Estimates of the inner radii of symmetric non-overlapping domains

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Let $\mathbb{N}$, $\mathbb{R}$ be the sets of natural and real numbers, respectively, $\mathbb{C}$ be the complex plane, $\overline{\mathbb{C}} = \mathbb{C}\bigcup\{\infty\}$ be a one point compactification and $\mathbb{R}^+ = (0, \infty)$. Let $r(B, a)$ be an inner radius of the domain $B \subset \overline{\mathbb{C}}$ relative to a point $a \in B$.

Theorem. Let $n \in \mathbb{N}$, $n \geqslant 2$, $\gamma \in (1, n]$. Then, for any system of different points $\{a_k\}_{k=1}^{n}$ of the unit circle and any mutually non-overlapping domains B_k, $a_k \in B_k \subset \overline{\mathbb{C}}$, $k = \overline{0, n}$, $a_0 = 0$, and B_k, $k = \overline{1, n}$, are symmetric about the unit circle $|a_k| = 1$, the following inequality holds

$$r^{\gamma}\left(B_0, 0\right) \prod_{k=1}^{n} r\left(B_k, a_k\right) \leqslant n^{-\frac{\gamma}{2}} \left(\prod_{k=1}^{n} r\left(B_k, a_k\right)\right)^{1-\frac{\gamma}{n}}.$$

Remark. If $\gamma = n$ then from above posed theorem the following inequality holds

$$r^{n}\left(B_0, 0\right) \prod_{k=1}^{n} r\left(B_k, a_k\right) \leqslant n^{-\frac{n}{2}}.$$

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