Asymptotic soliton-like solutions of the Benjamin-Bona-Mahony equation with variable coefficients

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The report deals with the singularly perturbed equation with variable coefficients

$$a(x,t,\varepsilon)u_t + b(x,t,\varepsilon)u_x + c(x,t,\varepsilon)uu_x - \varepsilon^n u_{xxt} = 0, \quad (1)$$

where $a(x, t, \varepsilon)$, $b(x, t, \varepsilon)$, $c(x, t, \varepsilon)$ are infinitely differentiable functions of variables (x, t, ε) , ε is a small parameter, n is natural.

Equation (1) is a generalization of Benjamin-Bona-Mahony (BBM) equation of the following form [1]

$$u_t + u_x + uu_x - u_{xxt} = 0 (2)$$

proposed as alternative to the Korteweg-de Vries equation in 1966. Equation (2) describes the propagation of long waves in nonlinear dispersive media and it is called the regularized long wave equation. The BBM equation has soliton solutions, but it is not integrable one since it has only three conservation laws.

We present the algorithm for construction of asymptotic solitonlike solutions to equation (1). The algorithm is based on the nonlinear WKB technique developed for constructing quasi-periodic solutions to the singularly perturbed KdV equation with constant coefficient [2] in 1974. The main results are presented in [3].

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