# Universal deformations of Poisson structures using the Kontsevich graph calculus

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There exist at least countably many deformations $\mathcal{P} \longmapsto \mathcal{P} + \varepsilon\mathcal{Q}(\mathcal{P}) + \bar{o}(\varepsilon)$ which are universal w.r.t. all Poisson brackets $\mathcal{P}$ on all finite-dimensional affine manifolds; in the course of every such symmetry of the Jacobi identity, the bi-vector $\mathcal{P} + \varepsilon\mathcal{Q}(\mathcal{P}) + \bar{o}(\varepsilon)$ remains Poisson up to $\bar{o}(\varepsilon)$. These flows are encoded using graphs with local wedge ordering of decorated edges.

Kontsevich's orientation morphism [1] is a way to construct the terms $\mathcal{Q}(\mathcal{P})$ from cocycles $\gamma$ in the unoriented graph complex. It guarantees the existence of factorisation $[\![\mathcal{P}, \mathcal{Q}(\mathcal{P})]\!] = \Diamond(\mathcal{P}, [\![\mathcal{P}, \mathcal{P}]\!])$ of the Poisson cocycle condition through differential consequences of the Jacobi identity $[\![\mathcal{P}, \mathcal{P}]\!] = 0$. We illustrate the work of the mechanism that produces a graph expansion of $\Diamond$ for the known flows. In this set-up, the operator $\Diamond$ is completely determined by the graph cocycle $\gamma$ from which the deformation was built.

The core open problem is the (non)triviality of velocities in Poisson cohomology, i.e. the solvability of equation $\mathcal{Q}(P) = [\![\mathcal{P}, X]\!] + \nabla(\mathcal{P}, [\![\mathcal{P}, \mathcal{P}]\!])$ w.r.t. vectors $X$ and operators $\nabla$. Countably many global vector fields $X$ would produce multivector deformations which are indistinguishable from the shifts under infinitesimal coordinate changes along integral trajectories of $X$, so that the affine, "quantum" manifold looks like a smooth one!

*The research of AVK is supported in part by the IHÉS (Bures-sur-Yvette, France) and Nokia Fund.*

1. Buring R., Kiselev A.V. The orientation morphism: from graph cocycles to deformations of Poisson structures. J. Phys.: Conf. Ser., 2019, 1194 (Paper 012017), 1–10. `arXiv:1811.07878`