Fractal Structures of Solar Supergranular Cells

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Abstract. We employ fractal analysis to study the complexity of supergranulation structure using the Solar and Heliospheric Observatory (SOHO) dopplergrams obtained during the solar minimum phase. Our data consists of 200 visually selected supergranular cells, for which we find a broad, slightly asymmetric dispersion in the size distribution, with the most probable size around $31.9\, Mm$. From the area-perimeter relation, we deduce a fractal dimension $D$ of about $1.25$. This is consistent with that for isobars, and suggests a possible turbulent origin of supergranulation. By relating this to the variances of kinetic energy, temperature and pressure, it is concluded that the supergranular network is close to being isobaric and that it has a possible turbulent origin. Also we are exploring a possible dependence of fractal dimension of the supergranulation structure on the solar maximum phase.

Keywords: turbulence- Sun: Supergranulation

Convection is the chief mode of transport of heat in the outer envelops of cool stars such as the Sun. The convection zone which lies in the sub-photospheric layers of the Sun has a thickness of about 30% of the solar radius. Here the opacity is so large that energy is carried by turbulent motions rather than by photon diffusion. The convective motions on the Sun are characterized by two prominent scales: the granulation with a typical size of 1000 km and the supergranulation with a typical size of 30000 km (Singh et. al 1994; Srikanth et. al 2000). The supergranules are regions of horizontal outflows along the surface diverging from the cell centre and subsiding flows at the cell borders. Such outflowing regions always show velocity of approach to the observer on the side close to the centre of the disk and velocity of recession on the side towards the limb. Near the centre of the disk where the horizontal outflows are transverse to the line-of-sight, there is less dopplershift and so the image is almost uniformly grey. These high photospheric large convective eddies sweep up any shreds of photospheric magnetic fields in their path from the declining active regions into the boundaries of the cell where they produce excess heating, resulting in the chromospheric network. The approximate lifespan of a supergranular cell is 24 hours. Broadly speaking supergranules are characterized by the three parameters namely length $L$, lifetime $T$ and horizontal flow velocity $v_h$.

The interrelationships amongst these parameters can shed light on the underlying convective processes.

A relationship between horizontal flow velocity and the size of a supergranular cell has been established as $v_h \propto L^{1/3}$ by Krishan et al. (2002). The corresponding dependence of the lifetime $T$ of the supergranular cell on its horizontal flow velocity is found to be $T \propto v_h^{-0.5}$. Here $T$, also the eddy turn-over time is estimated from the relation $T = L/v_h$ with $L$ as the distance from the centre to the edge of the cell (Paniveni et. al 2004).

Fractal analysis is a valuable mathematical tool to quantify the complexity of geometric structures and thus gain insight into the underlying dynamics. For example, statistical analyses like studies of the size distribution of active regions or of the fractal dimension of solar surface magnetic fields in the photosphere are useful for comparing observations and models. They can shed light on the turbulence of the magnetoconvective processes that generate the magnetic structures (Stenflo, Holzreuter 2003a; Lawrence, Ruzmaikin, Cadavid 1993).

For our purpose, the fractal dimension $D$ is characterized by the area-perimeter relation of the structures (Mandelbrot 1977). Self-similarity, or geometric scale-invariance, is expressed by a linear relationship between $\log P$ and $\log A$ (Eq. 1) over some range of scales.

Fractal analysis was first applied to a solar surface phenomena by Roudier and Muller (1987), who measured the fractal dimension of granular perimeters. From Pic du Midi data, they find a fractal dimension $D = 1.25$ for granular diameters of size $d \leq 1''3.7$ and $D \approx 2$ for larger granules.


We analysed 33 hour data of full disc dopplergrams obtained during the solar minimum phase on 28th and 29th June 1996 by the Michelson Doppler Interferometer (MDI) on board the Solar and Heliospheric observatory (SOHO) (Scherrer et al. 1995). Also we have analysed 48 hour full disc GONG dopplergrams obtained on 4th and 5th August 2001 during the solar maximum phase.
FIGURE 1.

Both the SOHO and the GONG full disc dopplergram data have been obtained with a resolution of 2″ each, which equals twice the granular scale. Further, the dopplergrams are time averaged over intervals of 10 min, which is about twice the 5-minute period of oscillations. Thus the signal due to granular velocity is averaged out. Similarly the contributions due to p-mode vibrations are reduced after the time averaging. Our analysis rests on the implicit belief that time averaging removes noise significantly, as judged from visual inspection and also as seen in the typical supergranular velocity profile for our data (cf. Fig. 1 of Paniveni et al. (2004)). After the averaging, the supergranular network is brought out with a fair clarity. This procedure yielded usually six images per hour of the data. Corrections due to solar rotation are applied to the dopplershifts. Two hundred well accentuated cells during the solar minimum phase and 81 distinct cells during the solar maximum phase, lying between 15° and 60° angular distance from the disc centre were selected.

Restricting to the above mentioned angular distance limits helps us discount weak supergranular flows as well as foreshortening effects. The solar maximum data is analysed in a smaller range of cell area namely 200- 600 Mm² for the fractal dimension depends on the cell area and should be studied over short ranges of area especially in the solar maximum phase (Meunier 2004).

The profile of a visually identified cell was scanned as follows: we chose a fiducial y-direction on the cell and performed velocity profile scans along the x-direction for all the pixel positions on the y-axis. In each scan, the cell extent is taken to be marked by two juxtaposed ‘crests’ (separated by a ‘trough’), expected in the dopplergrams. This set of data points was used to determine the area and perimeter of a given cell, and of the spectrum for all selected supergranules. The area-perimeter relation is used to evaluate the fractal dimension.

The main results pertaining to the maximum, minimum, mean, standard deviation and the skewness for cell area A and cell perimeter P for both solar minimum and solar maximum data are summarized in Table 1 and Table 2 respectively. A large dispersion in the area and perimeter was obtained in both the phases of the solar cycle.

We analyzed planar shapes by analyzing the area-perimeter relation,

\[ P \propto A^{D/2} \]  

The log(A) vs log(P) relation is linear as shown in the lower frame of Figure (1). A correlation co-efficient of 0.92 indicates strong correlation. Fractal dimension D, calculated as (2/slope), is found to be \( D = 1.345 \pm 0.082 \). If we interchange the log(A) and log(P) axes (upper frame, Figure 1), fractal dimension D here is 2 \times slope and found to be \( D = 1.136 \pm 0.070 \). The small difference in D values thus obtained may be because error bars in P and A are not symmetric. The average over the two methods is \( D = 1.24 \pm 0.076 \).

The log(A) vs log(P) relation is linear as shown in the lower frame of Figure (2) obtained for the solar maximum data. A correlation coefficient of 0.94 indicates strong correlation. Fractal dimension D, calculated as (2/slope), is found to be \( D = 1.563 \pm 0.13 \). If we interchange the log(A) and log(P) axes (upper frame, Figure 2), fractal dimension...
$D$ here is $2 \times$ slope and found to be $D = 1.365 \pm 0.113$. The small difference in $D$ values thus obtained may be because error bars in $P$ and $A$ are not symmetric. The average over the two methods is $D = 1.464 \pm 0.122$. For smooth shapes such as circles and squares $P \propto A^{1/2}$ and thus $D = 1$, the dimension of a line. As the perimeter becomes more and more contorted and tends to double back on itself filling the plane, so that $P \propto A$ and $D$ approaches the value 2, a maximum. The linear relation (Figures 1 and 2) suggests that supergranules are self-similar and may be regarded as fractal objects. Unlike the case of granules, we do not find any multifractal structure. So it seems likely that the entire distribution profile can be explained by a single physical phenomenon. Since $P \propto A^{D/2}$, we may expect $D$ to be an important parameter characterizing the processes which produces the solar supergranulation.

| Table 1. Maximum, minimum, mean, standard deviation and skewness for area ($A$) and Perimeter ($P$) using the solar minimum data. |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Max             | Min             | Mean            | $\sigma$       | $\text{skewness}$ |
| $A$ (Mm$^2$)    | 912.4           | 97.0            | 375.3           | $152.8 \pm 10.8$| 0.74 $\pm$ 0.17 |
| $P$ (Mm)        | 169.3           | 41.6            | 89.3            | $23.5 \pm 1.7$  | 0.83 $\pm$ 0.17 |

The spectral distribution of the temperature, a passive scalar, is related to the spectral distribution of kinetic energy. It can be easily shown that the Kolmogorov energy spectrum, $K^{-5/3}$, both in two and three dimensional turbulence leads to a temperature spectrum of $K^{-5/3}$ (Krishan 1991; 1996). Thus the temperature variance $\langle \theta^2 \rangle$ varies as $r^{2/3}$, as a function of the distance $r$ (Tennekes and Lumley 1970). According to Mandelbrot (1975), an isosurface has a fractal dimension given by $D_T = (\text{Euclidean dimension}) - 1/2$ (exponent of the variance). Thus for two dimensional supergranulation $D_T = 2 - (1/2 \times 2/3) = 5/3 = 1.66$ for an isotherm. The pressure variance $\langle p^2 \rangle$ on the other hand is proportional to the square of the velocity variance i.e. $\langle p^2 \rangle \propto r^{4/3}$ (Batchelor 1953). The fractal dimension of an isobar

| Table 2. Maximum, minimum, mean, standard deviation and skewness for area ($A$) and Perimeter ($P$) using the solar maximum data. |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Max             | Min             | Mean            | $\sigma$       | skewness        |
| $A$ (Mm$^2$)    | 1491            | 156.86          | 351.36          | $213.81 \pm 23.75$ | 1.8662 $\pm$ 0.272 |
| $P$ (Mm)        | 274.6           | 66.66           | 109.96          | $34.84 \pm 3.871$ | 2.0675 $\pm$ 0.272 |
is, therefore, found to be $D_p = 2 - \left(1/2 \times 4/3\right) = 1.33$. Our data furnishes a fractal dimension $D = 1.25$ for solar minimum data and $D = 1.464$, averaging to $D = 1.357$ which indicates that the supergranular network is close to being an isobar. It is interesting to note that Roudier and Muller (1987) obtained a similar dimension for smaller granules. To characterize the shape of the distributions for the area and perimeter, we computed skewness. Skewness is a measure of the extent of departure from the symmetry of the distribution about the mean. It is positive here indicating that cell area and perimeter values are bunched at lower values than the mean. The error on this statistic is computed as $\sqrt{6/N}$ (Brown 1996), where $N = 200$ for the solar minimum data and is $N = 81$ for the solar maximum data. The respective values for cell area and perimeter are given in Table 1 and Table 2.

It should be instructive to explore the relative merits and results of the different data sets such as dopplergrams, magnetograms and intensity patterns for a better understanding of the solar convective phenomena.

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