

CATEGORIES: BETWEEN CUBES AND GLOBES

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For a finite partially ordered set I we define an abstract polytope \mathcal{P}_I which is a cube or a globe in the cases of discrete or linear poset respectively. Then for a poset P we build a small category \diamond_P with finite lower subsets in P as objects. This category $\diamond_P = \diamond_P^+ \diamond_P^-$ is factorized into a product of two wide subcategories \diamond_P^+ of *faces* and \diamond_P^- of *degenerations*. One can imagine a degeneration from I to $J \subset I$ as a projection of abstract polytope \mathcal{P}_I to the subspace spanned by J . Morphisms in \diamond_P^+ with fixed target I are identified with faces of \mathcal{P}_I . Composition in \diamond_P admits natural geometric interpretation.

On the category $\widehat{\diamond}_I$ of presheaves on \diamond_I we construct a monad of free category in two steps: for a terminal presheaf the free category is obtained via a generalized nerve construction; for a general case cells of the nerve are colored by elements of the initial presheaf. Strict P -fold categories are defined as algebras over this monad.

All constructions are functorial in P . The usual theory of globular and cubical higher categories in natural way can be translated into our general context.