Weakly nonlinear BVP's for integro-differential equations. Critical Case of the Second Order

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The existence conditions and the structure of solutions of the weakly nonlinear boundary value problem in the critical case of the second order are obtained. For such systems, a general theory and effective methods of finding solutions are developed. Using the theory of orthoprojectors and pseudoinverse matrices in Moore-Penrose's sense [1], conditions for the existence of solutions to such problems are obtained, and an iterative algorithm for construction of the solution is proposed. It was shown that the existence of a solution depends on the conditions obtained by means of nonlinearities and the second approximation to the desired solution.

Consider a weakly nonlinear system of integrodifferential equations

$$\dot{x}(t)-\Phi(t)\int\limits_a^b [A(s)x(s)+B(s)\dot{x}(s)]ds = f(t)+\varepsilon\int\limits_a^b K(t,s)Z(x(s,\varepsilon),s,\varepsilon)ds, \tag{1}$$

$$\ell x(\cdot) = \alpha + \varepsilon J_2(x(\cdot,\varepsilon),\varepsilon), \quad \alpha \in \mathbb{R}^q. \tag{2}$$

Here, we use the assumptions and notation from [2].

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1. Boichuk A. A., Samoilenko A. M. Generalized inverse operators and Fredholm boundary value problems. – Utrecht, Boston: VSP, 2004, 317 p.; 2nd edition, Walter de Gruyter GmbH & Co KG, 2016, 314 p.
2. Boichuk O. A., Golovatska I. A. Weakly nonlinear systems of integrodifferential equations. Journal of mathematical sciences (Translation of Nonlinear Oscillations (Neliniini Kolyvannya), **16** (2013), No. 4), Springer, 2014, Vol. 3, No. 201, P. 288–295.