Output stream of leaky integrate-and-fire neuron without diffusion approximation

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Probability density function (pdf) of output interspike intervals (ISI) as well as its first and second moments are found in exact form for leaky integrate-and-fire (LIF) neuron stimulated with Poisson stream. The diffusion approximation is not used. The whole range of possible ISI values is represented as infinite union of disjoint intervals: $]0;\infty[=]0;T_2]+\sum_{m=0}^{\infty}]T_2+m\,T_3;T_2+(m+1)T_3]$, where T_2 and T_3 are defined by the LIF's physical parameters. Exact expression for the obtained pdf is different on different intervals and is given as finite sum of multiple integrals. The found distribution can be bimodal for some values of parameters. The moments found are as follows:

$$\bar{t}=\frac{2}{\lambda}+\frac{a^r}{\lambda(1-r\beta^r\Phi(\beta,1,r))},$$

$$\overline{t^2}=\frac{6}{\lambda^2}+\frac{2a^r}{\lambda^2(1-r\beta^r\Phi(\beta,1,r))}\Bigg(3+\lambda T_2+ \\ +\frac{r\beta^r\Phi(\beta,1,r)}{1-r\beta^r\Phi(\beta,1,r)}\left(\lambda T_3+r\frac{\Phi(\beta,2,r)}{\Phi(\beta,1,r)}\right)\Bigg),$$

where $r=\lambda\tau$, $a=(V_0-h)/h$, $\beta=a/(a+1)$, $T_2=\tau\log(h/(V_0-h))$, $T_3=\tau\log(V_0/(V_0-h))$. Here λ, V_0, h and τ are the physical parameters, and $\Phi(\beta,n,r)$ denote the Lerch transcendent.

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