On the one-dimensional boundary-value problems with parameter in Hölder spaces

H. O. Masliuk

National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute", Kyiv, Ukraine

masliukgo@ukr.net

We arbitrarily choose a compact interval $[a, b] \subset \mathbb{R}$, integers $m \geq 1, r \geq 2$, and $n \geq 0$ and a real number α subject to the condition $0 < \alpha \leq 1$. We use the complex Hölder spaces $(C^{n,\alpha})^m$ and $(C^{n,\alpha})^{m \times m}$ of indexes n and α .

Let $\varepsilon_0 > 0$. We consider the following linear boundary-value problem depending on the parameter $\varepsilon \in [0, \varepsilon_0)$:

$$L(\varepsilon)y(t,\varepsilon) \equiv y^{(r)}(t,\varepsilon) + \sum_{j=1}^{r} A_{r-j}(t,\varepsilon)y^{(r-j)}(t,\varepsilon) = f(t,\varepsilon), t \in [a,b]$$
$$B(\varepsilon)y(\cdot,\varepsilon) = c(\varepsilon).$$

We suppose for every $\varepsilon \in [0, \varepsilon_0)$ that $y(\cdot, \varepsilon) \in (C^{n+r,\alpha})^m$ is an unknown vector-valued function, whereas the matrix-valued functions $A_{r-j}(\cdot, \varepsilon) \in (C^{n,\alpha})^{m \times m}$ for each $j \in \{1, ..., r\}$, vectorvalued function $f(\cdot, \varepsilon) \in (C^{n,\alpha})^m$, continuous linear operator $B(\varepsilon) : (C^{n+r,\alpha})^m \to \mathbb{C}^{rm}$ and vector $c(\varepsilon) \in \mathbb{C}^{rm}$ are arbitrarily given.

We obtain a constructive criterion under which the solutions to these problems are continuous with respect to the parameter in the normed space $C^{n+r,\alpha}$ [2]. Also a two-sided estimate for the degree of convergence of these solutions was obtained there.

For system of first-order linear differential equations, these problems were investigated in [1].

- Mikhailets V. A., Murach A. A., Soldatov V. Continuity in a parameter of solutions to generic boundary-value problems. Electron. J. Qual. Theory Differ. Equ., 2016, no. 87, 1–16.
- Masliuk H., Soldatov V. One-dimensional parameter-dependent boundary-value problems in Holder spaces. Methods of Functional Analysis and Topology, 2018, V. 24, 2, 143 – 151.