Derivative theta constants in singular half-periods

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A general Thomae formula was obtained, which is a development of second Thomae formula in hyperelliptic case. Namely, values at zero of the lowest non-vanishing derivatives of theta functions with singular characteristics of arbitrary multiplicity are expressed in terms of branch points and period matrix. The general Thomae formula gives first and second Thomae formulas [1] as particular cases.

Some further results are derived. It is proven that matrices of second derivative theta constants (Hessian matrices of zero-values of theta functions with characteristics of multiplicity two) have rank three in any genus. Similar result about the structure of order 3 tensor of third derivative theta constants is obtained, and a conjecture regarding higher multiplicities is made. As a byproduct a generalization of Bolza formulas [2] is deduced.

For more details see `arXiv:1904.09333`

1. Thomae C. J. Beitrag zur Bestimmung von $\theta(0, 0, \ldots 0)$ durch die Klassenmuduln algebraischer Functionen. *J. reine angew. Math.*, 71 : 201–222, 1870.
2. Bolza O. Uber die Reduction hyperelliptischer Integrale erster Ordnung und erster Gattung auf elliptische durch eine Transformation vierten Grades. *Math. Ann.*, 28(3) : 447–456, 1887.