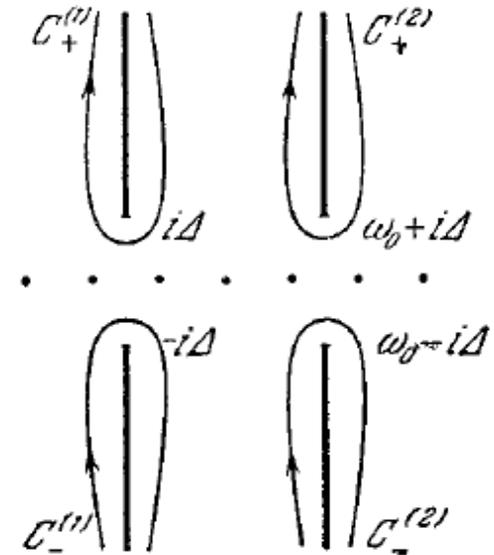
We derive an equation describing the behavior of superconductors in a high frequency field. With the aid of this equation the frequency and temperature dependence of the impedance of a bulk superconductor have been evaluated.

$$j(x) = - \int Q(x-y) A(y) d^4y$$

$$\bar{Q}(k, \omega_0) = \frac{3\pi T}{4} \sum_{\omega'} \int_{-1}^1 (1 - \mu^2) d\mu \times$$

$$\times \left\{ \frac{i(\omega_+ + V\sqrt{\omega_+^2 + \Delta^2}) [i(\omega_- + V\sqrt{\omega_+^2 + \Delta^2}) - v|k|\mu] + \Delta^2}{V\sqrt{\omega_+^2 + \Delta^2} [\omega_-^2 + \Delta^2 + (v|k|\mu - iV\sqrt{\omega_+^2 + \Delta^2})^2]} + \right.$$

$$\left. + \frac{i(\omega_- + V\sqrt{\omega_-^2 + \Delta^2}) [i(\omega_+ + V\sqrt{\omega_-^2 + \Delta^2}) + v|k|\mu] + \Delta^2}{V\sqrt{\omega_-^2 + \Delta^2} [\omega_+^2 + \Delta^2 + (v|k|\mu + iV\sqrt{\omega_-^2 + \Delta^2})^2]} \right\}. \quad (37.24)$$



A A Abrikosov in co-authorship with L P Gor'kov explained the Knight shift in superconductors.

*SPIN-ORBIT INTERACTION AND THE KNIGHT SHIFT IN SUPERCONDUCTORS*

A. A. ABRIKOSOV and L. P. GOR'KOV

Institute for Physical Problems, Academy of Sciences, U.S.S.R.

Submitted to JETP editor November 4, 1961

J. Exptl. Theoret. Phys. (U.S.S.R.) **42**, 1088-1096 (April, 1962)



A theory of the Knight shift in superconductors is presented. It is shown that the effect can be completely explained if the spin-orbit part of the interaction in the scattering of electrons on the crystal boundaries is taken into account. The agreement between the theory and the experimental data<sup>[8]</sup> is found to be good.

The Knight shift measures the relative shift in NMR frequency due to spin polarization in a magnetic field.

the nuclear spin is acted on by an extra magnetic field:

$$\frac{\chi_s(0)}{\chi_n} \approx \frac{1}{6}\pi^2 \frac{\xi_0}{l_{so}}, \quad \xi_0 \ll l_{so},$$

$$\frac{\chi_s(0)}{\chi_n} \approx 1 - \frac{3}{4} \frac{l_{so}}{\xi_0}, \quad \xi_0 \gg l_{so}$$

$$\Delta H = \frac{8}{3}\pi\chi \frac{|\psi_s(0)|^2}{n} H.$$

Abrikosov in co-authorship with L. Falkovsky calculated the Raman scattering intensity in the newly formulated microscopic theory of superconductivity.

*RAMAN SCATTERING OF LIGHT IN SUPERCONDUCTORS*

A. A. ABRIKOSOV and L. A. FAL'KOVSKIĬ

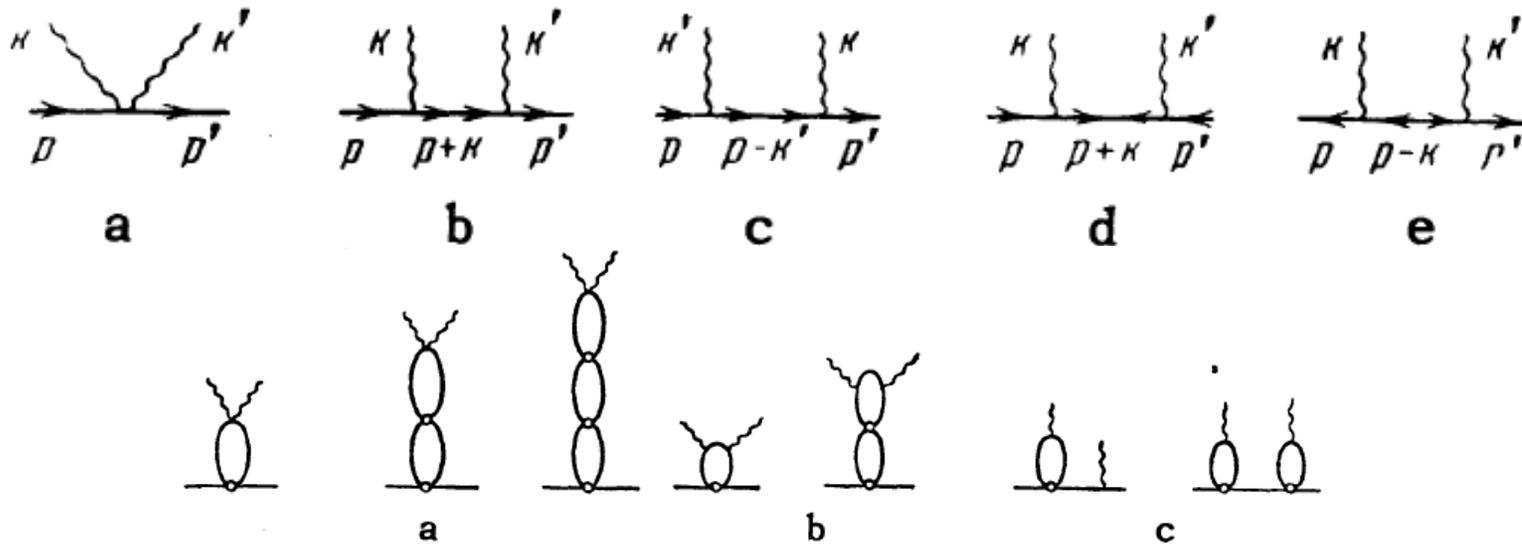
Institute for Physics Problems, Academy of Sciences, U.S.S.R.

Submitted to JETP editor July 25, 1960

J. Exptl. Theoret. Phys. (U.S.S.R.) 40, 262-270 (January, 1961)



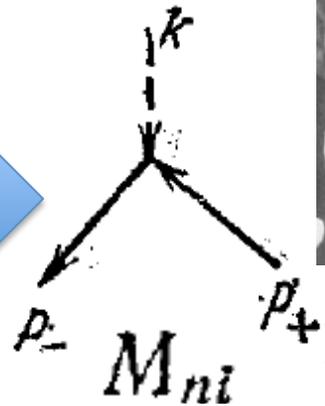
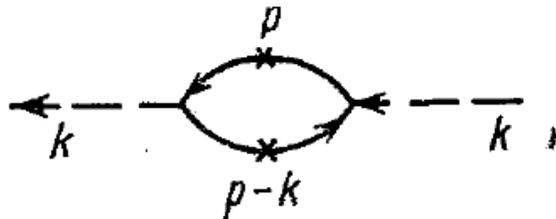
A computation is carried out of Raman scattering upon reflection of light from the surface of a superconductor. The distribution with angle and frequency of the scattered light and the absolute magnitude of the effect are found.



## VII. “Moscow Zero” and development of the diagrammatic technics

The Green's functions and the effective cross sections of the Compton effect and mutual electron and positron scattering at high energies were calculated.

Dirac, Feynman, Schwinger



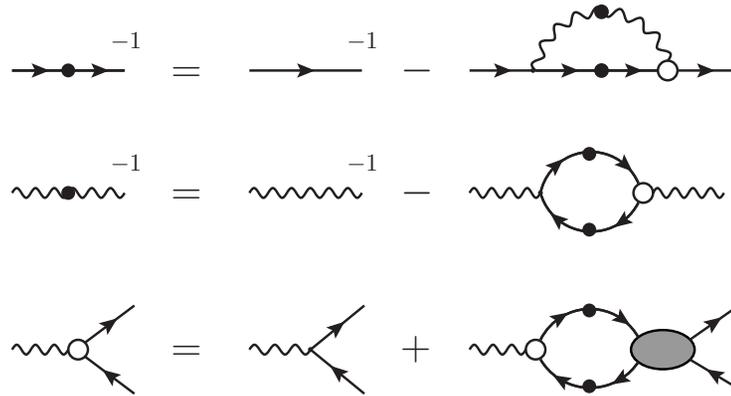
$$\mathcal{P}(k^2) = \frac{\alpha}{3\pi} k^2 \ln \frac{|k^2|}{m^2} \quad \frac{\alpha}{3\pi} \ln \frac{|k^2|}{m^2} \ll 1$$

QED suffered from the divergence problems. The idea to smear electron charge in space was proposed, but the answers depended on the “electron radius”

# Asymptotic behavior of the photon propagator at large momenta

A. A. Abrikosov, I. M. Khalatnikov and L. D. Landau, 1954

$$\mathcal{P}(k^2) = \frac{\alpha}{3\pi} k^2 \ln \frac{|k^2|}{m^2} \quad \text{Is valid up to} \quad \frac{\alpha}{3\pi} \ln \frac{|k^2|}{m^2} \leq 1.$$



Selection of diagrams by  $\alpha^n \ln^n (|k^2|/m^2)$

$$\alpha = \frac{\alpha_0}{1 + \frac{\alpha_0}{3\pi} \ln \frac{\Lambda^2}{m^2}}$$

LAKh solved the Schwinger-Dyson equations

$$\mathcal{D}(k^2) = \frac{4\pi}{k^2} \frac{1}{1 - \frac{\alpha}{3\pi} \ln \frac{|k^2|}{m^2}} \quad \text{QED}$$

LAKh: Renormalized photon Propagator does not depend on the size of electron smearing

$$\mathcal{D}(k^2) = \frac{4\pi}{k^2} \frac{1}{1 + \frac{\alpha}{3\pi} \ln \frac{|k^2|}{m^2}} \quad \text{QCD}$$

Gross, Wilczek, Politzer, 1973  
Asymptotic freedom

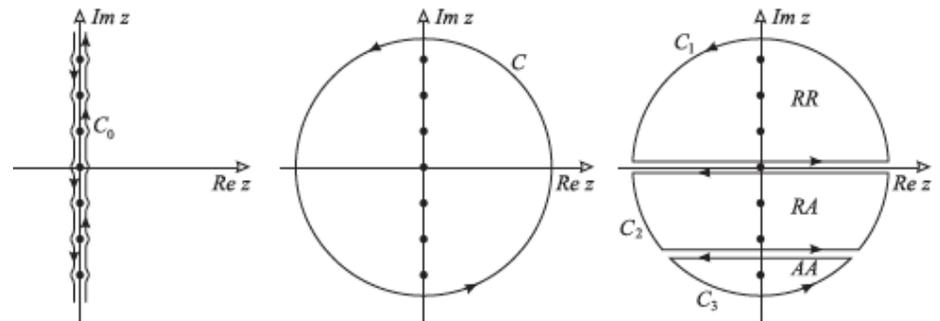
In 1959, following the works Matsubara, Migdal and Galitski, the new temperature diagrammatic technique was developed by Abrikosov along with Gor'kov and Dzyaloshinskii. It was based on their achievements in QED and some other beautiful ideas: the method of analytical continuation of Feynman diagrams from imaginary to real frequencies, etc.

On the application of quantum-field-theory methods to problems of quantum statistics at finite temperatures, Sov. Phys. JETP 9(3), 636-641 (1959)

$$\tilde{\psi}_\alpha(\mathbf{r}, \tau) = e^{\tau(\hat{H} - \mu\hat{N})} \psi_\alpha(\mathbf{r}) e^{-\tau(\hat{H} - \mu\hat{N})},$$

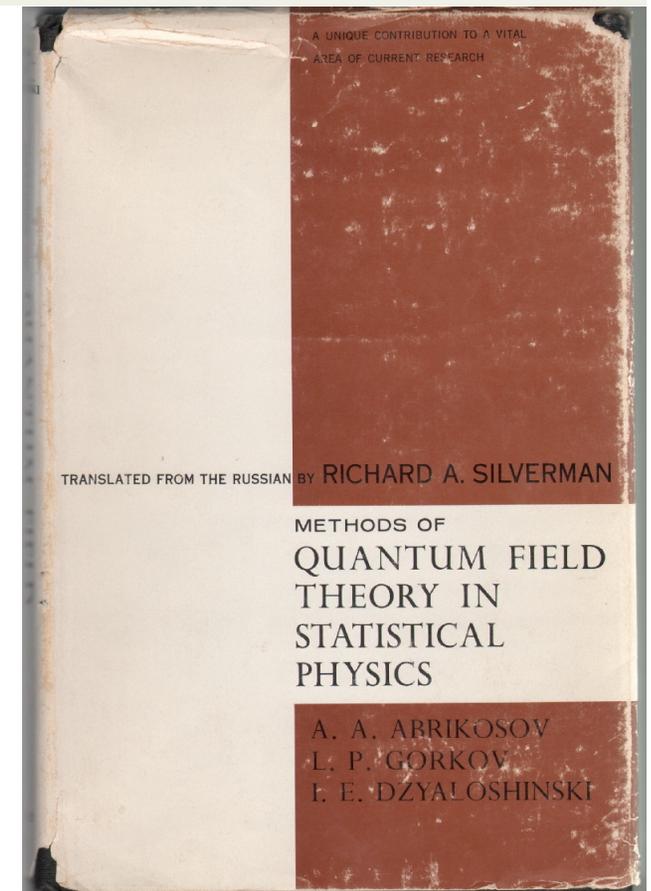
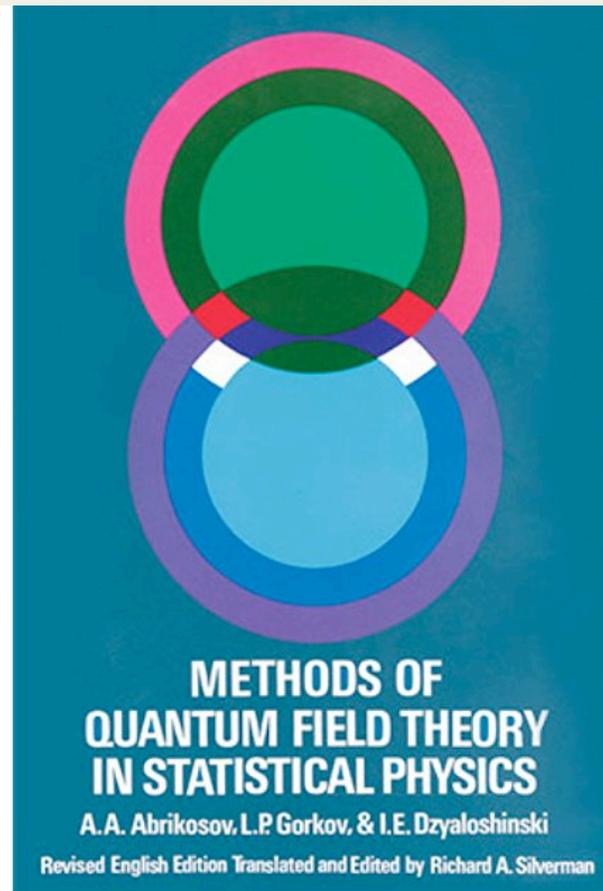
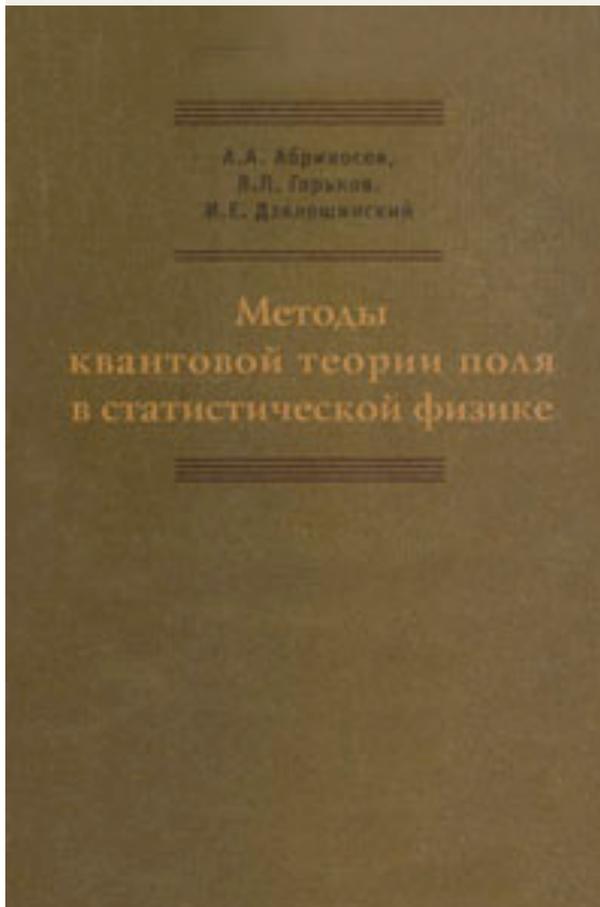
$$\tilde{\bar{\psi}}_\alpha(\mathbf{r}, \tau) = e^{\tau(\hat{H} - \mu\hat{N})} \psi_\alpha^\dagger(\mathbf{r}) e^{-\tau(\hat{H} - \mu\hat{N})},$$

$$\tilde{\varphi}(\mathbf{r}, \tau) = e^{\tau\hat{H}} \varphi(\mathbf{r}) e^{-\tau\hat{H}}.$$



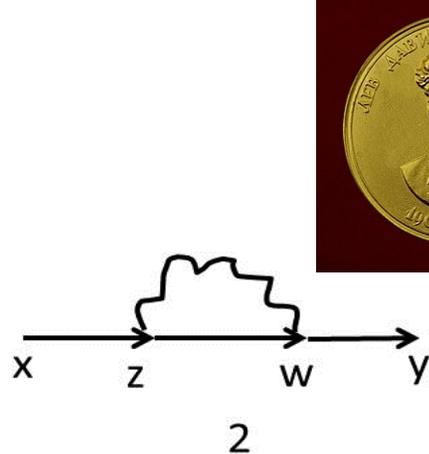
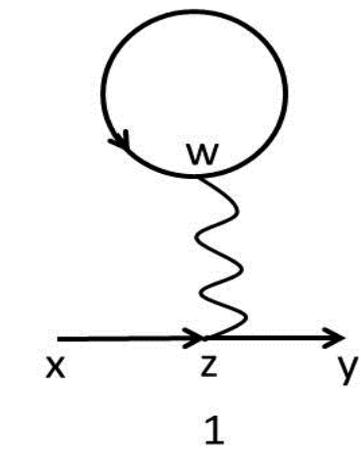
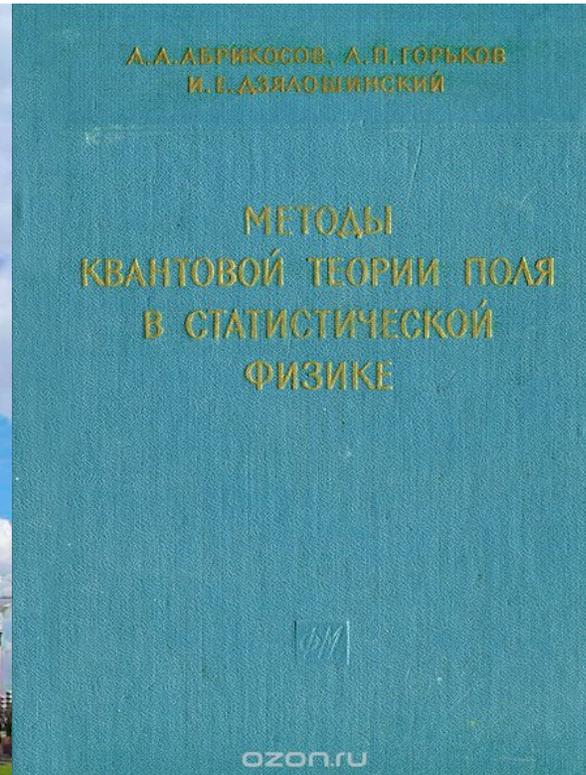
$$\begin{aligned} \mathcal{G}_{\alpha\beta}(\mathbf{r}_1, \tau_1; \mathbf{r}_2, \tau_2) &= -\text{Sp} \left\{ e^{\frac{\Omega + \mu\hat{N} - \hat{H}}{T}} T_\tau(\tilde{\psi}_\alpha(\mathbf{r}_1, \tau_1) \tilde{\bar{\psi}}_\beta(\mathbf{r}_2, \tau_2)) \right\} \\ &\equiv -\langle T_\tau(\tilde{\psi}_\alpha(\mathbf{r}_1, \tau_1) \tilde{\bar{\psi}}_\beta(\mathbf{r}_2, \tau_2)) \rangle. \quad (11). \end{aligned}$$

VIII. A.A. Abrikosov, L.P. Gor'kov, I.E. Dzyaloshinskiy, 1961 :  
“Methods of Quantum Field Theory in Statistical Physics”.  
This book became a handbook for theoretical physicists in many countries where it was translated and published.



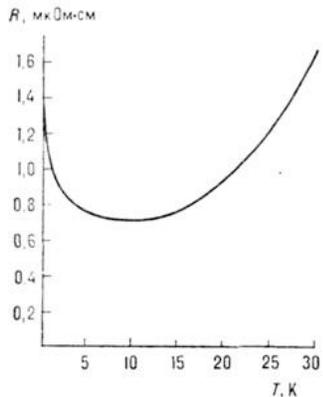


<http://ernycomp.chat.ru/>

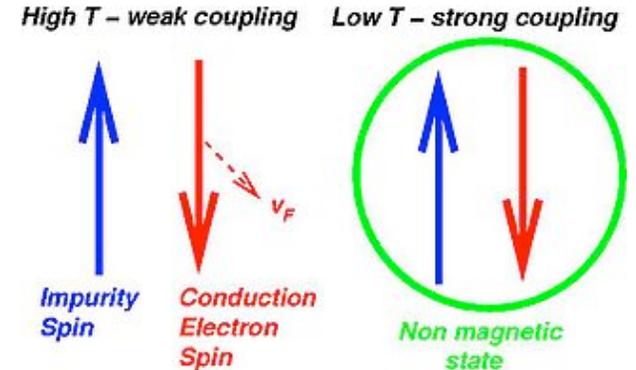


# IX. Kondo problem: Abrikosov-Suhl resonance

In 1960's Abrikosov's scientific interests moved towards the theory of normal metals, semimetals, and semiconductors. He was engaged in the Kondo problem and found that, depending on the sign of exchange interaction, the effective scattering either vanishes or increases greatly.



The Kondo resonance forms due to a coherent many-body interaction between itinerant electrons and local moments. It is the signature of the formation of a coherent many-body state.



The Abrikosov pseudo-fermion representation for spin operators with spin  $S = 1/2$  is given by

$$\mathbf{S}_i = \frac{1}{2} \begin{pmatrix} c_{i\uparrow}^\dagger & c_{i\downarrow}^\dagger \end{pmatrix} \boldsymbol{\sigma} \begin{pmatrix} c_{i\uparrow} \\ c_{i\downarrow} \end{pmatrix}, \quad c_{i\uparrow}^\dagger c_{i\uparrow} + c_{i\downarrow}^\dagger c_{i\downarrow} = 1,$$

where the operators  $c_{i\sigma}$  and  $c_{i\sigma}^\dagger$  are fermionic operators obeying the usual fermionic anticommutation relations, and  $\boldsymbol{\sigma} = (\sigma^x, \sigma^y, \sigma^z)$  is the vector of the Pauli matrices

# X. Physical Properties of Bismuth

Abrikosov with Falkovsky formulated the theory of bismuth type semimetals . As a result, the crystal structure of semimetals was explained.

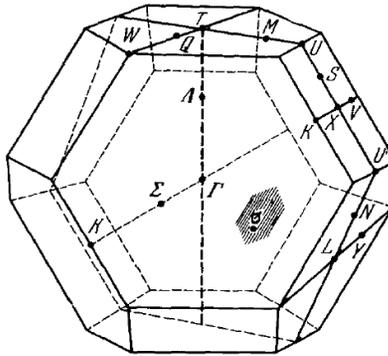
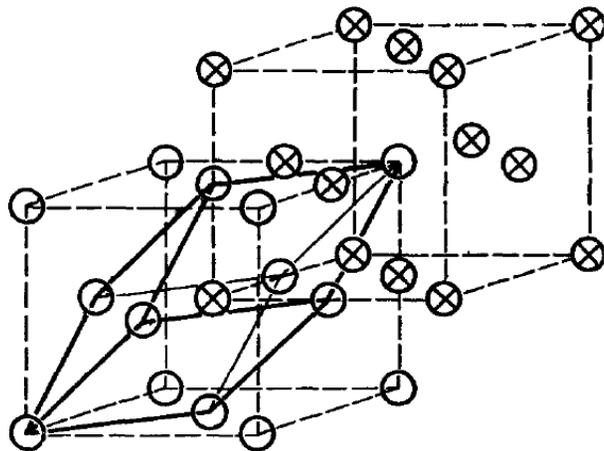
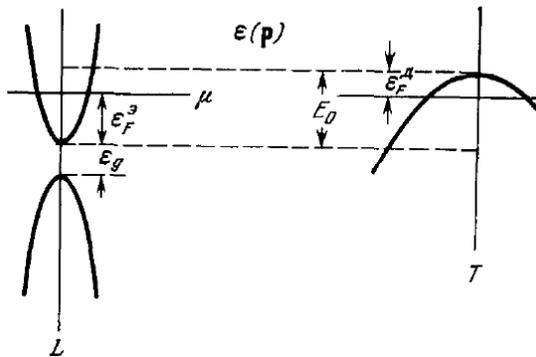


Рис. 2. Первая зона Бриллюэна.



## 3. Deformation Theory

We proceed to describe the concepts developed in <sup>[9]</sup>. To obtain the energy spectrum, the lattice considered in <sup>[9]</sup> was infinitesimally close to a cubic one, and was then deformed into the true lattice of bismuth. In order that the change of the spectrum occur without a phase transition, the “dielectric” phase (see p. 3) is con-



$$\tilde{\mathcal{H}} = \begin{pmatrix} -\epsilon_g I & H_1 \\ H_1^+ & 0 \end{pmatrix}, \quad H_1 = \mathbf{p} \begin{pmatrix} \mathbf{t} & \mathbf{u} \\ -\mathbf{u}^* & \mathbf{t}^* \end{pmatrix}$$

# XI Semimetals. Prototype of the future Dirac materials

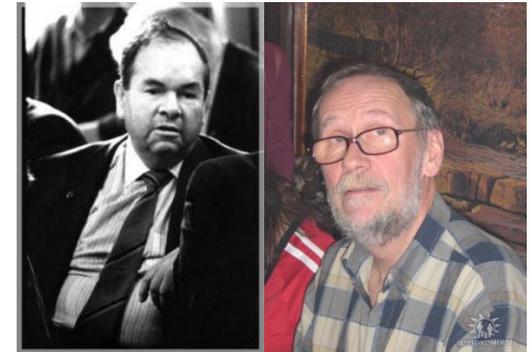
## *POSSIBLE EXISTENCE OF SUBSTANCES INTERMEDIATE BETWEEN METALS AND DIELECTRICS*

A. A. ABRIKOSOV and S. D. BENESLAVSKIĬ

L. D. Landau Institute of Theoretical Physics

Submitted April 13, 1970

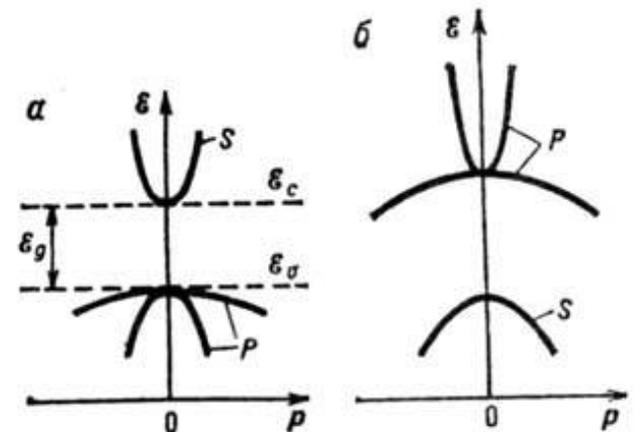
Zh. Eksp. Teor. Fiz. 59, 1280—1298 (October, 1970)



The question of the possible existence of substances having an electron spectrum without any energy gap and, at the same time, not possessing a Fermi surface is investigated. First of all the question of the possibility of contact of the conduction band and the valence band at a single point is investigated within the framework of the one-electron problem. It is shown that the symmetry conditions for the crystal admit of such a possibility. A complete investigation is carried out for points in reciprocal lattice space with a little group which is equivalent to a point group, and an example of a more complicated little group is considered. It is shown that in the neighborhood of the point of contact the spectrum may be linear as well as quadratic.



The types of symmetry allowing a gapless spectrum were found, the spectrum of carriers and its behavior under pressure were analyzed. This work is especially topical today in connection with the discovery of graphene and the related prospects of the development of nanoelectronics.



# Soviet physicists in vacations: hike to the taiga (1970)



Yuri Galperin

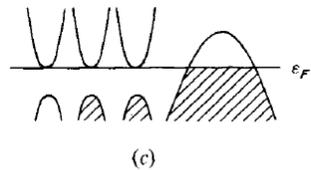
Alexei Abrikosov

Vadim Gurevich

# XII. Transition of a bismuth-type semimetal to an excitonic insulator in a strong magnetic field

Transition of a bismuth-type semimetal to an excitonic insulator in a strong magnetic field

A. A. Abrikosov, [Journal of Low Temperature Physics](#)  
January 1973, Volume 10, [Issue 1-2](#), pp 3-34

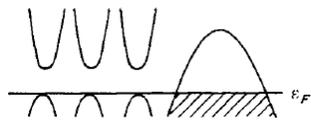


$$G = \frac{1}{3} \left\{ \frac{(i\bar{\omega}_2 + \xi_h)}{[(i\bar{\omega}_1 - \xi_e)(i\bar{\omega}_2 + \xi_h) - 3|\bar{\Delta}_1|^2]} \right\} + \frac{2}{3} \frac{1}{i\bar{\omega}_1' - \xi_e},$$

$$D = \frac{1}{3} \left\{ \frac{(i\bar{\omega}_2 + \xi_h)}{[(i\bar{\omega}_1 - \xi_e)(i\bar{\omega}_2 + \xi_h) - 3|\bar{\Delta}_1|^2]} \right\} - \frac{1}{3} \frac{1}{i\bar{\omega}_1' - \xi_e},$$

$$\mathcal{L}(-\omega) = - \frac{(i\bar{\omega}_1 - \xi_e)}{[(i\bar{\omega}_1 - \xi_e)(i\bar{\omega}_2 + \xi_h) - 3|\bar{\Delta}_1|^2]},$$

$$F^+(\omega) = \frac{\bar{\Delta}_1^*}{[(i\bar{\omega}_1 - \xi_e)(i\bar{\omega}_2 + \xi_h) - 3|\bar{\Delta}_1|^2]} = -F(-\omega) \frac{\bar{\Delta}_1^*}{\bar{\Delta}_1},$$



Investigation of the Excitonic Insulator Phase in Bismuth-Antimony Alloys

N. B. Brandt and S. M. Chudinov

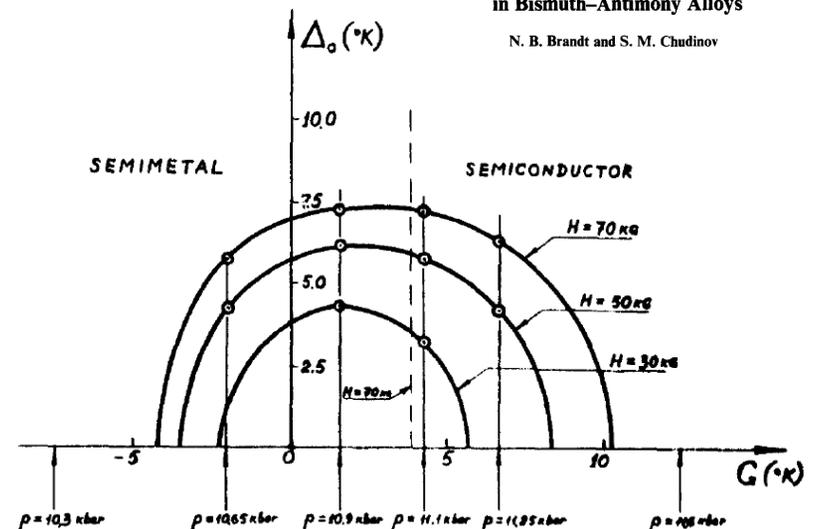


Fig. 10. Schematic diagram of the exciton insulator gap  $\Delta_0$  for various values of the magnetic field  $H$  vs. the "overlap-gap"  $G$  at  $H = 0$  in the unrearranged spectrum of the alloy  $\text{Bi}_{0.95}\text{Sb}_{0.049}$  ( $T = 2.0$  K). The circles indicate the values of  $\Delta_0$  calculated for  $p = 10.65, 10.9, 11.1, \text{ and } 11.3$  kbar. The dashed line corresponds to the metal-semiconductor transition ( $G = 0$ ) in the unrearranged system at  $H = 70$  kOe.

# XIII. Contribution to quasi-one-dimensional physics

In the 1970s-1980s, Abrikosov constructs the original method for calculating the conductivity of a quasi-one-dimensional metal, which permits account for electron jumps between filaments and electron scattering by phonons and impurities

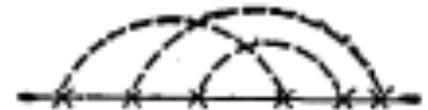
## Electrical properties of one-dimensional metals

A. A. Abrikosov and I. A. Ryzhkin



Instead of the interaction with the impurities we introduce random fields<sup>1)</sup>  $\eta(z)$  and  $\zeta(z)$ . The interaction with the field  $\eta(z)$  leaves an electron in the vicinity of  $p_0$  if it was there, i. e., this interaction is diagonal in the indices 1 and 2. We can assume this interaction to be real and to have the following properties:

$$\langle \eta(z) \rangle = 0, \quad \langle \eta(z) \eta(z') \rangle = \delta(z-z') v / \tau_1. \quad (5)$$

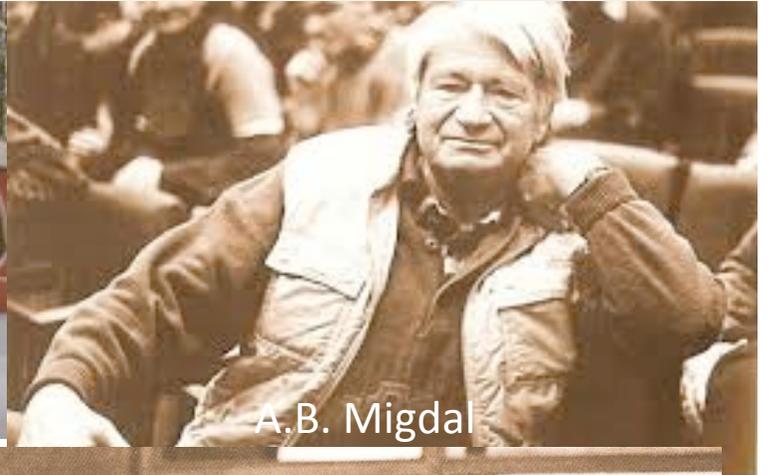


The second field  $\zeta$  transfers an electron from the vicinity of  $p_0$  to the vicinity of  $-p_0$ . This field must be assumed to be complex

$$\langle \zeta(z) \zeta^*(z') \rangle = \delta(z-z') v / \tau_2, \quad \langle \zeta \rangle = 0, \quad \text{as a "quantum" of such a field}$$

$$\langle \zeta(z) \zeta(z') \rangle = 0. \quad \text{can be "emitted" only in the transition } 1 \rightarrow 2 \text{ and}$$

XIV. Abrikosov carried on active scientific, organizational and pedagogical work at the L. D. Landau Institute of Theoretical Physics (he was one of the founders of this Institute, 1965)



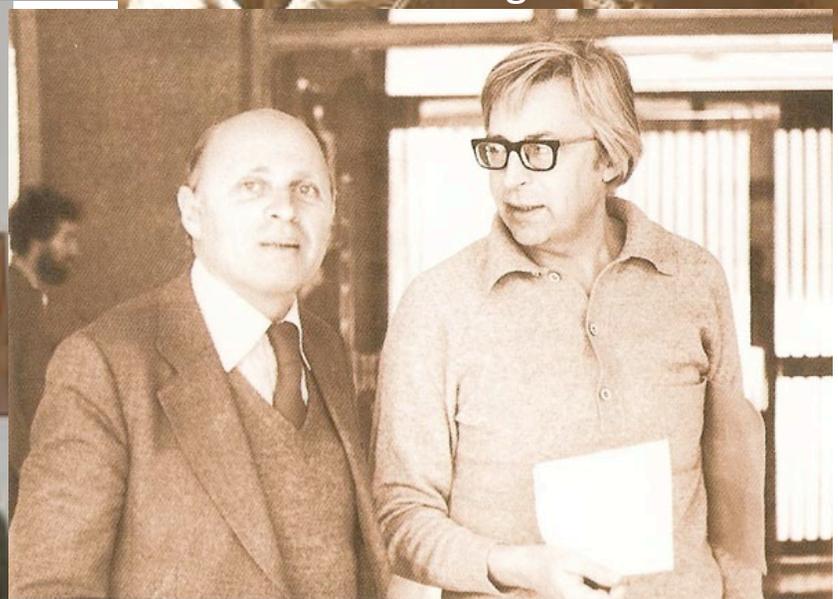
A.B. Migdal



A. A. Abrikosov



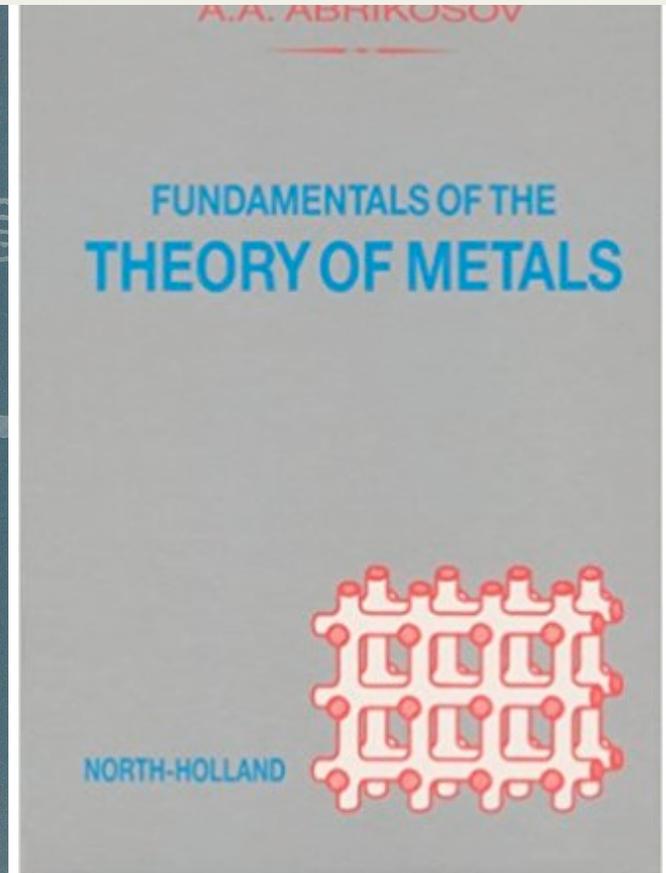
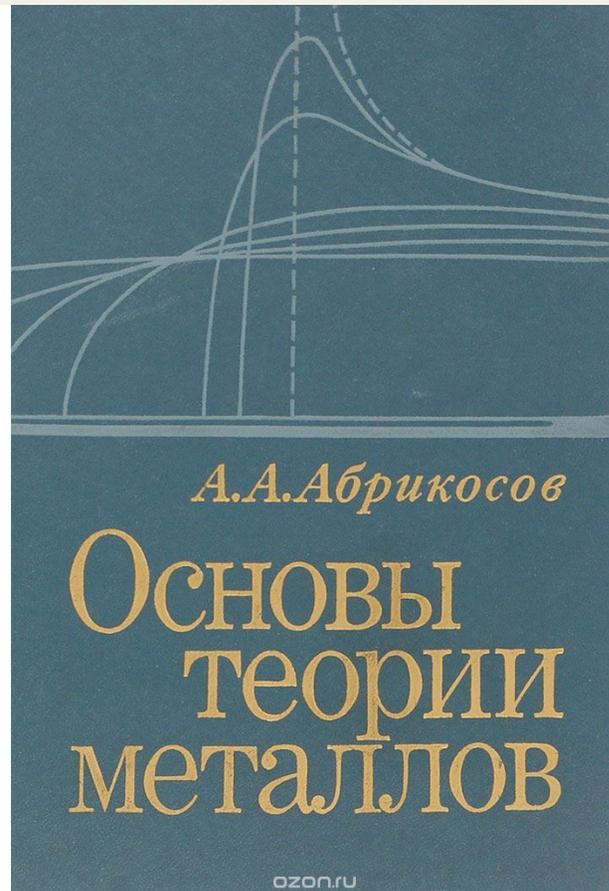
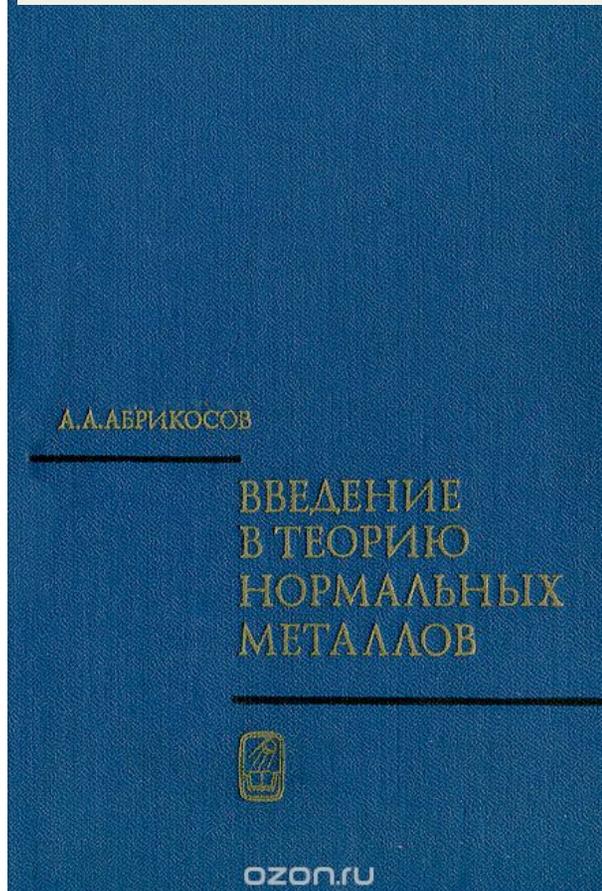
I.E. Dzyaloshinskiy



I.M. Khalatnikov

L.P. Gor'kov

XV. In 1987 Abrikosov published a book “Fundamentals of the Theory of Metals” which was based on his brilliant lecture courses delivered at the Moscow Institute of Physics and Technology and Moscow Institute of Steel and Alloys (MISA). This book became an encyclopedia of the theory of normal metals and superconductors.

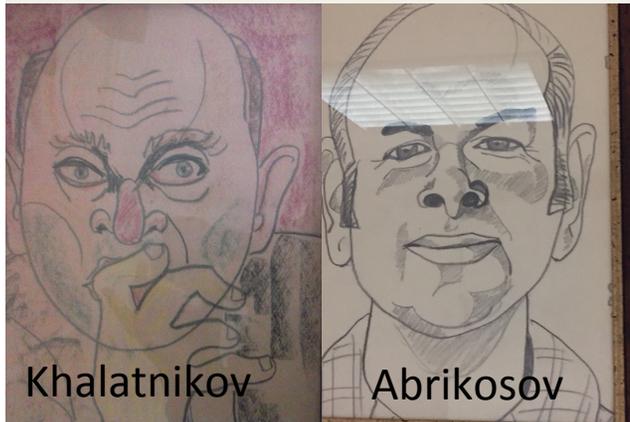


# XVI. Since 1976 Abrikosov headed the Department of Theoretical Physics at the Moscow Institute of Steel and Alloys (MISA)



Y. Yul'kin, A. Abrikosov, L. Aslamazov

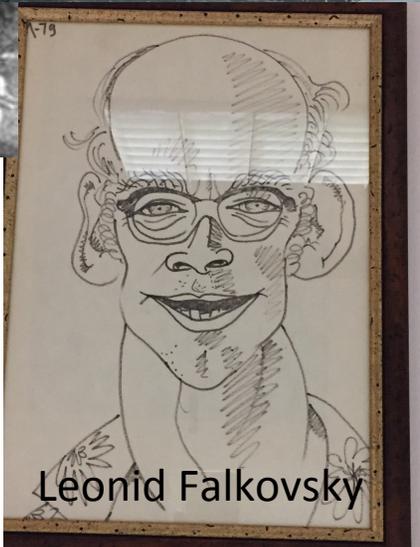
XVII. Odessa symposiums in theoretical physics of Landau Institute organized by Abrikosov & Khalatnikov since 1961 'till the end of 80s



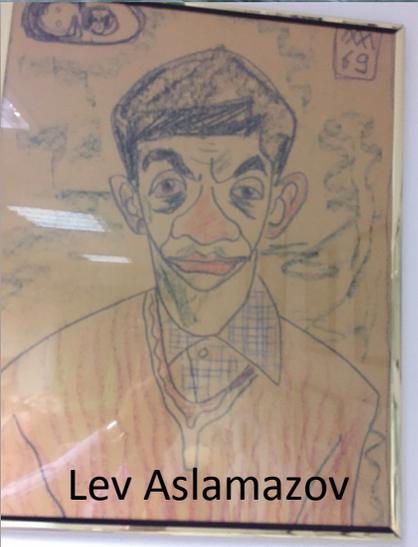
Odessa, Arkadia



Anybody who had occasion to meet Abrikosov, to work with him, to participate in the regular symposia on theoretical physics, organized by him, remembers his erudition and adherence to principles, his teller's gift.



Leonid Falkovsky



Lev Aslamazov

# Abrikosov & Khaikin's schools "Physics of Metals", Kazan, 1976



# Abrikosov & Khaikin's school "Physics of Metals", Tadgikistan, 1987

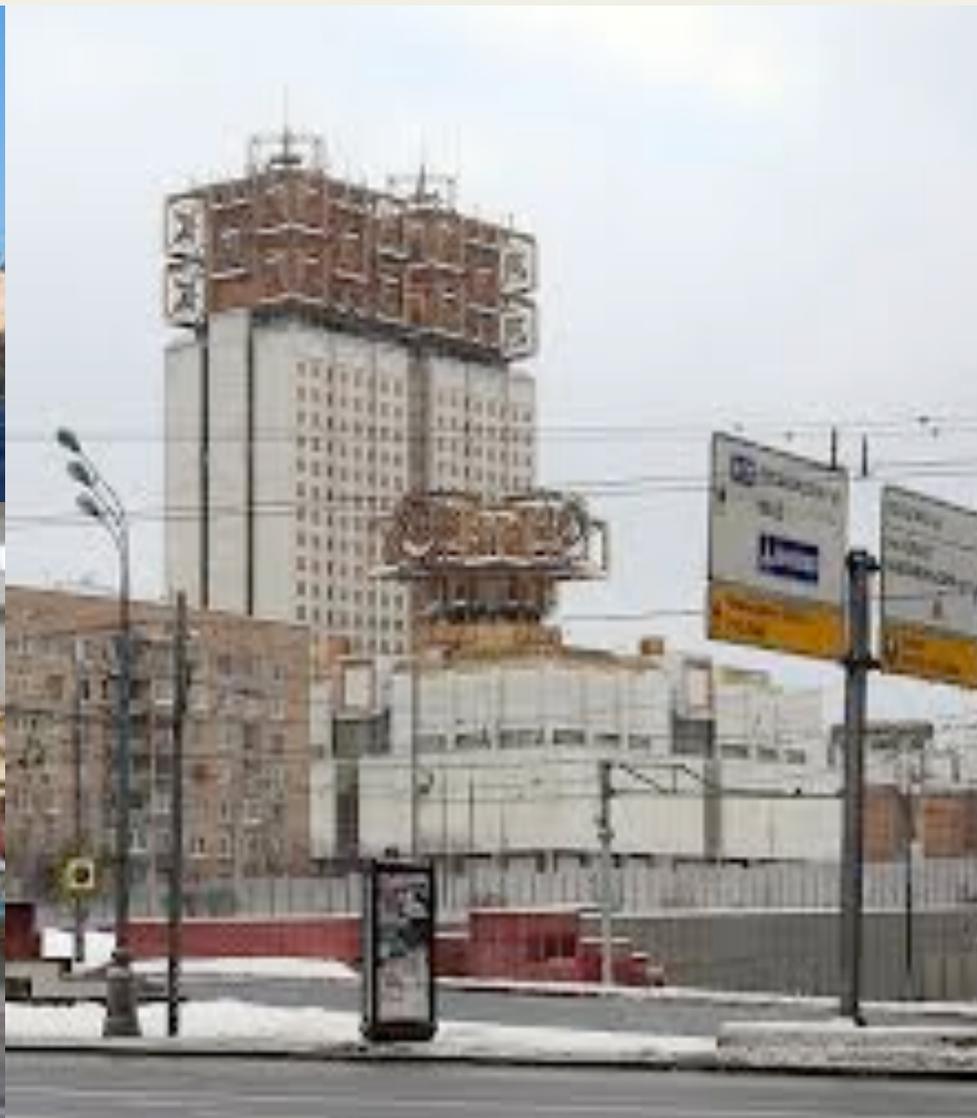


M. Gershenzon, R. Shekhter,  
B. Altshuler, A. Varlamov,  
V. Shaternik, A. Shpigel

# Abrikosov with pupils (A.Tsvelik, 1978)



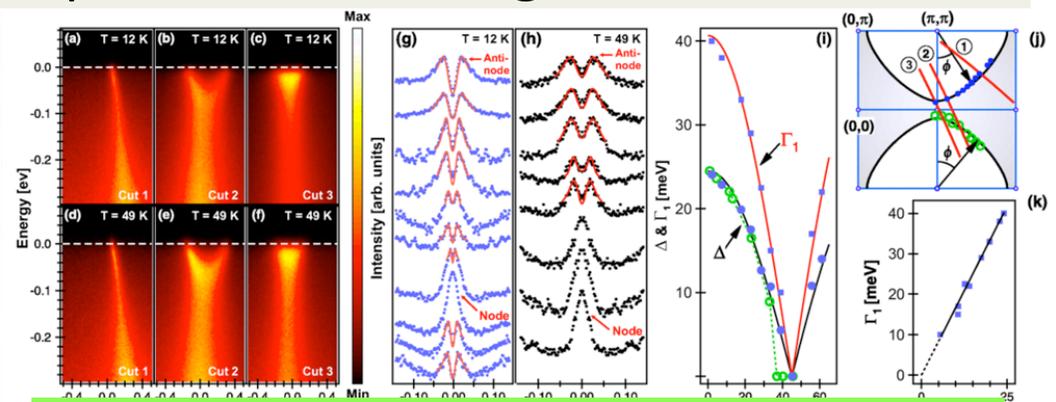
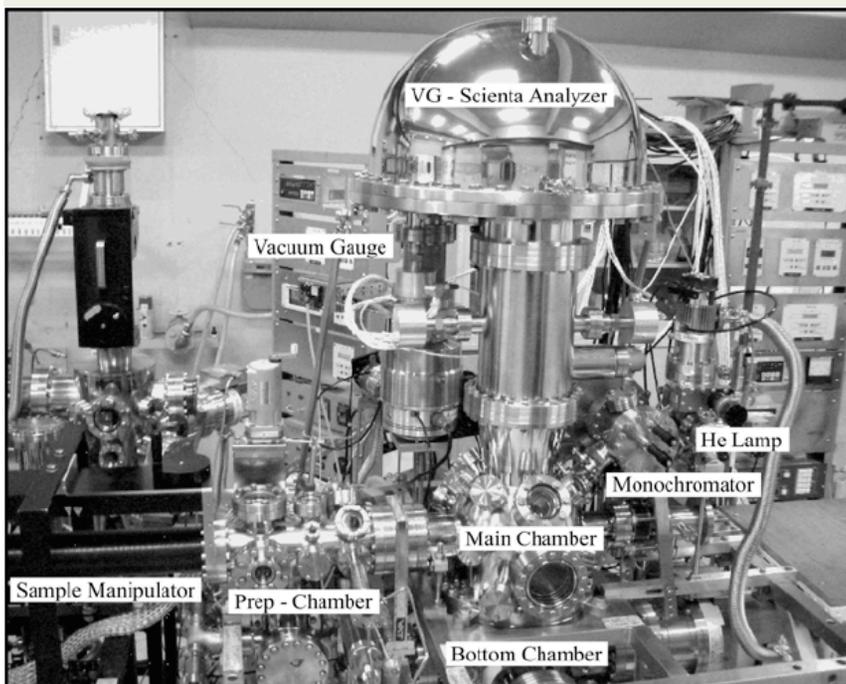
XVIII. In the period from 1988 to 1991, Abrikosov headed the Institute for High-Pressure Physics of RAS



XIX. At the beginning of the 1990s, Abrikosov accepted an offer to head the Theoretical Group in the Argonne National Laboratory and left for the USA



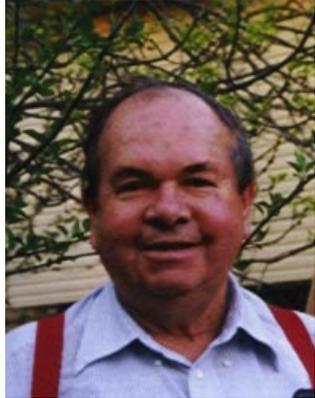
XX. The most intriguing problem in condensed matter physics in those days was the explanation of the phenomenon of high-temperature superconductivity, and Abrikosov, in close contact with the experimentalists of the Laboratory, devoted himself to studies in this field. They revealed the existence of a specific singularity in the spectrum of cuprate superconductors, after which Abrikosov proposed his version of the theory of high-temperature superconductivity that explained the variety of existing experimental findings.



## Main ingredients of Abrikosov's theory of HTS

- BCS nature
- Quasi-two dimensionality
- Van Hove singularity
- Resonance tunneling along c-axis

# XXI. The last foresight... Quantum magnetoresistance

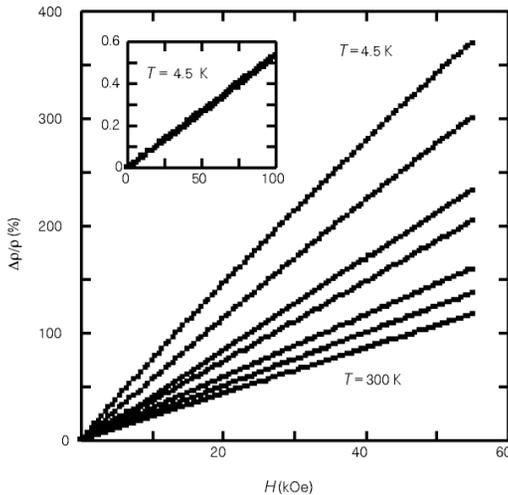


A. A. Abrikosov

Materials Science Division, Argonne National Laboratory, 9700 South Cass Avenue, Argonne, Illinois 60439

(Received 26 September 1997; revised manuscript received 9 March 1998)

An explanation is proposed of the unusual magnetoresistance, linear in magnetic field and positive, observed recently in nonstoichiometric silver chalcogenides. The idea is based on the assumption that these substances are basically gapless semiconductors with a linear energy spectrum. Most of the excess silver atoms form metallic clusters which are doping the remaining material to a very small carrier concentration, so that even in a magnetic field as low as 10 Oe, only one Landau band participates in the conductivity.



$$H = \int \psi^\dagger v \left[ \sigma \left( \mathbf{p} - \frac{e}{c} \mathbf{A} \right) \right] \psi dV, \quad -i \frac{\partial}{\partial z} \psi_1 + \left( -i \frac{\partial}{\partial x} - \frac{\partial}{\partial y} + i \frac{eH}{c} x \right) \psi_2 = \frac{\epsilon}{v} \psi_1,$$

$$\left( -i \frac{\partial}{\partial x} + \frac{\partial}{\partial y} - i \frac{eH}{c} x \right) \psi_1 + i \frac{\partial}{\partial z} \psi_2 = \frac{\epsilon}{v} \psi_2.$$

$$G_{\alpha\beta}^{(0)}(p_z, p_y, x, x', \omega_m) = \sum_n \frac{\psi_{n\alpha}(x - cp_y/eH) \psi_{n\beta}^*(x' - cp_y/eH)}{i\omega_m + \mu - \epsilon_n(p_z)}, \quad Q_{ik}(i\omega_0) = \frac{2e^2 v^2}{c} T \sum_m \int \frac{dp_y}{2\pi} \int \frac{dp_z}{2\pi} \int dx'$$

$$\times \text{Tr}[\sigma_i G(p_y, p_z, x, x', \omega_m + \omega_0) \times \sigma_k G(p_y, p_z, x', x, \omega_m)],$$

**Large magnetoresistance in non-magnetic silver chalcogenides**, R. Xu, A. Husmann, T. F. Rosenbaum, M.-L. Saboungi, J. E. Enderby and P. B. Littlewood, Nature 390, 57, 1997

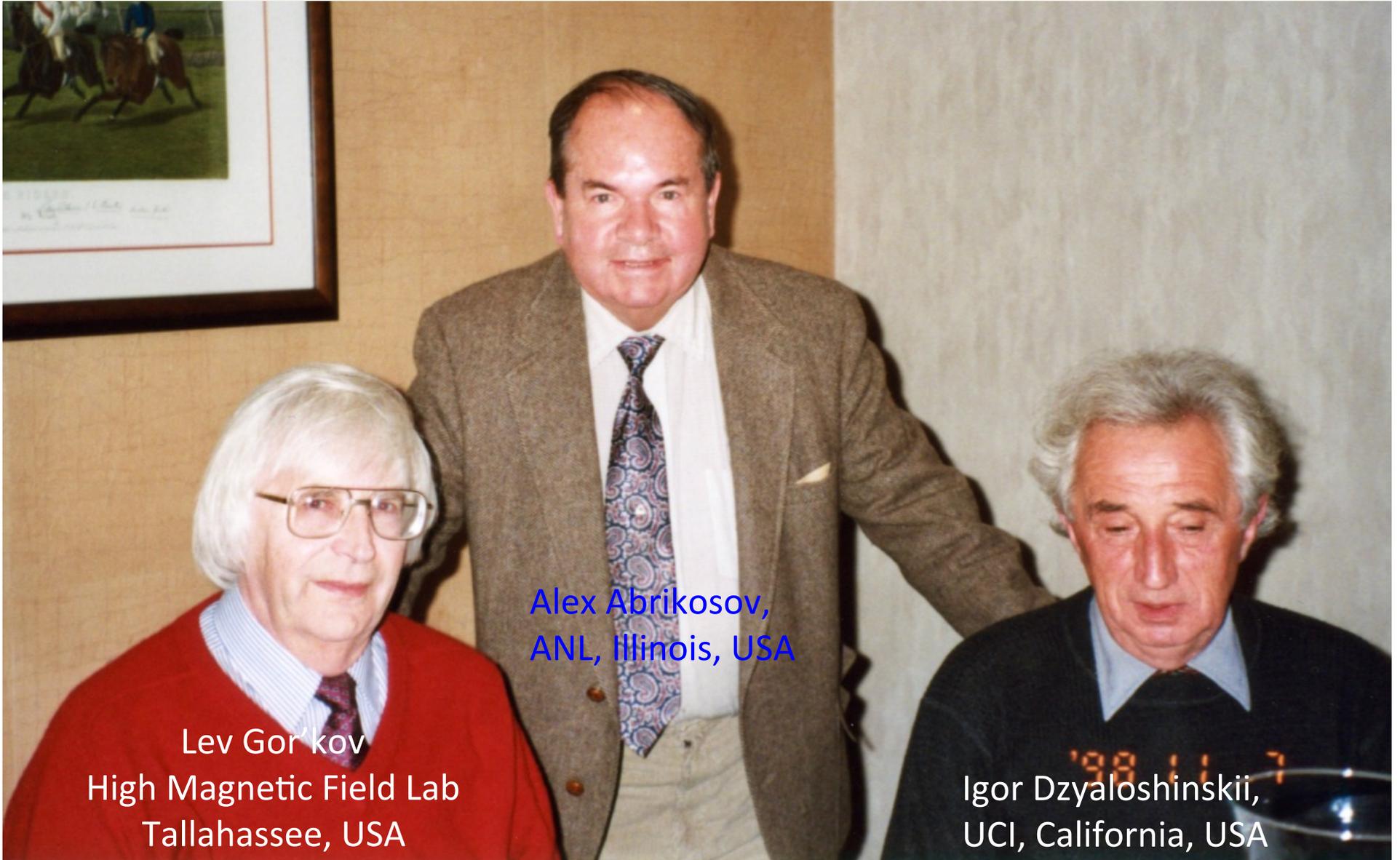
$$\rho_{xx} = \frac{1}{2\pi} \left( \frac{e^2}{\epsilon_\infty v} \right)^2 \ln \epsilon_\infty \frac{N_i}{ec n_0^2} H.$$

Richard Klemm	George Crabtree	Ernst Helmut Brandt	Lev Gor'kov	David Bishop	David Nelson	Michael Tinkham	Valerii Vinokur	David Khmel'nitskii
Grigorii Volovik	Boris Shklovskii	Boris Altshuler	Alexei Abrikosov	Phil W. Anderson	Igor' Dzyaloshinskii			

Abrikosov's 70th Birthday Symposium, 6 Nov 1998 in Argonne



# The meeting of three coauthors in forty years, 1998 , Argonne, USA



Alex Abrikosov,  
ANL, Illinois, USA

Lev Gor'kov  
High Magnetic Field Lab  
Tallahassee, USA

Igor Dzyaloshinskii,  
UCI, California, USA

Visit to Italy in 2000: Camerino



Visit to Italy in 2000: Frascati



Vortex conference, Crete, September 2003



# Argonne, October, 5, 2003, next day after the Nobel Committee announcement



10 Dec 2003 Stockholm





# Nobel prize Winner in third Generation

## The Nobel Prize in Physics 1922



The Nobel Prize in Physics 1922 was awarded to **Niels Bohr** "*for his services in the investigation of the structure of atoms and of the radiation emanating from them*".



## The Nobel Prize in Physics 1962



The Nobel Prize in Physics 1962 was awarded to **Lev Landau** "*for his pioneering theories for condensed matter, especially liquid helium*".

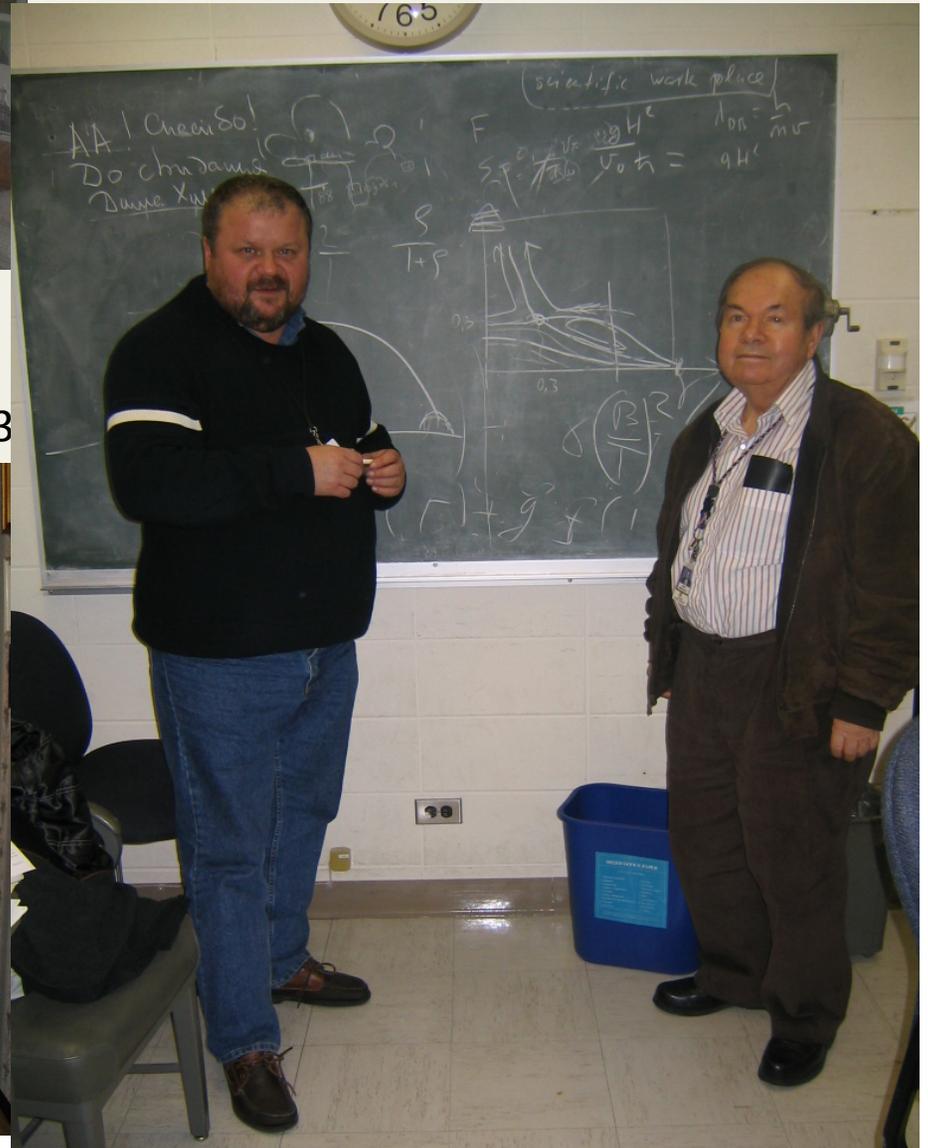


The Nobel Prize in Physics 2003 was awarded to **Alexei Abrikosov, Vitaly L. Ginzburg, Anthony J. Leggett** "*for pioneering contributions to the theory of superconductors and superfluids*".



Abrikosov was always surrounded by colleagues and pupils

Dmitri Livanov, the Minister of Science and Education of RF, alumni of the Department of Theoretical Physics of MISA, visits Abrikosov, 2013



A. Glatz

K. Kadowaki

A. Buzdin

M. Tachiki

Abrikosov's 80th Birthday Symposium, 8 Nov 2008 in Argonne



Tony Leggett  
Alexei Abrikosov

# Last Prize in his life: Vernadsky Gold Medal



**Abrikosov receives  
Ukrainian Gold Medal**

By [Lynn Tefft Hoff](#) • July 28, 2015

**Alexei Abrikosov**, a distinguished scientist at the U.S. Department of Energy's Argonne National Laboratory and a Nobel Prize recipient, has been recognized again for his groundbreaking work. Abrikosov has received the Gold Medal of Vernadsky of the National Academy of Sciences of the Ukraine.

As part of the medal presentation, **Director Peter Littlewood** read from a letter to Abrikosov from **Ukrainian National Academy President Boris Paton**. "The National Academy of Sciences values your outstanding contributions in the development of the theory of normal and superconducting metals and your long, active cooperation with Ukrainian scientists," Paton wrote.

The Academy has awarded its Vernadsky Gold Medal annually since 2004 to the most distinguished academicians.

2003 Borys Paton

...

2010 Mikhail Lisitsa and Manuel Cardona

2011 Borys Oliynyk and Blaže Ristovski

2013 Oleksandr Huz and Herbert Mang

2014 Vadim M. Loktev

**2015 Alexei Abrikosov**



9 ноября 2015 г.

Президенту Национальной Академии Наук Украины  
Академику Б. Е. Патону

Глубокоуважаемый Борис Евгеньевич!

Прежде всего, я хочу выразить мою благодарность Академии Наук Украины и Вам лично за оказанную честь – присуждение мне высшей награды Национальной академии наук Украины – Золотой медали имени В. И. Вернадского. Диплом и медаль были вручены мне директором Аргонской Национальной лаборатории Питером Литлвудом, что стало праздником и для меня, и для моих многочисленных коллег.

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Alexei Alexeevich Abrikosov will be remembered as a great scientist whose work influenced many fields of physics and paved the way to a new comprehension of superconductivity. He was a very bright man with a brilliant sense of humor, which helped him to overcome the difficult periods of a life so full of events.