Equivalence groupoids in the group classification problems

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It is widely known that there is no general theory for integration of nonlinear partial differential equations (PDEs). Nevertheless, many special cases of complete integration or finding particular solutions are related to appropriate changes of variables. Nondegenerate point transformations that leave a differential equation invariant and form a connected Lie group are called Lie symmetries of this equation. In many cases, the algorithmic Lie reduction method, which uses known Lie symmetries, results in the construction of group-invariant solutions for a given PDE. This places the transformation methods among the most powerful analytic tools currently available in the study of nonlinear PDEs.

Many nonlinear PDEs that are important for applications are parameterized by arbitrary elements (constants or functions) and constitute classes of PDEs. The problem of classification of Lie symmetries for a given class of PDEs is called the *group classification problem*. Another important task is to study transformational properties of such classes, i.e. to describe explicitly nondegenerate point transformations that link members of the class and not necessarily form a group. Such transformations, which are called admissible transformations, appear to be a useful tool not only for finding exact solutions but also for exhaustive solving group classifications problems, design of physical parameterization schemes, and study of integrability. The set of admissible transformations considered with the standard operation of composition of transformations is also called the *equivalence groupoid*.

We will discuss the role of equivalence groupoids in the solving group classification problems using illustrative examples.

1. Vaneeva O., Pošta S. Equivalence groupoid of a class of variable coefficient Korteweg–de Vries equations. J. Math. Phys., 2017, 58, 101504.